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A Spiral Dynamic Optimised Hybrid Fuzzy Logic Controller for a Unicycle Mobile Robot on Irregular Terrains

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***Abstract*—**This paper presents a hybrid fuzzy logic control strategy for a unicycle trajectory following robot on irregular terrains. In literature, researchers have presented the design of path tracking controllers of mobile robots on non-frictional surface. In this work, the robot is simulated to drive on irregular terrains with contrasting frictional profiles of peat and rough gravel. A hybrid fuzzy logic controller is utilised to stabilise and drive the robot precisely with the predefined trajectory and overcome the frictional impact. The controller gains and scaling factors were optimised using spiral dynamics optimisation algorithm to minimise the mean square error of the linear and angular velocities of the unicycle robot. The robot was simulated on various frictional surfaces and terrains and the controller was able to stabilise the robot with a superior performance that is shown via simulation results.

***Keywords*—**Fuzzy logic control, mobile robot, trajectory tracking, spiral dynamic algorithm.

I. Introduction

MOBILE robots have been of great interest to many researchers due to their various applications. Mobile robots have been used widely in industry for goods transportation, surveillance and security. Various path following and tracking controllers with high precision were developed by researchers [1]-[3].

An adaptive controller was developed by Martins et al. (2014) as in [1] based on both the kinematic and dynamic model of a unicycle. The authors utilized a robust updating law to avoid the drifting of the robot and kept the control error in bounded region to overcome instability problems. The authors presented a successful control results both in simulation and experimentally using Pioneer 2-DX robot platforms.

Fuzzy logic control (FLC) strategy has been adopted by Castillo et al. [2] and Martinez et al. [3] to control a unicycle robot. The authors have utilised a FLC based on back stepping control to ensure system stability.

Petrov and Dimitrov [4] presented a path tracking nonlinear controller for a differential drive robot. A backstepping control has been utilised to control the robot system and resulted in a fast exponential stability toward the predefined path. Stability analyses were reported using Lyapunov stability theory. The results have presented the feasibility of the proposed control strategy with a superior control performance. Previous studies have focused on the design of path tracking controllers of mobile robots on non-frictional surfaces.

In this paper, we utilise a hybrid fuzzy logic control strategy developed by Almeshal et al. [5], [6] for path tracking of a unicycle mobile robot on an irregular terrain. Studying the impact of irregular terrains on the performance of path tracking strategy of the robot will contribute to the literature and thus enables the development of more mobility applications that are based on the differential drive and unicycle robots.

The paper is organised as follows, section two is the description of the system and the mathematical modeling of the unicycle robot. Section three is the hybrid fuzzy logic control strategy. Section four is the spiral dynamic optimisation algorithm and the optimal controller gains. Section five is the irregular terrain and environment modeling. Section six presents the simulation results and discussion and finally the conclusion in section seven.

II. System Description

The robot considered in this paper is a unicycle mobile robot, reported by Klancar et al. [7], and is described by the following equation of motion.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

where v is the linear velocity and w is the angular velocity of the robot. The right and left wheel motor velocities can be expressed as $v_R = v + \frac{\omega L}{2}$ and $v_L = v - \frac{wL}{2}$ respectively. Given a reference path $(x_r(t), y_r(t))$, the required inputs can be calculated as

$$v_r(t) = \pm\sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \tag{2}$$

$$\omega_r = v_r(t)\kappa(t) \tag{3}$$

$$\theta_r(t) = \arctan 2(\dot{y}_r(t), \dot{x}_r(t)) + \beta\pi \tag{4}$$

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where $\kappa(t)$ represents the path curvature and β= 0,1 for forward or backward movement. The angular velocity is derived by differentiating (4) with respect to time as

$$\omega_r(t) = \frac{\dot{x}_r(t)\ddot{y}_r(t) - \dot{y}_r(t)\ddot{x}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \tag{5}$$

III. Hybrid Fuzzy Logic Control Strategy

Fuzzy logic controllers have been proven to be powerful in controlling non-linear systems. The FLC comprises model-free control approach for complex systems that are difficult to be described and modelled mathematically. A hybrid fuzzy logic control (FLC) strategy was developed by Almeshal et al. [8]-[10].

