World Academy of Science, Engineering and Technology
International Journal of Physical and Mathematical Sciences
Vol:15, No:09, 2021

A Combinatorial Representation for the Invariant Measure of Diffusion Processes on Metric Graphs

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Abstract : We study a generalization to a continuous setting of the classical Markov chain tree theorem. In particular, we consider an irreducible diffusion process on a metric graph. The unique invariant measure has an atomic component on the vertices and an absolutely continuous part on the edges. We show that the corresponding density at x can be represented by a normalized superposition of the weights associated to metric arborescences oriented toward the point x. A metric arborescence is a metric tree oriented towards its root. The weight of each oriented metric arborescence is obtained by the product of the exponential of integrals of the form $\int a/b^2$, where b is the drift and σ^2 is the diffusion coefficient, along the oriented edges, for a weight for each node determined by the local orientation of the arborescence around the node and for the inverse of the diffusion coefficient at x. The metric arborescences are obtained by cutting the original metric graph along some edges.

Keywords : diffusion processes, metric graphs, invariant measure, reversibility
Conference Title : ICPTS 2021 : International Conference on Probability Theory and Statistics
Conference Location : Vancouver, Canada
Conference Dates : September 23-24, 2021