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Analytical Description of Disordered Structures in Continuum Models of Pattern Formation

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Abstract : Even though numerical simulations indeed have a significant precursory/supportive role in exploring the disordered phase displaying no long-range order in pattern formation models, studying the stability properties of this phase and determining the order of the ordered-disordered phase transition in these models necessitate an analytical description of the disordered phase. First, we will present the results of a comprehensive statistical analysis of a large number (1,000-10,000) of numerical simulations in the Swift-Hohenberg model, where the bulk disordered (or amorphous) phase is stable. We will show that the average free energy density (over configurations) converges, while the variance of the energy density vanishes with increasing system size in numerical simulations, which suggest that the disordered phase is a thermodynamic phase (i.e., its properties are independent of the configuration in the macroscopic limit). Furthermore, the structural analysis of this phase in the Fourier space suggests that the phase can be modeled by a colored isotropic Gaussian noise, where any instant of the noise describes a possible configuration. Based on these results, we developed the general mathematical framework of finding a pool of solutions to partial differential equations in the sense of continuous probability measure, which we will present briefly. Applying the general idea to the Swift-Hohenberg model we show, that the amorphous phase can be found, and its properties can be determined analytically. As the general mathematical framework is not restricted to continuum theories, we hope that the proposed methodology will open a new chapter in studying disordered phases.

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