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The Shannon Entropy and Multifractional Markets

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Abstract : Introduced by Shannon in 1948 in the field of information theory as the average rate at which information is produced by a stochastic set of data, the concept of entropy has gained much attention as a measure of uncertainty and unpredictability associated with a dynamical system, eventually depicted by a stochastic process. In particular, the Shannon entropy measures the degree of order/disorder of a given signal and provides useful information about the underlying dynamical process. It has found widespread application in a variety of fields, such as, for example, cryptography, statistical physics and finance. In this regard, many contributions have employed different measures of entropy in an attempt to characterize the financial time series in terms of market efficiency, market crashes and/or financial crises. The Shannon entropy has also been considered as a measure of the risk of a portfolio or as a tool in asset pricing. This work investigates the theoretical link between the Shannon entropy and the multifractional Brownian motion (mBm), stochastic process which recently is the focus of a renewed interest in finance as a driving model of stochastic volatility. In particular, after exploring the current state of research in this area and highlighting some of the key results and open questions that remain, we show a well-defined relationship between the Shannon (log)entropy and the memory function H(t) of the mBm. In details, we allow both the length of time series and time scale to change over analysis to study how the relation modify itself. On the one hand, applications are developed after generating surrogates of mBm trajectories based on different memory functions; on the other hand, an empirical analysis of several international stock indexes, which confirms the previous results, concludes the work.

Keywords : Shannon entropy, multifractional Brownian motion, Hurst–Holder exponent, stock indexes
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