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Stochastic Matrices and Lp Norms for Ill-Conditioned Linear Systems

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Abstract : In quite diverse application areas such as astronomy, medical imaging, geophysics or nondestructive evaluation, many problems related to calibration, fitting or estimation of a large number of input parameters of a model from a small amount of output noisy data, can be cast as inverse problems. Due to noisy data corruption, insufficient data and model errors, most inverse problems are ill-posed in a Hadamard sense, i.e. existence, uniqueness and stability of the solution are not guaranteed. A wide class of inverse problems in physics relates to the Fredholm equation of the first kind. The ill-posedness of such inverse problem results, after discretization, in a very ill-conditioned linear system of equations, the condition number of the associated matrix can typically range from 109 to 1018. This condition number plays the role of an amplifier of uncertainties on data during inversion and then, renders the inverse problem difficult to handle numerically. Similar problems appear in other areas such as numerical optimization when using interior points algorithms for solving linear programs leads to face ill-conditioned systems of linear equations. Devising efficient solution approaches for such system of equations is therefore of great practical interest. Efficient iterative algorithms are proposed for solving a system of linear equations. The approach is based on a preconditioning of the initial matrix of the system with an approximation of a generalized inverse leading to a stochastic preconditioned matrix. This approach, valid for non-negative matrices, is first extended to hermitian, semi-definite positive matrices and then generalized to any complex rectangular matrices. The main results obtained are as follows: 1) We are able to build a generalized inverse of any complex rectangular matrix which satisfies the convergence condition requested in iterative algorithms for solving a system of linear equations. This completes the (short) list of generalized inverse having this property, after Kaczmarz and Cimmino matrices. Theoretical results on both the characterization of the type of generalized inverse obtained and the convergence are derived. 2) Thanks to its properties, this matrix can be efficiently used in different solving schemes as Richardson-Tanabe or preconditioned conjugate gradients. 3) By using Lp norms, we propose generalized Kaczmarz’s type matrices. We also show how Cimmino's matrix can be considered as a particular case consisting in choosing the Euclidian norm in an asymmetrical structure. 4) Regarding numerical results obtained on some pathological well-known test-cases (Hilbert, Nakasaka, ...), some of the proposed algorithms are empirically shown to be more efficient on ill-conditioned problems and more robust to error propagation than the known classical techniques we have tested (Gauss, Moore-Penrose inverse, minimum residue, conjugate gradients, Kaczmarz, Cimmino). We end on a very early prospective application of our approach based on stochastic matrices aiming at computing some parameters (such as the extreme values, the mean, the variance, ...) of the solution of a linear system prior to its resolution. Such an approach, if it were to be efficient, would be a source of information on the solution of a system of linear equations.

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