

# Hybrid Artificial Bee Colony and Least Squares Method for Rule-Based Systems Learning

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**Abstract**—This paper deals with the problem of automatic rule generation for fuzzy systems design. The proposed approach is based on hybrid artificial bee colony (ABC) optimization and weighted least squares (LS) method and aims to find the structure and parameters of fuzzy systems simultaneously. More precisely, two ABC based fuzzy modeling strategies are presented and compared. The first strategy uses global optimization to learn fuzzy models, the second one hybridizes ABC and weighted least squares estimate method. The performances of the proposed ABC and ABC-LS fuzzy modeling strategies are evaluated on complex modeling problems and compared to other advanced modeling methods.

**Keywords**—Automatic design, learning, fuzzy rules, hybrid, swarm optimization.

## I. INTRODUCTION

THE construction of process models automatically from numerical data is still devoting special attention in the literature. This data-driven modeling paradigm can be emphasized as an alternative approach to most general physical modeling methods that use physical laws or even rule-based modeling techniques. Generally, these techniques involve experts' knowledge which is not always easy to acquire and sometimes becomes impossible to obtain mainly for complex systems. Fuzzy models showed interesting results in dealing with several complex systems modeling problems [6], [8], [14]. Designing fuzzy models from numerical data is also of substantial interest in the field of fuzzy modeling. It consists of developing fuzzy rules, fuzzy sets and associated parameters so that the complete structure and parameters of the so-called data-driven fuzzy model are entirely specified. The modeling procedure can be achieved through consecutive structure and parameter identification. It can be formulated as a nonlinear optimization problem which can be solved using clustering algorithms [9], [15], partition-based methods [13], least squares and nonlinear optimization for model parameters tuning [16]. Equivalently, the construction of fuzzy models can be regarded as a search problem in multidimensional space where each point corresponds to a potential fuzzy model. Obviously, the objective is to find an optimal or near optimal location on the search space according to specific performance criteria and predefined constraints. To this end, evolutionary algorithms [1]

and swarm intelligence based techniques [10] have been extensively investigated in recent literatures with a common focus on achieving enhanced fuzzy model design.

In this paper, a methodology for extracting TS-type fuzzy models from data is addressed using artificial bee colony (ABC) optimization. A study on the same topic was considered in our previous work [11] where the issue of data-driven fuzzy modeling using ABC optimization is tackled starting with a predefined fuzzy model structure. The present contribution deals with a novel approach which finds the structure and the parameters of a fuzzy model simultaneously and aims to achieve enhanced fuzzy modeling without knowing the rule number as a priori. To this end, two different modeling strategies are presented and compared. The first one is based on global optimization where the whole parameters of the fuzzy model are identified simultaneously. The second strategy referred to as ABC-LS hybridizes ABC algorithm and weighted least squares method. Benchmark modeling problems are considered to check the performances of both algorithms.

Briefly, the paper is organized as follows. Section II introduces the concept of artificial bee colony optimization. Section III describes the proposed methodology of extracting fuzzy rules directly from numerical data using ABC concept. The performances of the proposed modeling approaches are evaluated through simulations by considering benchmark systems and the results are compared to other meta-heuristic algorithms in Section IV. Finally, concluding remarks are given in Section V.

## II. GENERAL ABC OPTIMIZATION STRATEGY

Artificial bee colony (ABC) optimization is a swarm intelligence based technique which was originally proposed by Karaboga [3], [4] to solve numerical function optimization. ABC algorithm simulates the foraging behavior of honey bees that are categorized into three main groups: employed bees, onlooker bees and scout bees. Based on two essential leading modes of honey bee colony which are recruitment to a food source and abandonment of a source, the process of bees seeking for sources with high amount of nectar is the one applied to find the optimal solution for a given optimization problem.

