

Extensions of Schwarz Lemma in the Half-Plane

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Abstract : Aside from being a fundamental tool in Complex analysis, Schwarz Lemma-which was finalized in its most complete form at the beginning of the last century-generated an important area of research in various fields of mathematics, which continues to advance even today. We present some properties of analytic functions in the half-plane which satisfy the conditions of the classical Schwarz Lemma (Carathéodory functions) and obtain a generalization of the well-known Aleksandrov-Sobolev Lemma for analytic functions in the half-plane (the correspondent of Schwarz-Pick Lemma from the unit disk). Using this Schwarz-type lemma, we obtain a characterization for the entire class of Carathéodory functions, which might be of independent interest. We prove two monotonicity properties for Carathéodory functions that do not depend upon their normalization at infinity (the hydrodynamic normalization). The method is based on conformal mapping arguments for analytic functions in the half-plane satisfying appropriate conditions, in the spirit of Schwarz lemma. According to the research findings in this paper, our main results give estimates for the modulus and the argument for the entire class of Carathéodory functions. As applications, we give several extensions of Julia-Wolf-Carathéodory Lemma in a half-strip and show that our results are sharp.

Keywords : schwarz lemma, Julia-wolf-caratéodory lemma, analytic function, normalization condition, caratéodory function

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