The system consists of two control loops with two hybrid FLC controllers. Each hybrid FLC controller is composed a proportional-derivative plus integral controller followed by a fuzzy controller that work together to fine tune the control signal and thus driving the robot to the desired reference path. The hybrid FLC is presented in Fig. 1.

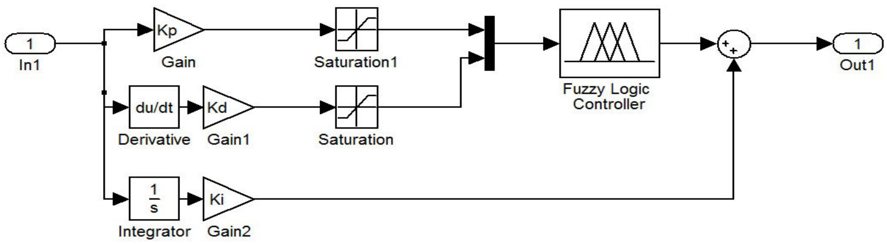

Fig. 1 Hybrid FLC controller block diagram

The inputs for the hybrid FLC are the error signal, change of error and the sum of previous errors. The FLC is of Mamdani-type with Gaussian membership functions, presented in Fig. 2, to smooth the outputs. The linguistic variables describing the inputs and outputs were chosen as Positive Big (PB), Positive Small (PS), Zero (Z), Negative Big (NB) and Negative Small (NS) with 25 fuzzy rule base described in Table I

TABLE I
Fuzzy Rule Base

e \ e'	NB	NS	Z	PS	PB
NB	NB	NB	NB	NS	Z
NS	NB	NB	NS	Z	PS
Z	NB	NS	Z	PS	PB
PS	NS	Z	PS	PB	PB
PB	Z	PS	PB	PB	PB

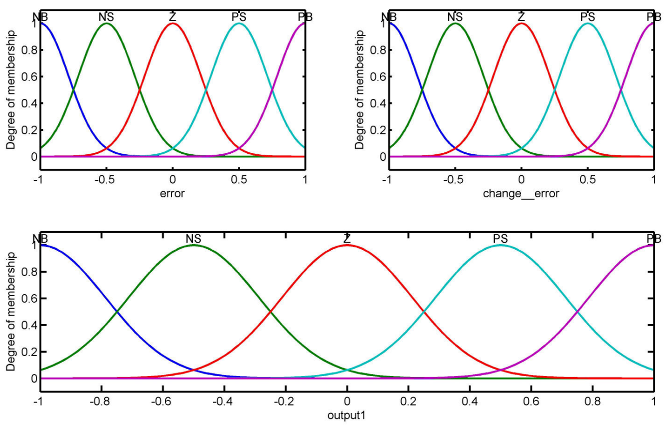

Fig. 2 Mamdani Fuzzy membership functions for the inputs and outputs of the system

In order to achieve an optimal control with the minimum control efforts with feasible gains, a spiral dynamic optimisation algorithm (SDA) will be adopted to optimise the controller gains. In the next section, the SDA optimisation will be integrated into the control system.

IV. Spiral Dynamic Optimisation Algorithm

Spiral dynamic algorithm is a metaheuristic optimisation algorithm inspired by the spiral phenomena in nature, such as whirling current, developed by Tamura and Yasuda [11]. It is relatively new optimisation algorithm and was reported first for two dimensional search spaces [11]. Later, it was derived into a general model for n-dimension problem. The common feature of logarithmic spirals motivated the authors in developing the algorithm, which they believed could make an effective search strategy. The algorithm was tested and compared with other proven search strategies, such as Particle Swarm Optimisation (PSO), and showed either a better or equal performance in terms of accuracy and speed of convergence.