In ABC model, three main phases are considered: employed bee phase, onlooker phase and scout phase. Employed bees investigate their food sources and share the nectar and the position information of these sources with onlooker bees. Based on a greedy selection, onlooker bees will have to choose food sources with high profitability. The employed bee whose food source has been abandoned by the bees becomes a scout

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bee. The algorithmic structure of ABC concept defines the position of a food source as a possible solution to the optimization problem. The nectar amount of that source represents the fitness of the associated solution. Food source positions are generated using the following equation [4], [21]:

$$x_{ij} = x_j^{\min} + \text{rand}(0,1) \cdot (x_j^{\max} - x_j^{\min}) \quad (1)$$

Each solution  $x_i$  ( $i = 1, 2, \dots, SN$ ) is a D-dimensional vector of optimization parameters. In the employed phase, an employed bee produces a modification on the position of the food source in her memory and finds a neighboring food source according to the following expression:

$$v_{ij} = x_{ij} + \phi_{ij} \cdot (x_{ij} - x_{kj}) \quad (2)$$

where  $\phi_{ij}$  is a random number in the interval  $[-1, 1]$  and  $k \in \{1, 2, \dots, SN\}$  with  $k \neq i$  and  $j \in \{1, 2, \dots, D\}$  are randomly chosen indexes. Greedy selection between the old and the updated food source position is performed by the employed bee based on fitness value evaluation. This valuable information about the position and the quality of the food sources are shared with the onlooker bees.

In the onlooker phase, an onlooker bee evaluates the information provided by the employed bees and selects a food source depending on its probability value  $p_i$  which is calculated by

$$p_i = \frac{fit_i}{\sum_{l=1}^{SN} fit_l} \quad (3)$$

where  $fit_i$  denotes the fitness of the  $i$ th solution in the population. The probability of a food source being selected by the onlooker bees increases as the fitness value of a food source increases. After selecting the food source, an onlooker bee produces a modification on the position of that site using the same mechanism as in (2). Greedy selection is also applied by onlooker bees so that new food sources with high nectar are memorized. During scout phase, any solution that cannot be improved through a predefined number of generations will be abandoned and replaced by a new position that is randomly determined by a scout bee according to (1).

### III. LEARNING FUZZY SYSTEMS THROUGH ABC AND ABC-LS OPTIMIZATION

Among many types of fuzzy systems, the Takagi-Sugeno (TS) type model [7] attracted particular attention. As a universal approximator [2], it consists of IF-THEN fuzzy rules with linguistic antecedents and functional consequent parts. Generally, the fuzzy rule base comprises  $N_R$  IF-THEN rules of the form:

$$R^i: \text{ If } z_1 \text{ is } A_1^i \text{ and } z_2 \text{ is } A_2^i \text{ and } \dots \text{ and } z_n \text{ is } A_n^i \quad (4)$$

$$\text{Then } y^i = a_1^i z_1 + \dots + a_n^i z_n + a_0^i \quad (1 \leq i \leq N_R)$$

where  $z = [z_1 \ z_2 \ \dots \ z_n]^T$  is the premise input vector,  $A_j^i$ , ( $1 \leq j \leq n$ ), is the  $i$ th linguistic value of rule-premise variable  $z_j$  and  $a_0^i, a_1^i, \dots, a_n^i$  are consequent parameters.

Given the premise input vector  $z \in \mathcal{R}^n$ , the output of the TS fuzzy model is inferred by a weighted average defuzzification method as follows:

$$\hat{y} = \frac{\sum_{i=1}^{N_R} \prod_{j=1}^n \mu(A_j^i) y^i}{\sum_{i=1}^{N_R} \prod_{j=1}^n \mu(A_j^i)} \quad (5)$$

where  $\mu(A_j^i)$  is the grade of the  $j$ th Gaussian-type membership function  $A_j^i$  which is fully described by the center  $c_j^i$  and the width  $\sigma_j^i$ .

The construction of the TS fuzzy model (4) is achieved in two consecutive steps: structure identification and parameter estimation. As mentioned above, this paper investigates two different ABC-based fuzzy modeling algorithms. The first modeling strategy deals with global optimization of the whole fuzzy model parameters where all premise and consequent parameters of the fuzzy model evolve together through a stochastic strategy that simulates the foraging behavior of honey bee swarms so that optimal solutions are achieved. The second modeling strategy hybridizes ABC optimization and least squares. The premise parameters are optimized using ABC concept and the consequent parameters are identified by using weighted least squares estimate method.

