

Neural Network Approach for Solving Integral Equations

Authors : Bhavini Pandya

Abstract : This paper considers $H_\eta: T^2 \rightarrow T^2$ the Perturbed Cerbelli-Giona map. That is a family of 2-dimensional nonlinear area-preserving transformations on the torus $T^2 = [0,1] \times [0,1] = \mathbb{R}^2 / \mathbb{Z}^2$. A single parameter η varies between 0 and 1, taking the transformation from a hyperbolic toral automorphism to the “Cerbelli-Giona” map, a system known to exhibit multifractal properties. Here we study the multifractal properties of the family of maps. We apply a box-counting method by defining a grid of boxes $B_i(\delta)$, where i is the index and δ is the size of the boxes, to quantify the distribution of stable and unstable manifolds of the map. When the parameter is in the range $0.51 < \eta < 0.58$ and $0.68 < \eta < 1$ the map is ergodic; i.e., the unstable and stable manifolds eventually cover the whole torus, although not in a uniform distribution. For accurate numerical results we require correspondingly accurate construction of the stable and unstable manifolds. Here we use the piecewise linearity of the map to achieve this, by computing the endpoints of line segments which define the global stable and unstable manifolds. This allows the generalized fractal dimension D_q , and spectrum of dimensions $f(\alpha)$, to be computed with accuracy. Finally, the intersection of the unstable and stable manifold of the map will be investigated, and compared with the distribution of periodic points of the system.

Keywords : feed forward, gradient descent, neural network, integral equation

Conference Title : ICFIE 2021 : International Conference on Fredholm Integral Equation

Conference Location : London, United Kingdom

Conference Dates : December 09-10, 2021