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Structural Invertibility and Optimal Sensor Node Placement for Error and Input Reconstruction in Dynamic Systems

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Abstract : Understanding and modelling of real-world complex dynamic systems in biology, engineering and other fields is often made difficult by incomplete knowledge about the interactions between systems states and by unknown disturbances to the system. In fact, most real-world dynamic networks are open systems receiving unknown inputs from their environment. To understand a system and to estimate the state dynamics, these inputs need to be reconstructed from output measurements. Reconstructing the input of a dynamic system from its measured outputs is an ill-posed problem if only a limited number of states is directly measurable. A first requirement for solving this problem is the invertibility of the input-output map. In our work, we exploit the fact that invertibility of a dynamic system is a structural property, which depends only on the network topology. Therefore, it is possible to check for invertibility using a structural invertibility algorithm which counts the number of node disjoint paths linking inputs and outputs. The algorithm is efficient enough, even for large networks up to a million nodes. To understand structural features influencing the invertibility of a complex dynamic network, we analyze synthetic and real networks using the structural invertibility algorithm. We find that invertibility largely depends on the degree distribution and that dense random networks are easier to invert than sparse inhomogeneous networks. We show that real networks are often very difficult to invert unless the sensor nodes are carefully chosen. To overcome this problem, we present a sensor node placement algorithm to achieve invertibility with a minimum set of measured states. This greedy algorithm is very fast and also guaranteed to find an optimal sensor node-set if it exists. Our results provide a practical approach to experimental design for open, dynamic systems. Since invertibility is a necessary condition for unknown input observers and data assimilation filters to work, it can be used as a preprocessing step to check, whether these input reconstruction algorithms can be successful. If not, we can suggest additional measurements providing sufficient information for input reconstruction. Invertibility is also important for systems design and model building. Dynamic models are always incomplete, and synthetic systems act in an environment, where they receive inputs or even attack signals from their exterior. Being able to monitor these inputs is an important design requirement, which can be achieved by our algorithms for invertibility analysis and sensor node placement.

Keywords : data-driven dynamic systems, inversion of dynamic systems, observability, experimental design, sensor node placement
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