The algorithm is based on spiral search trajectories. One of the strength features of the algorithm is in in the diversification and intensification at the early and later phases of the search trajectory. Diversification is to search for better solutions by searching in a wider area of the search space, while intensification is to search for better solutions by searching intensively around a good solution. Table II presents a description of parameters used in the SDA optimisation pseudo code.

TABLE II
SDA OPTIMISATION NOMENCLATURE

Parameter	Description
θ	Rotation angle, $0 \le \theta \le 2\pi$
$k_{\max}$	Maximum iteration number.
r	Convergence rate of distance between a point and the origin, $0 \le r \le 1$
$R_{i,j}$	Rotation matrix between $x_i - x_j$ planes
m	Dimension of the search space

where the rotation matrix for the n-dimension SDA algorithm is defined as

$$R^{n}_{i,j}(\theta_{i,j}) = \begin{array}{cc} & \begin{array}{ccccccccccc} & & & i & & & & j & & & \end{array} \\ \begin{array}{c} \\ \\ \\ i \\ \\ \\ \\ j \\ \\ \\ \\ \end{array} & \begin{bmatrix} 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & \cos\theta_{i,j} & & \cdots & & -\sin\theta_{i,j} & & & \\ & & & & 1 & & & & & & \\ & & & \vdots & & \ddots & & \vdots & & & \\ & & & & & & 1 & & & & \\ & & & \sin\theta_{i,j} & & \cdots & & \cos\theta_{i,j} & & & \\ & & & & & & & & 1 & & \\ & & & & & & & & & \ddots & \\ & & & & & & & & & & 1 \end{bmatrix} \end{array}$$

The n-dimension spiral dynamic model is expressed using the rotational matrix as:

$$x_i(k+1) = S_n(r,\theta)x_i(k) - (S_n(r,\theta) - I_n)x^* \tag{6}$$

where

$$S_n(r,\theta)x(k) = \prod_{i=1}^{n-1}\left(\prod_{j=1}^{i} R^{n}_{n-i,n+1-j}(\theta_{n-i,n+1-j})\right)$$

The n-dimension SDA optimisation pseudo code reported in [11] is as follows:

Step 0: Preparation
Select the number of search points $m \ge 2$, parameters $0 \le \theta < 2\pi,\ \ 0 < r < 1$ of $S_n(r,\theta)$, and maximum number of iterations $k_{\max}$
Set $k = 0$.
Step 1: Initialization
Set initial points $x_i(0) \in R^n,\ i = 1,2,\ldots m$ in the feasible region at random and centre x^* as $x^* = x_{i_g}(0)$,
$i_g = \arg\min_i\ f(x_i(0)),\ i = 1,2,\ldots,m$.
Step 2: Updating x_i
$x_i(k+1) = S_n(r,\theta)x_i(k) - (S_n(r,\theta) - I_n)x^*$
$i = 1,2,\ldots,m$.
Step 3: Updating x^*
$x^* = x_{i_g}(k+1)$,
$i_g = \arg\min_i\ f(x_i(k+1)),\ i = 1,2,\ldots,m$.
Step 4: Checking termination criterion
If $k = k_{\max}$ then terminate. Otherwise set $k = k+1$, and return to step 2.

The performance index of the system is chosen as the minimum mean squared error (MSE) of system response. The MSE is calculated for each control loop in the system as follows:

$$MSE\ 1 = \min\left\{\frac{1}{N}\sum_{i=1}^{N}(x_d - x_a)^2\right\} \tag{7}$$

$$MSE\ 2 = \min\left\{\frac{1}{N}\sum_{i=1}^{N}(y_d - y_a)^2\right\} \tag{8}$$

The objective function is chosen as the summation of the MSE of the system which can be expressed as the summation

$$J = MSE_1 + MSE_2 \tag{9}$$

Hence; minimizing the objective function J will result in finding the optimum control parameters with the minimum mean square error of the overall system response

In order to restrict the optimisation algorithms to search within the feasibility region of the system, which is the stability region of the vehicle, system constraints must be defined to ensure stability.

