Allocation of Mobile Units in an Urban Emergency Service System

Dimitra Alexiou

Abstract—In an urban area the location allocation of emergency services mobile units, such as ambulances, police patrol cars must be designed so as to achieve a prompt response to demand locations.

In this paper the partition of a given urban network into distinct sub-networks is performed such that the vertices in each component are close and simultaneously the sums of the corresponding population in the sub-networks are almost uniform. The objective here is to position appropriately in each sub-network a mobile emergency unit in order to reduce the response time to the demands.

A mathematical model in framework of graph theory is developed.

In order to clarify the corresponding method a relevant numerical example is presented on a small network.

Keywords—Distances, Emergency Service, Graph Partition, location.

I. INTRODUCTION

THERE is a wide range of work that has been done on mobile emergency services due to the importance of their taking a prompt and effective response. Emergency services are dealt with in [1] using goal programming, simulation model is proposed in [2], [8], [10], [13] and mathematical programming formulation is given in [3]. An integer linear programming method is formulated in [6], while a non-linear integer programming model is applied in [7] and a P-median model is discussed in [11], [12].

Here a given urban network is partitioned into distinct sub-networks in such a way so that the vertices in each sub-network are close to each other and the size of the population in the sub-networks is almost uniform.

The problem is dealt within the framework of graph theory. More specifically, the vertices of the corresponding network represent habitat sites and candidate positions for an emergency service vehicle; to every vertex the corresponding population magnitude is assigned.

The direct distances of neighboring sites are known.

Next section presents an explanation of preliminary notions and notations used here in order for the paper to be self contained, Section III analyzes the pursued method so as to solve the described problem. Section IV gives a detailed numerical example on a small network that express an urban area in order to clarify the proposed approach. The conclusions are the content of the last section.

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II. PRELIMINARIES

A finite nonempty set of vertices $V = \{v_1, v_2, \ldots v_n\}$ and links $E = \{e_1, e_2, \ldots e_q\}$ joining a subset of unordered pairs in V called edges consist of a graph $G = (V, E)[4, 9]$. In case where the elements of E are ordered pairs the graph is a directed graph. Here for reason of clarification and without loss of generality we will deal with simple graphs. A network N may be regarded as a weighted graph to which usually non-negative numbers are associated to every vertex or edge or to both, such a weighted graph can be denoted as $G(V, E, D, W)$ where D and E express the weights of the edges and vertices respectively.

A pair of vertices $x, y \in V$ are said to be adjacent if $\{x, y\} \in E$.

A path is a sequence of adjacent vertices where all its vertices and edges are distinct. The distance $d_{(u, v)}$ between two nodes u and v in V is the length of the shortest path joining them. A graph is connected if every pair of vertices is joined by a path.

A sub-graph of G is a graph having all of its vertices and edges in G.

III. NETWORK PARTITION

Let $G(V, E, D, W)$ be the weighted graph representing a network N that will be partitioned into sub-networks. The elements $d(i, j) \in D$, $i = 1, 2, \ldots n$, $j = 1, 2, \ldots n$, $i \neq j$ is the distance between vertices $i, j \in V, n$ is the number of vertices in V. The square matrix D is obtained by Floyd's algorithm [5]. The weights $w(i) \in W$ denote the population size that correspond to vertex i.

Let nv be the number of emergency vehicles to be positioned in N. The eccentricity $e(i), i \in V$ is the longest distance of i to any vertex j in $V, i.e.$ $e(i), = \max\{d(i, j), j \in V\}$. The vertices with the smallest eccentricity are the centres of the network [4], [9], while the maximum eccentricity is the diameter of N.

We seek to partition G into nv sub-graphs $G_i(V_i, E_i)$ that acquire the properties referred in the introduction. $SW(V) = \sum_{i \in V} W(i)$ is the total sum of the population in G.

Therefore to obtain an almost uniform distribution among the

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nv sub-graphs each one must possess a population magnitude near the mean $MW(V) = SW(V)/nv$.

The vertices already assigned to a sub-graph G_i are the elements of set F.

The elements of $V_i = \{v_i^1, v_i^2, ..., v_i^{n_i}\}$ are produced so that the sums of the distances of a vertex $v_i^j \in V - F$ to all vertices $\{v_i^1, v_i^2, ..., v_i^{j-1}\}$ is the minimum among all vertices in V and not in F. Thus a vertex v_i^j is inserted in $V_i = \{v_i^1, v_i^2, ..., v_i^{j-1}\}$ if

$$v_i^j = \min\{v_i^k : \sum d(v_i^j, v_i^k), v_i^k \in (V - F), k = 1, 2, ..., j-1\} \quad (1)$$

The above procedure is performed in order for the vertices of V_i, $i = 1, 2, ... nv$ to be close to each other.

