

On Lie-Central Derivations and Almost Inner Lie-Derivations of Leibniz Algebras

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Abstract : The Liezation functor is a map from the category of Leibniz algebras to the category of Lie algebras, which assigns a Leibniz algebra to the Lie algebra given by the quotient of the Leibniz algebra by the ideal spanned by the square elements of the Leibniz algebra. This functor is left adjoint to the inclusion functor that considers a Lie algebra as a Leibniz algebra. This environment fits in the framework of central extensions and commutators in semi-abelian categories with respect to a Birkhoff subcategory, where classical or absolute notions are relative to the abelianization functor. Classical properties of Leibniz algebras (properties relative to the abelianization functor) were adapted to the relative setting (with respect to the Liezation functor); in general, absolute properties have the corresponding relative ones, but not all absolute properties immediately hold in the relative case, so new requirements are needed. Following this line of research, it was conducted an analysis of central derivations of Leibniz algebras relative to the Liezation functor, called as Lie-derivations, and a characterization of Lie-stem Leibniz algebras by their Lie-central derivations was obtained. In this paper, we present an overview of these results, and we analyze some new properties concerning Lie-central derivations and almost inner Lie-derivations. Namely, a Leibniz algebra is a vector space equipped with a bilinear bracket operation satisfying the Leibniz identity. We define the Lie-bracket by $[x, y]_{\text{lie}} = [x, y] + [y, x]$, for all x, y . The Lie-center of a Leibniz algebra is the two-sided ideal of elements that annihilate all the elements in the Leibniz algebra through the Lie-bracket. A Lie-derivation is a linear map which acts as a derivative with respect to the Lie-bracket. Obviously, usual derivations are Lie-derivations, but the converse is not true in general. A Lie-derivation is called a Lie-central derivation if its image is contained in the Lie-center. A Lie-derivation is called an almost inner Lie-derivation if the image of an element x is contained in the Lie-commutator of x and the Leibniz algebra. The main results we present in this talk refer to the conditions under which Lie-central derivation and almost inner Lie-derivations coincide.

Keywords : almost inner Lie-derivation, Lie-center, Lie-central derivation, Lie-derivation

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