Detailed Quantum Circuit Design and Evaluation of Grover's Algorithm for the Bounded Degree Traveling Salesman Problem Using the Q# Language

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Abstract : The Traveling Salesman problem is famous in computing and graph theory. In short, it asks for the Hamiltonian cycle of the least total weight in a given graph with N nodes. All variations on this problem, such as those with K-bounded-degree nodes, are classified as NP-complete in classical computing. Although several papers propose theoretical high-level designs of quantum algorithms for the Traveling Salesman Problem, no quantum circuit implementation of these algorithms has been created up to our best knowledge. In contrast to previous papers, the goal of this paper is not to optimize some abstract complexity measures based on the number of oracle iterations, but to be able to evaluate the real circuit and time costs of the quantum computer. Using the emerging quantum programming language Q# developed by Microsoft, which runs quantum circuits in a quantum computer simulation, an implementation of the bounded-degree problem and its respective quantum circuit were created. To apply Grover's algorithm to this problem, a quantum oracle was designed, evaluating the cost of a particular set of edges in the graph as well as its validity as a Hamiltonian cycle. Repeating the Grover algorithm with an oracle that finds successively lower cost each time allows to transform the decision problem to an optimization problem, finding the minimum cost of Hamiltonian cycles. $N \log_2 K$ qubits are put into an equiprobablistic superposition by applying the Hadamard gate on each qubit. Within these $N \log_2 K$ qubits, the method uses an encoding in which every node is mapped to a set of its encoded edges. The oracle consists of several blocks of circuits: a custom-written edge weight adder, node index calculator, uniqueness checker, and comparator, which were all created using only quantum Toffoli gates, including its special forms, which are Feynman and Pauli X. The oracle begins by using the edge encodings specified by the qubits to calculate each node that this path visits and adding up the edge weights along the way. Next, the oracle uses the calculated nodes from the previous step and check that all the nodes are unique. Finally, the oracle checks that the calculated cost is less than the previously-calculated cost. By performing the oracle an optimal number of times, a correct answer can be generated with very high probability. The oracle of the Grover Algorithm is modified using the recalculated minimum cost value, and this procedure is repeated until the cost cannot be further reduced. This algorithm and circuit design have been verified, using several datasets, to generate correct outputs.

Keywords : quantum computing, quantum circuit optimization, quantum algorithms, hybrid quantum algorithms, quantum programming, Grover's algorithm, traveling salesman problem, bounded-degree TSP, minimal cost, Q# language
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