The sum of the population that correspond to a sub-graph G_i is denoted by $SW(V_i)$, i.e.

$$SW(V_i) = \sum_{i \in V_i} w(i) \quad (2)$$

The extension of sub-graphs $V_i = \{v_i^1, v_i^2, ..., v_i^{j-1}\}$ continues until the inclusion of any vertex $u \in V - F$ that verifies (1) surpasses the value $MW(V)$, meaning that $SW(V_i \cup u) > MW(V)$. At this stage u will be the first vertex inserted in sub-graph V_{i+1}. The initial vertex inserted in sub-graph V_1 is a vertex that is included in a pair $i, j \in V$ such that $d(i, j)$ complies with the diameter of G.

With the use of matrix D, for each sub-graph G_i the eccentricity of all its vertices are detected. The vertices with the smallest eccentricity define its centres. An emergency vehicle is positioned at a selected centre of G_i. In general some vertices $U = \{u_1, u_{2,...}, u_r\} \in (V - F)$ may not be included in any G_i. In this case the item $u_k \in U$ will be inserted in the sub-graph Gk which the selected centre c_k is closer to u_k, that is,

$$u_k = \{u_j : \min\{d(u_j, c_q), u_j \in U, c_q \in G_q\}$$

IV. NUMERICAL EXAMPLE
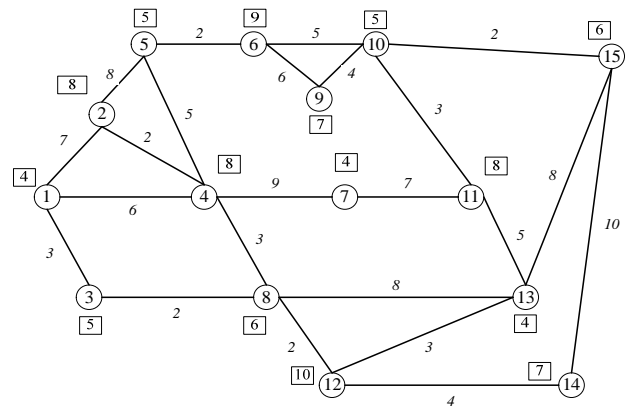
Fig. 1 represents an urban area network.

Fig. 1 Urban area network

The numbers in italic near the edges represent their lengths, while the boxed numbers beside the vertices express their corresponding population magnitude.

Table I shows the square matrix D that gives the distances between all pair of vertices of the graph of Fig. 1. The last column indicates the eccentricity of the corresponding vertex. We observe that the pairs {1,9}, {9,14) and {7,15} define the diameter of the graph while vertex 13 is its centre.

TABLE I
DISTANCES/ ECCENTRICITY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	0	7	3	6	11	13	15	5	19	18	15	7	10	11	18	19
2	7	0	7	2	7	9	11	5	15	14	15	7	10	11	18	18
3	3	7	0	5	10	12	14	2	18	15	12	4	7	8	15	18
4	6	2	5	0	5	7	9	3	13	12	13	5	8	9	16	16
5	11	7	10	5	0	2	14	8	8	7	10	10	13	14	16	16
6	13	9	12	7	2	0	15	10	6	5	8	12	13	16	14	16
7	15	11	14	9	14	15	0	12	14	10	7	14	12	18	19	19
8	5	5	2	3	8	10	12	0	16	13	10	2	5	6	13	16
9	19	15	18	13	8	6	14	16	0	4	7	15	12	19	13	19
10	18	14	15	12	7	5	10	13	4	0	3	11	8	15	9	18
11	15	15	12	13	10	8	7	10	7	3	0	8	5	12	12	15
12	7	7	4	5	10	12	14	2	15	11	8	0	3	4	11	15
13	10	10	7	8	13	13	12	5	12	8	5	3	0	7	8	13
14	11	11	8	9	14	16	18	6	19	15	12	4	7	0	10	19
15	18	18	15	16	16	14	19	13	13	9	12	8	10	0		19

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The method described in the previous section for $nv = 2$ will give the following results. The sum of the population of all vertices of network in Fig. 1 is $SW(V) = 96$ and the mean is

$$MW(V) = (SW(V) / nv) = 48.$$

In Table I we observe that vertices 9, 1, 14, 7, 15 form distances that lead to maximum eccentricity because $d(1,9) = 19$, $d(9,14) = 19$ and $d(7,15) = 19$.