Constrained optimisation is used to limit the system within the stability region while searching for optimum parameters. The stability bounds are defined by using trial and error and defining a feasible interval for each control parameter; shown in Table III, that assures the stability of the system.

TABLE III
BOUNDARY LIMITS OF CONTROLLER GAINS

Boundaries	**Loop 1**			**Loop 2**		
	Kp	Kd	Ki	Kp	Kd	Ki
Lower	0.5	0.5	0.1	0.5	4	0.01
Upper	100	8	2	100	30	5

Penalty methods are used to convert the constrained optimisation problem into an unconstrained problem by adding a penalty function P(x) to the objective function when the constraint is violated. The cost function of the system can be rewritten as:

$$J(x) = \begin{cases} J(x) & x \in \textit{feasible region} \\ J(x) + P(x) & x \notin \textit{feasible region} \end{cases}$$

where J(x) represents the cost function and P(x) represents the penalty function. The penalty function is added to the cost function to result in a very high cost whenever a constraint is violated. The penalty function is defined as ten times the cost function of the unfeasible region and zero otherwise. This can be expressed as:

$$P(x) = \begin{cases} 0 & x \in feasible \;\; region \\ 10\,.\,J(x) & x \notin feasible \;\; region \end{cases}$$

V. Irregular Terrains and Environment Modelling

To simulate the robot to drive on various outdoor environments of contrastive frictional profils, environment modelling must be integrated into the simulation blocks. Environment modelling is based on study of soil mechanics and was reported in literature, specifically in locomotion systems; by many researchers to enable them study the foot-ground interaction forces. Silva et al. [12] have used a modified spring-damper dashpot system to simulate different types of grounds to study the foot-ground interaction for locomotion systems. They have presented the modification by changing the parameters of damping and stiffness B and K respectively for both horizontal and vertical deflection forces. The contact of the foot and ground can be described by the nonlinear equations:

$$f_{i\eta F} = -K_{\eta F}(\eta_{iF} - \eta_{iF0}) - B'_{\eta F}[-(y_{iF} - y_{iF0})]^{v_n}(\dot{\eta}_{iF} - \dot{\eta}_{iF0}) \tag{10}$$

$$-B'_{\eta F}(-\Delta_{iyFMax})^{v_\eta} = -B_{nF} \tag{11}$$

where

$K_{\eta F}$ = Linear stiffness factor

$B'_{\eta F}$ = Nonlinear damping factor

η = Directions in x and y

x_{if0}, y_{if0} = Coordinates of the wheel-ground touchdown

v_η = A parameter dependent on ground characteristics with $0.9 < v_\eta < 1.0$

Δ_{iyFMax} = Maximum penetration depth of wheel into ground

The linear damping and stiffness parameter values for different ground profiles, extracted from soil mechanics and Young's modulus of elasticity of gravel and sand soil types is presented in Table IV.

TABLE IV
YOUNG'S MODULUS OF ELASTICITY FOR GRAVEL AND SAND SOIL TYPES

Soil type	K_{xF} (Nm^{-1})	B_{xF} (Nsm^{-1})	K_{yF} (Nm^{-1})	B_{xF} (Nsm^{-1})
Gravel	17362028	12500	22735988	14305
Sand	6944811	7906	9094395	9047

In this work, the robot will be simulated on a soil that contains a mixture of gravel and sand to allow increasing the roughness of the terrain.

VI. Simulations and Results

The robot model will be simulated using Matlab/Simulink environment and using Runge-Kutta method for solving the model differentially. The system was simulated with 100 iterations to find the optimal gains of the control system. Fig. 3 shows the convergence of the cost function. The SDA optimisation found the optimal values of the gains that lead to the minimum cost function within 12 iterations.

