

Introduction of Para-Sasaki-Like Riemannian Manifolds and Construction of New Einstein Metrics

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Abstract : The concept of almost paracontact Riemannian manifolds (abbr., apcR manifolds) was introduced by I. Sato in 1976 as an analogue of almost contact Riemannian manifolds. The notion of an apcR manifold of type (p,q) was defined by S. Sasaki in 1980, where p and q are respectively the numbers of the multiplicity of the structure eigenvalues 1 and -1. It also has a simple eigenvalue of 0. In our work, we consider $(2n+1)$ -dimensional apcR manifolds of type (n,n) , i.e., the paracontact distribution of the studied manifold can be considered as a $2n$ -dimensional almost paracomplex Riemannian distribution with almost paracomplex structure and structure group $O(n) \times O(n)$. The aim of the present study is to introduce a new class of apcR manifolds. Such a manifold is obtained using the construction of a certain Riemannian cone over it, and the resulting manifold is a paraholomorphic paracomplex Riemannian manifold (abbr., phpcR manifold). We call it a para-Sasaki-like Riemannian manifold (abbr., pSIR manifold) and give some explicit examples. We study the structure of pSIR spaces and find that the paracontact form η is closed and each pSIR manifold locally can be considered as a certain product of the real line with a phpcR manifold, which is locally a Riemannian product of two equidimensional Riemannian spaces. We also obtain that the curvature of the pSIR manifolds is completely determined by the curvature of the underlying local phpcR manifold. Moreover, the ξ -directed Ricci curvature is equal to $-2n$, while in the Sasaki case, it is $2n$. Accordingly, the pSIR manifolds can be interpreted as the counterpart of the Sasaki manifolds; the skew-symmetric part of $\nabla\eta$ vanishes, while in the Sasaki case, the symmetric part vanishes. We define a hyperbolic extension of a (complete) phpcR manifold that resembles a certain warped product, and we indicate that it is a (complete) pSIR manifold. In addition, we consider the hyperbolic extension of a phpcR manifold and prove that if the initial manifold is a complete Einstein manifold with negative scalar curvature, then the resulting manifold is a complete Einstein pSIR manifold with negative scalar curvature. In this way, we produce new examples of a complete Einstein Riemannian manifold with negative scalar curvature. Finally, we define and study para contact conformal/homothetic deformations by deriving a subclass that preserves the para-Sasaki-like condition. We then find that if we apply a paracontact homothetic deformation of a pSIR space, we obtain that the Ricci tensor is invariant.

Keywords : almost paracontact Riemannian manifolds, Einstein manifolds, holomorphic product manifold, warped product manifold

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