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A Theorem Related to Sample Moments and Two Types of Moment-Based Density Estimates

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Abstract : Numerous statistical inference and modeling methodologies are based on sample moments rather than the actual observations. A result justifying the validity of this approach is introduced. More specifically, it will be established that given the first n moments of a sample of size n, one can recover the original n sample points. This implies that a sample of size n and its first associated n moments contain precisely the same amount of information. However, it is efficient to make use of a limited number of initial moments as most of the relevant distributional information is included in them. Two types of density estimation techniques that rely on such moments will be discussed. The first one expresses a density estimate as the product of a suitable base density and a polynomial adjustment whose coefficients are determined by equating the moments of the density estimate to the sample moments. The second one assumes that the derivative of the logarithm of a density function can be represented as a rational function. This gives rise to a system of linear equations involving sample moments, the density estimate is then obtained by solving a differential equation. Unlike kernel density estimation, these methodologies are ideally suited to model 'big data' as they only require a limited number of moments, irrespective of the sample size. What is more, they produce simple closed form expressions that are amenable to algebraic manipulations. They also turn out to be more accurate as will be shown in several illustrative examples.

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