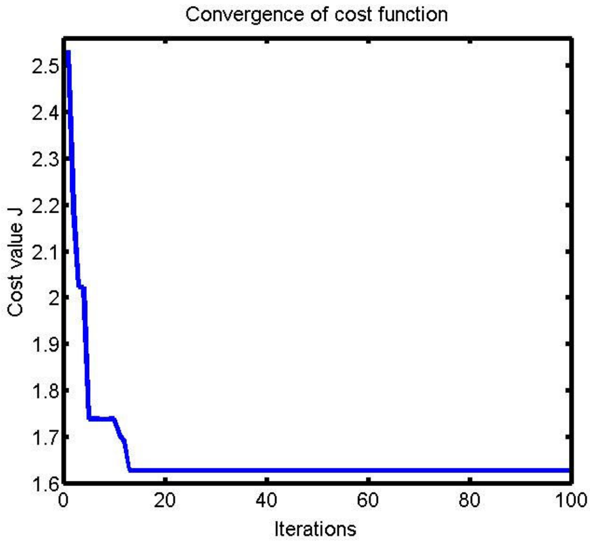

Fig. 3 Cost convergence plot

The optimal gains were then used to simulate the vehicle on an irregular terrain of mixed sand and gravel with a noticeable roughness. Fig. 4 presents the system response in path following a predefined 8-shape-trajectory over an irregular terrain while Fig. 5 shows the error convergence of the angular and linear velocities of the unicycle robot.

The robot was able to drive on the terrain and follow the predefined 8-shape-trajectory with minimal oscillations that are noticeable on the turn points. This is due to the effect of the irregularity of the ground that affects the robot's yaw angle and cause drifts at some points.

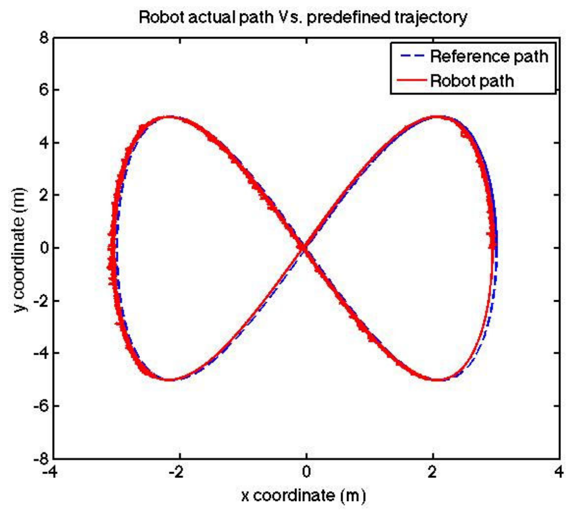

Fig. 4 Path following movement scenario over an irregular terrain

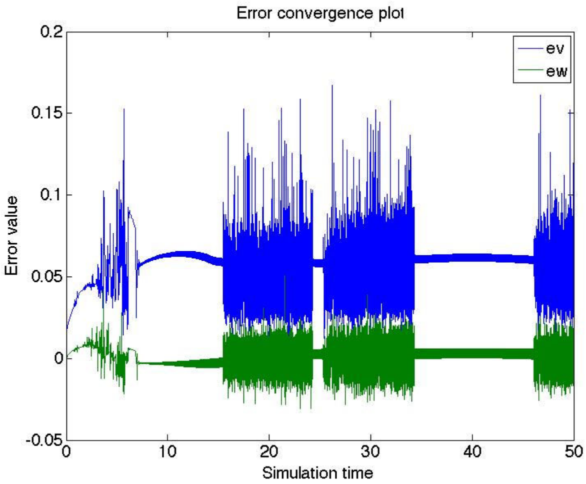

Fig. 5 Error convergences of the angular and linear velocities of the robot while moving on the frictional terrain

VII. Conclusion

This paper presented a hybrid fuzzy logic control strategy to control a path following unicycle robot moving on irregular terrains of different soil types. A spiral dynamic optimisation algorithm was used to optimise the hybrid FLC control gains and minimise the mean square error of the system. The SDA algorithm optimised the controller with the optimal gains within 12 iterations. The optimal controller gains were used to simulate the unicycle robot over a mixed soil type of sand and gravel with irregular profile. The robot was able to drive on the terrain and follow the predefined trajectory with a high degree of robustness. The presented simulation results show the effectiveness of the hybrid FLC control strategy to stabilise the robot. Further work will be carried out experimentally to incorporate testing the hybrid FLC over a real prototype of the unicycle robot.