Following, the outcomes are presented for each vertex when it is used as the initial one in the creation of sub-graph G_i in order to start the partition of G. Every element in V_1 and V_2 verifies (1), while the population sum of their vertices satisfies (2). The expression $e_{G_i}(v) = k$ means that an emergency vehicle will be positioned at vertex v since k is the minimum eccentricity in sub-graph G_i. The outcomes can easily be verified in Table I.

Sub-graph G_1- Initial vertex 1.

$V_1 = \{1,3,8,12,4,2,13\}$, $SW(V_1) = 45 \le 48$, $e_{G_i}(8) = 5$

Sub-graph G_2

$V_2 = \{14,15,10,11,9,6,5\}$, $SW(V_2) = 47 \le 48$, $e_{G_2}(15) = 10$

Vertex 7 was not yet included in any sub-graph. The distances of 7 from the two centres 8 and 15 are $d(7,8) = 12$ and $d(7,15) = 19$, therefore we include 7 in G_1 and finally $V_1 = \{1,3,8,12,4,2,13,7\}$ with $SW(V_1) = 51$.

Sub-graph G_1- Initial vertex 9.

$V_1 = \{9,10,15,11,6,5,13\}$, $SW(V_1) = 44 \le 48$, $e_{G_i}(10)=8$

Sub-graph G_2

$V_2 = \{12,8,3,4,1,2,14\}$, $SW(V_2) = 48 \le 48$, $e_{G_2}(8) = 6$

Vertex 7 was not yet included in any sub-graph. The distances of 7 from the two centres 10 and 8 are $d(7,10) = 10$ and $d(7,8) = 12$, therefore we include 7 in G_1 and finally $V_1 = \{9,10,15,11,6,5,13,7\}$ with $SW(V_1) = 48$.

Sub-graph G_1- Initial vertex 14.

$V_1 = \{14,12,8,3,4,13,1\}$, $SW(V_1) = 44 \le 48$, $e_{G_i}(8)=6$

Sub-graph G_2

$V_2 = \{2,5,6,10,9,15,11\}$, $SW(V_2) = 48 \le 48$, $e_{G_2}(6) = 9$

Vertex 7 was not yet included in any sub-graph. The distances of 7 from the two centres 8 and 6 are $d(7,8) = 12$ and $d(7,6) = 15$, therefore we include 7 in G_1 and finally $V_1 = \{14,12,8,3,4,13,1,7\}$ with $SW(V_1) = 48$.

Sub-graph G_1- Initial vertex 7

$V_1 = \{7,11,10,15,9,6,5,13\}$, $SW(V_1) = 48 \le 48$, $e_{G_i}(10) = 11$

Sub-graph G_2

$V_2 = \{4,2,8,3,12,1,14\}$, $SW(V_2) = 48 \le 48$, $e_{G_2}(8) = 6$

Sub-graph G_1- Initial vertex 15.

$V_1 = \{15,10,11,9,6,5,13\}$, $SW(V_1) = 44 \le 48$, $e_{G_i}(10) = 8$

Sub-graph G_2

$V_2 = \{12,8,3,4,1,2,14\}$, $SW(V_2) = 48 \le 48$, $e_{G_2}(8) = 6$

Vertex 7 was not yet included in any sub-graph. The distances of 7 from the two centres 8 and 6 are $d(7,10) = 10$ and $d(7,8) = 12$, therefore we include 7 in G_1 and finally $V_1 = \{14,12,8,3,4,13,1,7\}$ with $SW(V_1) = 48$.

Similar results are derived when the initial vertex in V_1 is 9 or 15.

The selection of a vertex with a maximum eccentricity to be initially included in V_1 is made in order to reduce the number of vertices not contained in some V_i during the partition process at large networks.

V. Conclusions

The method presented here has been implemented in a Fortran computer program. The complexity of Floyds's algorithm [5] that determine the distances of all pair of vertices in the urban network is $O(n^3)$ and it is use once. The main part of the method presented here has a complexity of $O(n)$, which means that it can be applied efficiently to real-world urban networks using as many emergency vehicles needed so as to locate ambulances, police patrol vehicles, roadside assistance and fire stations. Whenever an emergency vehicle is on duty and not at its fixed location, a backup emergency vehicle should replace it. Therefore, the most reasonable station for applying such a backup is to position emergency vehicles at a centre of sub-graph G_c that comprises all the center vertices already assigned in the Network.

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