

Closed Form Solution for 4-D Potential Integrals for Arbitrary Coplanar Polygonal Surfaces

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Abstract : A closed-form solution for 4-D double surface integrals arising in boundary integrals equations of a potential theory is obtained for arbitrary coplanar polygonal surfaces. The solution method is based on the construction of exact differential forms followed by the application of Stokes' theorem for each surface integral. As a result, the 4-D double surface integral is reduced to a 2-D double line integral. By an appropriate change of variables, the integrand is transformed into a separable function of integration variables. The closed-form solutions to the corresponding 1-D integrals are readily available in the integration tables. Previously closed-form solutions were known only for the case of coincident triangle surfaces and coplanar rectangles. Solutions for these cases were obtained by surface-specific ad-hoc methods, while the present method is general. The method also works for non-polygonal surfaces. As an example, we compute in closed form the 4-D integral for the case of coincident surfaces in the shape of a circular disk. For an arbitrarily shaped surface, the proposed method provides an efficient quadrature rule. Extensions of the method for non-coplanar surfaces and other than $1/R$ integral kernels are also discussed.

Keywords : boundary integral equations, differential forms, integration, stokes' theorem

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