

A Fourier Method for Risk Quantification and Allocation of Credit Portfolios

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Abstract : Herewith we present a Fourier method for credit risk quantification and allocation in the factor-copula model framework. The key insight is that, compared to directly computing the cumulative distribution function of the portfolio loss via Monte Carlo simulation, it is, in fact, more efficient to calculate the transformation of the distribution function in the Fourier domain instead and inverting back to the real domain can be done in just one step and semi-analytically, thanks to the popular COS method (with some adjustments). We also show that the Euler risk allocation problem can be solved in the same way since it can be transformed into the problem of evaluating a conditional cumulative distribution function. Once the conditional or unconditional cumulative distribution function is known, one can easily calculate various risk metrics. The proposed method not only fills the niche in literature, to the best of our knowledge, of accurate numerical methods for risk allocation but may also serve as a much faster alternative to the Monte Carlo simulation method for risk quantification in general. It can cope with various factor-copula model choices, which we demonstrate via examples of a two-factor Gaussian copula and a two-factor Gaussian-t hybrid copula. The fast error convergence is proved mathematically and then verified by numerical experiments, in which Value-at-Risk, Expected Shortfall, and conditional Expected Shortfall are taken as examples of commonly used risk metrics. The calculation speed and accuracy are tested to be significantly superior to the MC simulation for real-sized portfolios. The computational complexity is, by design, primarily driven by the number of factors instead of the number of obligors, as in the case of Monte Carlo simulation. The limitation of this method lies in the "curse of dimension" that is intrinsic to multi-dimensional numerical integration, which, however, can be relaxed with the help of dimension reduction techniques and/or parallel computing, as we will demonstrate in a separate paper. The potential application of this method has a wide range: from credit derivatives pricing to economic capital calculation of the banking book, default risk charge and incremental risk charge computation of the trading book, and even to other risk types than credit risk.

Keywords : credit portfolio, risk allocation, factor copula model, the COS method, Fourier method

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