

## Planning a Haemodialysis Process by Minimum Time Control of Hybrid Systems with Sliding Motion

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**Abstract :** The aim of the paper is to provide a computational tool for planning a haemodialysis process. It is shown that optimization methods can be used to obtain the most effective treatment focused on removing both urea and phosphorus during the process. In order to achieve that, the IV-compartment model of phosphorus kinetics is applied. This kinetics model takes into account a rebound phenomenon that can occur during haemodialysis and results in a hybrid model of the process. Furthermore, vector fields associated with the model equations are such that it is very likely that using the most intuitive objective functions in the planning problem could lead to solutions which include sliding motions. Therefore, building computational tools for solving the problem of planning a haemodialysis process has required constructing numerical algorithms for solving optimal control problems with hybrid systems. The paper concentrates on minimum time control of hybrid systems since this control objective is the most suitable for the haemodialysis process considered in the paper. The presented approach to optimal control problems with hybrid systems is different from the others in several aspects. First of all, it is assumed that a hybrid system can exhibit sliding modes. Secondly, the system's motion on the switching surface is described by index 2 differential-algebraic equations, and that guarantees accurate tracking of the sliding motion surface. Thirdly, the gradients of the problem's functionals are evaluated with the help of adjoint equations. The adjoint equations presented in the paper take into account sliding motion and exhibit jump conditions at transition times. The optimality conditions in the form of the weak maximum principle for optimal control problems with hybrid systems exhibiting sliding modes and with piecewise constant controls are stated. The presented sensitivity analysis can be used to construct globally convergent algorithms for solving considered problems. The paper presents numerical results of solving the haemodialysis planning problem.

**Keywords :** haemodialysis planning process, hybrid systems, optimal control, sliding motion

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