Efficient encoding of a fuzzy model is the key issue in a population-based optimization method. There exist several encoding schemes that have been proposed in the literature, showing in the most of application studies good performance in finding a compact number of rules with optimized model parameters [12]. However, it is clear that providing an efficient method to encode a fuzzy model is closely related to the optimization strategy being used for fuzzy model building. In the proposed fuzzy modeling methodology, a fuzzy model is encoded into a food source, to be visited by a forager honey bee, by selecting an appropriate string representation so that a TS fuzzy model is entirely specified. A fuzzy model is realized when the number of rules and membership functions, the rule structure, and parameters of premise fuzzy sets and rule-consequents are specified. The proposed encoding scheme uses a food structure that consists of three parts. The first part deals with the rule selection and the optimization of the number of rules, the second part deals with the optimization of rule premise fuzzy sets and the third part deals with the optimization

of the rule-consequent parameters. This encoding scheme is reduced to only two parts in ABC-LS based fuzzy modeling strategy. The fitness of a food source is taken as the mean square error (MSE) and is given by:

$$MSE = \frac{1}{N} \sum_{k=1}^N [y_d(k) - \hat{y}(k)]^2 \quad (6)$$

where  $N$  is the number of training data,  $y_d(k)$  is the desired output and  $\hat{y}(k)$  is the fuzzy model output.

The general framework of the ABC and ABC-LS optimization based modeling methodology developed in this paper for automatic generation of TS rule-based fuzzy models is described as follows.

Step1. Encode all the parameters into food source.

Step2. Generate randomly an initial population of TS fuzzy models.

Step3. Evaluate the model accuracy of each individual {  
Perform rule selection mechanism

Calculate rule consequents using weighted LS estimates (If ABC-LS is used)

Calculate model output using fuzzy inference engine

Calculate the fitness value (MSE)}

Step4. Perform the three-step procedure of ABC optimization as described in [11].

#### IV. EXPERIMENTS AND RESULTS

In this Section, validation of the proposed ABC-based fuzzy modeling methodology is illustrated on nonlinear system identification problems. The proposed ABC and ABC-LS based fuzzy modeling strategies are applied to the Box-Jenkins gas furnace benchmark process and a nonlinear plant modeling problem. The performances of both strategies are compared to other meta-heuristic algorithms, namely PSO, CRPSO, GA and DE.

##### A. Identification of the Box-Jenkins Gas Furnace

The well-known Box-Jenkins gas furnace benchmark is widely used to validate the approximation ability of modeling methods. The data set, which was recorded from a combustion process of a methane-air mixture, is composed of  $N = 296$  pairs of input-output measurements of  $\text{CO}_2$  concentration  $y(k)$ , and methane flow rate  $u(k)$ . Several model structures have been considered in previous studies to model the dynamic behavior of this system. In this study, the following target structure is used:

$$y(k) = f(y(k-1), u(k-4)) \quad (7)$$

The parameters of ABC algorithm are initialized as follows: the bee colony is equally partitioned into employed bees and onlooker bees. The number of employed bees is set to 30, the limit parameter  $limit = 200$ , and the maximum number of cycles is 2000. The experiment is repeated 50 times, and the average MSE is taken as the performance index. The resulting means and standard deviations of MSEs are listed in Table I.

