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# Robust Numerical Method for Singularly Perturbed Semilinear Boundary Value Problem with Nonlocal Boundary Condition

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**Abstract :** In this work, our primary interest is to provide ε-uniformly convergent numerical techniques for solving singularly perturbed semilinear boundary value problems with non-local boundary condition. These singular perturbation problems are described by differential equations in which the highest-order derivative is multiplied by an arbitrarily small parameter ε (say) known as singular perturbation parameter. This leads to the existence of boundary layers, which are basically narrow regions in the neighborhood of the boundary of the domain, where the gradient of the solution becomes steep as the perturbation parameter tends to zero. Due to the appearance of the layer phenomena, it is a challenging task to provide ε-uniform numerical methods. The term 'ε-uniform' refers to identify those numerical methods in which the approximate solution converges to the corresponding exact solution (measured to the supremum norm) independently with respect to the perturbation parameter ε. Thus, the purpose of this work is to develop, analyze, and improve the ε-uniform numerical methods for solving singularly perturbed problems. These methods are based on nonstandard fitted finite difference method. The basic idea behind the fitted operator, finite difference method, is to replace the denominator functions of the classical derivatives with positive functions derived in such a way that they capture some notable properties of the governing differential equation. A uniformly convergent numerical method is constructed via nonstandard fitted operator numerical method and numerical integration methods to solve the problem. The non-local boundary condition is treated using numerical integration techniques. Additionally, Richardson extrapolation technique, which improves the first-order accuracy of the standard scheme to second-order convergence, is applied for singularly perturbed convection-diffusion problems using the proposed numerical method. Maximum absolute errors and rates of convergence for different values of perturbation parameter and mesh sizes are tabulated for the numerical example considered. The method is shown to be ε-uniformly convergent. Finally, extensive numerical experiments are conducted which support all of our theoretical findings. A concise conclusion is provided at the end of this work.

**Keywords :** nonlocal boundary condition, nonstandard fitted operator, semilinear problem, singular perturbation, uniformly convergent
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