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Multiple Version of Roman Domination in Graphs

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Abstract : In 2004, it was introduced the concept of Roman domination in graphs. This concept was initially inspired and related to the defensive strategy of the Roman Empire. An undefended place is a city so that no legions are established on it, whereas a strong place is a city in which two legions are deployed. This situation may be modeled by labeling the vertices of a finite simple graph with labels {0, 1, 2}, satisfying the condition that any 0-vertex must be adjacent to, at least, a 2-vertex. Roman domination in graphs is a variant of classic domination. Clearly, the main aim is to obtain such labeling of the vertices of the graph with minimum cost, that is to say, having minimum weight (sum of all vertex labels). Formally, a function f: V (G) → {0, 1, 2} is a Roman dominating function (RDF) in the graph G = (V, E) if f(u) = 0 implies that f(v) = 2 for, at least, a vertex v which is adjacent to u. The weight of an RDF is the positive integer w(f)= ∑_(v∈V)▒f(v). The Roman domination number, γ_R (G), is the minimum weight among all the Roman dominating functions? Obviously, the set of vertices with a positive label under an RDF f is a dominating set in the graph, and hence γ(G)≤γ_R (G). In this work, we start the study of a generalization of RDF in which we consider that any undefended place should be defended from a sudden attack by, at least, k legions. These legions can be deployed in the city or in any of its neighbours. A function f: V → {0, 1, . . . , k + 1} such that f(N[u]) ≥ k + |AN(u)| for all vertex u with f(u) < k, where AN(u) represents the set of active neighbours (i.e., with a positive label) of vertex u, is called a [k]-multiple Roman dominating functions and it is denoted by [k]-MRDF. The minimum weight of a [k]-MRDF in the graph G is the [k]-multiple Roman domination number ([k]-MRDN) of G, denoted by γ_[kR] (G). First, we prove that the [k]-multiple Roman domination decision problem is NP-complete even when restricted to bipartite and chordal graphs. A problem that had been resolved for other variants and wanted to be generalized. We know the difficulty of calculating the exact value of the [k]-MRD number, even for families of particular graphs. Here, we present several upper and lower bounds for the [k]-MRD number that permits us to estimate it with as much precision as possible. Finally, some graphs with the exact value of this parameter are characterized.

Keywords : multiple roman domination function, decision problem np-complete, bounds, exact values
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