

Multifractal Behavior of the Perturbed Cerbelli-Giona Map: Numerical Computation of ω -Measure

Authors : Ibrahim Alsendid, Rob Sturman, Benjamin Sharp

Abstract : In this paper, we consider a family of 2-dimensional nonlinear area-preserving transformations on the torus. A single parameter η varies between 0 and 1, taking the transformation from a hyperbolic toral automorphism to the “Cerbelli-Giona” map, a system known to exhibit multifractal properties. Here we study the multifractal properties of the family of maps. We apply a box-counting method by defining a grid of boxes $B_i(\delta)$, where i is the index and δ is the size of the boxes, to quantify the distribution of stable and unstable manifolds of the map. When the parameter is in the range $0.51 < \eta < 0.58$ and $0.68 < \eta < 1$ the map is ergodic; i.e., the unstable and stable manifolds eventually cover the whole torus, although not in a uniform distribution. For accurate numerical results, we require correspondingly accurate construction of the stable and unstable manifolds. Here we use the piecewise linearity of the map to achieve this, by computing the endpoints of line segments that define the global stable and unstable manifolds. This allows the generalized fractal dimension D_q , and spectrum of dimensions $f(\alpha)$, to be computed with accuracy. Finally, the intersection of the unstable and stable manifold of the map will be investigated and compared with the distribution of periodic points of the system.

Keywords : Discrete-time dynamical systems, Fractal geometry, Multifractal behaviour of the Perturbed map, Multifractal of Dynamical systems

Conference Title : ICFGAM 2021 : International Conference on Fractals, Geometry and Applied Mathematics

Conference Location : London, United Kingdom

Conference Dates : December 09-10, 2021