

## The Malfatti's Problem in Reuleaux Triangle

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**Abstract :** The Malfatti's Problem is to ask for fitting 3 circles into a right triangle such that they are tangent to each other, and each circle is also tangent to a pair of the triangle's side. This problem has been extended to any triangle (called general Malfatti's Problem). Furthermore, the problem has been extended to have  $1+2+\dots+n$  circles, we call it extended general Malfatti's problem, these circles whose tangency graph, using the center of circles as vertices and the edge connect two circles center if these two circles tangent to each other, has the structure as Pascal's triangle, and the exterior circles of these circles tangent to three sides of the triangle. In the extended general Malfatti's problem, there are closed-form solutions for  $n=1, 2$ , and the problem becomes complex when  $n$  is greater than 2. In solving extended general Malfatti's problem ( $n>2$ ), we initially give values to the radii of all circles. From the tangency graph and current radii, we can compute angle value between two vectors. These vectors are from the center of the circle to the tangency points with surrounding elements, and these surrounding elements can be the boundary of the triangle or other circles. For each circle  $C$ , there are vectors from its center  $c$  to its tangency point with its neighbors (count clockwise)  $p_i, i=0, 1, 2, \dots, n$ . We add all angles between  $cp_i$  to  $cp_{(i+1) \bmod (n+1)}, i=0, 1, \dots, n$ , call it  $\text{sumangle}(C)$  for circle  $C$ . Using  $\text{sumangle}(C)$ , we can reduce/enlarge the radii for all circles in next iteration, until  $\text{sumangle}(C)$  is equal to  $2\pi$  for all circles. With a similar idea, this paper proposed an algorithm to find the radii of circles whose tangency has the structure of Pascal's triangle, and the exterior circles of these circles are tangent to the unit Reuleaux Triangle.

**Keywords :** Malfatti's problem, geometric constraint solver, computer-aided geometric design, circle packing, data visualization

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