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On the Cyclic Property of Groups of Prime Order

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Abstract : The study of finite groups is a central topic in algebraic structures, and one of the most fundamental questions in this field is the classification of finite groups up to isomorphism. In this paper, we investigate the cyclic property of groups of prime order, which is a crucial result in the classification of finite abelian groups. We prove the following statement: If p is a prime, then every group G of order p is cyclic. Our proof utilizes the properties of group actions and the class equation, which provide a powerful tool for studying the structure of finite groups. In particular, we first show that any non-identity element of G generates a cyclic subgroup of G. Then, we establish the existence of an element of order p, which implies that G is generated by a single element. Finally, we demonstrate that any two generators of G are conjugate, which shows that G is a cyclic group. Our result has significant implications in the classification of finite groups, as it implies that any group of prime order is isomorphic to the cyclic group of the same order. Moreover, it provides a useful tool for understanding the structure of more complicated finite groups, as any finite abelian group can be decomposed into a direct product of cyclic groups. Our proof technique can also be extended to other areas of group theory, such as the classification of finite p-groups, where p is a prime. Therefore, our work has implications beyond the specific result we prove and can contribute to further research in algebraic structures.

Keywords : group theory, finite groups, cyclic groups, prime order, classification.
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