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References

[1] Martins, F. N., Celeste, W. C., Carelli, R., Sarcinelli-Filho, M., & Bastos-Filho, T. F. (2008). An adaptive dynamic controller for autonomous mobile robot trajectory tracking. *Control Engineering Practice*, *16*(11), 1354-1363

[2] Castillo, O., Martínez-Marroquín, R., Melin, P., Valdez, F., & Soria, J. (2012). Comparative study of bio-inspired algorithms applied to the optimization of type-1 and type-2 fuzzy controllers for an autonomous mobile robot. *Information Sciences*, *192*, 19-38.

[3] Martínez, R., Castillo, O., & Aguilar, L. T. (2009). Optimization of interval type-2 fuzzy logic controllers for a perturbed autonomous wheeled mobile robot using genetic algorithms. Information Sciences, 179(13), 2158-2174.

[4] Petrov, P., & Dimitrov, L. (2010). Nonlinear path control for a differential-drive mobile robot. RECENT Journal, 11(1), 41-45.

[5] A M Almeshal, M O Tokhi, K M Goher, Robust hybrid fuzzy logic control of a novel two-wheeled robotic vehicle with a movable payload under various operating conditions, *Proceedings of the United Kingdom Automatic Control Council International Conference on Control (UKAAC 2012),* University of Glamorgan, Cardiff, UK, 3-5 Sept 2012. pp 747-752

[6] A M Almeshal, K M Goher, A N K Nasir, M O Tokhi, Steering and dynamic performance of a new configuration of a wheelchair on two wheels in various indoor and outdoor environments, *Proceedings of 18th International Conference on Methods and Models in Automation and Robotics (MMAR),* Miedzyzdroje, Poland, 26-29 Aug. 2013.

[7] Klancar, G., Matko, D., & Blazic, S. (2005). Mobile robot control on a reference path. In Intelligent Control. Proceedings of the 2005 IEEE International Symposium on, Mediterrean Conference on Control and Automation (pp. 1343-1348).

[8] A M Almeshal, K M Goher, A N K Nasir, M O Tokhi, S A Agouri, Fuzzy logic optimized control of a novel structure two-wheeled robotic vehicle using HSDBC, SDA and BFA: a comparative study, *Proceedings of 18th International Conference on Methods and Models in Automation and Robotics (MMAR),* Miedzyzdroje, Poland, 26-29 Aug. 2013.

[9] A M Almeshal, M O Tokhi and K M Goher, Stabilization of a new configurable two-wheeled machine using a PD-PID and a hybrid FL control strategies: a comparative study, *Proceedings of the 32nd International Conference on Control, Automation and Robotics (ICCAR 2012)*, Dubai, UAE, 8-9 Oct. 2012. pp 65-72

[10] A M Almeshal, K M Goher, M O Tokhi, O Sayidmarie, S A Agouri, Hybrid fuzzy logic control of a two wheeled double inverted pendulum-like robotic vehicle, *Proceedings of 15th International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines (CLAWAR 2012),* Baltimore, USA, 23-26 Jul. 2012. pp 681-688

[11] Tamura, K., & Yasuda, K. (2011). Primary study of spiral dynamics inspired optimization. IEEJ Transactions on Electrical and Electronic Engineering, 6(S1), S98–S100.

[12] Silva, M. F., Machado, J. A. T., & Lopes, A. M. (2005). Modelling and simulation of artificial locomotion systems. Robotica, 23(5), 595–606.