Integral Domains and Alexandroff Topology

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Abstract : Let S be an integral domain which is not a field, let F be its field of fractions, and let A be an F-algebra. An S-subalgebra R of A is called S-nice if $R \cap F = S$ and F R = A. A topological space whose set of open sets is closed under arbitrary intersections is called an Alexandroff space. Inspired by the well-known Zariski-Riemann space and the Zariski topology on the set of prime ideals of a commutative ring, we define a topology on the set of all S-nice subalgebras of A. Consequently, we get an interplay between Algebra and topology, that gives us a better understanding of the S-nice subalgebras of A. It is shown that every irreducible subset of S-nice subalgebras of A has a supremum; and a characterization of the irreducible components is given, in terms of maximal S-nice subalgebras of A.

Keywords : Alexandroff topology, integral domains, Zariski-Riemann space, S-nice subalgebras

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