

## A Combinatorial Representation for the Invariant Measure of Diffusion Processes on Metric Graphs

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**Abstract :** We study a generalization to a continuous setting of the classical Markov chain tree theorem. In particular, we consider an irreducible diffusion process on a metric graph. The unique invariant measure has an atomic component on the vertices and an absolutely continuous part on the edges. We show that the corresponding density at  $x$  can be represented by a normalized superposition of the weights associated to metric arborescences oriented toward the point  $x$ . A metric arborescence is a metric tree oriented towards its root. The weight of each oriented metric arborescence is obtained by the product of the exponential of integrals of the form  $f a/b^2$ , where  $b$  is the drift and  $\sigma^2$  is the diffusion coefficient, along the oriented edges, for a weight for each node determined by the local orientation of the arborescence around the node and for the inverse of the diffusion coefficient at  $x$ . The metric arborescences are obtained by cutting the original metric graph along some edges.

**Keywords :** diffusion processes, metric graphs, invariant measure, reversibility

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