Convergence Results of Two-Dimensional Homogeneous Elastic Plates from Truncation of Potential Energy

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Abstract : Plates are important engineering structures which have attracted extensive research since the 19th century. The subject of this work is statical analysis of a linearly elastic homogenous plate under small deformations. A 'thin plate' is a threedimensional structure comprising of a small transverse dimension with respect to a flat mid-surface. The general aim of any plate theory is to deduce a two-dimensional model, in terms of mid-surface quantities, to approximately and accurately describe the plate's deformation in terms of mid-surface quantities. In recent decades, a common starting point for this purpose is to utilize series expansion of a displacement field across the thickness dimension in terms of the thickness parameter (h). These attempts are mathematically consistent in deriving leading-order plate theories based on certain a priori scaling between the thickness and the applied loads; for example, asymptotic methods which are aimed at generating leading-order twodimensional variational problems by postulating formal asymptotic expansion of the displacement fields. Such methods rigorously generate a hierarchy of two-dimensional models depending on the order of magnitude of the applied load with respect to the plate-thickness. However, in practice, applied loads are external and thus not directly linked or dependent on the geometry/thickness of the plate; thus, rendering any such model (based on a priori scaling) of limited practical utility. In other words, the main limitation of these approaches is that they do not furnish a single plate model for all orders of applied loads. Following analogy of recent efforts of deploying Fourier-series expansion to study convergence of reduced models, we propose two-dimensional model(s) resulting from truncation of the potential energy and rigorously prove the convergence of these twodimensional plate models to the parent three-dimensional linear elasticity with increasing truncation order of the potential energy.

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