

## The Bernstein Expansion for Exponentials in Taylor Functions: Approximation of Fixed Points

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**Abstract :** Bernstein's expansion for exponentials in Taylor functions provides lower and upper optimization values for the range of its original function. these values converge to the original functions if the degree is elevated or the domain subdivided. Taylor polynomial can be applied so that the exponential is a polynomial of finite degree over a given domain. Bernstein's basis has two main properties: its sum equals 1, and positive for all  $x \in (0; 1)$ . In this work, we prove the existence of fixed points for exponential functions in a given domain using the optimization values of Bernstein. The Bernstein basis of finite degree  $T$  over a domain  $D$  is defined non-negatively. Any polynomial  $p$  of degree  $t$  can be expanded into the Bernstein form of maximum degree  $t \leq T$ , where we only need to compute the coefficients of Bernstein in order to optimize the original polynomial. The main property is that  $p(x)$  is approximated by the minimum and maximum Bernstein coefficients (Bernstein bound). If the bound is contained in the given domain, then we say that  $p(x)$  has fixed points in the same domain.

**Keywords :** Bernstein polynomials, Stability of control functions, numerical optimization, Taylor function

**Conference Title :** ICMCT 2023 : International Conference on Mathematics and Control Theory

**Conference Location :** Berlin, Germany

**Conference Dates :** May 11-12, 2023