TABLE I  
COMPARISON RESULTS FOR BOX-JENKINS GAS FURNACE

Method	Rules number (mean)	MSE (mean)	MSE (std.)
PSO [5,17]	3.64	0.1550	0.0427
CRPSO [5,17]	3.96	0.1428	0.0138
GA [5,17]	4.32	0.1474	0.0109
DE [5,17]	4.96	0.1768	0.0602
VGA-FCM [17]	3.74	0.1515	0.0312
VABC-FCM [17]	4	0.1256	0.77e-5
Xu and Yong [18]	25	0.3280	-
Chen et al. [19]	3	0.2678	-
Li et al. [20]	4	0.4260	-
Our method (ABC)	4.08	<b>0.1160</b>	0.0312
Our method (ABC-LS)	4.22	<b>0.0734</b>	<b>4.51e-4</b>

As can be clearly seen, ABC and ABC-LS results outperform the results obtained by other advanced methods such as PSO, CRPSO, GA, DE, VABC-FCM, etc. The resulting optimal TS fuzzy model determined by ABC consists of six rules with an average approximation error  $MSE = 0.0789$ . This result is slightly improved through ABC-LS which gives an optimal fuzzy model composed of five rules with  $MSE = 0.0725$ .

Moreover, the best fuzzy model identified by VABC-FCM algorithm has four rules with  $MSE = 0.125547$ , while the model found by CRPSO has three rules with  $MSE = 0.1275$ . Thus, we can conclude that the ABC based modeling methodology can achieve much better modeling accuracy with appropriate number of rules.

TABLE II  
COMPARISON RESULTS FOR NONLINEAR PLANT MODELING

Method	Rules number (mean)	MSE (training)		MSE (testing)	
		Mean	Std.	Mean	Std.
PSO [5]	5.64	0.0055	0.0055	0.0063	0.0072
CRPSO [5]	5.60	0.0020	0.0016	0.0035	0.0044
GA [5]	3.62	0.0038	0.0053	0.0045	0.0051
DE [5]	5.32	0.0081	0.0041	0.0086	0.0062
Our method (ABC)	4.98	<b>0.0014</b>	0.0005	<b>0.0019</b>	0.0012
Our method (ABC-LS)	4.88	<b>4.27e-4</b>	<b>1.17e-4</b>	<b>3.17e-4</b>	<b>8.77e-5</b>

##### B. Nonlinear Plant Modeling Problem

The nonlinear dynamic plant is described by a second order nonlinear difference equation:

$$y(k) = g(y(k-1), y(k-2)) + u(k) \quad (8)$$

where

$$g(y(k-1), y(k-2)) = \frac{y(k-1)y(k-2)(y(k-1)-0.5)}{1 + y^2(k-1) + y^2(k-2)} \quad (9)$$

The aim is to approximate the unforced system  $g(\cdot)$  by using a TS fuzzy model. For this purpose, 400 simulated data points were generated from the nonlinear plant model (8). The first 200 training data points are obtained using a random input signal uniformly distributed in  $[-1.5, 1.5]$ . The last 200 samples of validation data are obtained using a sinusoidal input signal  $u(k) = \sin(2\pi k/25)$ . As in several studies,  $y(k-1)$  and  $y(k-2)$  are selected as input variables.

The ABC algorithm employs the same parameters as that used in Section IV A. The results averaged over 50 runs in terms of MSEs and standard deviations are shown and compared with different modeling methods in Table II. As can be noticed, ABC and ABC-LS based modeling algorithms perform considerably well on both training and testing data in comparison with PSO, CRPSO, GA and DE. The average MSE over 50 trials found by using ABC based fuzzy modeling strategy is 0.0014, which gives an optimal fuzzy model of six rules with a training MSE of  $3.08 \times 10^{-4}$ , while ABC-LS fuzzy modeling algorithm improved considerably this result with an average MSE =  $4.27 \times 10^{-4}$  and a best model composed of five fuzzy rules with a training MSE of  $1.92 \times 10^{-4}$ . These results are to be compared to those obtained by CRPSO for instance, which determined an optimal fuzzy model with a training MSE =  $2.63 \times 10^{-4}$ . Thus, we can say that the ABC-based modeling strategies can construct enhanced fuzzy models with good approximation and generalization abilities.

## V. CONCLUSION

This paper addresses the problem of automatic fuzzy rule generation for fuzzy systems design by using artificial bee colony optimization method. The impressive features of ABC model are exploited to develop a hybrid ABC and least squares algorithm for learning rule-based models from data. The adopted encoding scheme was efficient enough to guarantee the simultaneous evolution of the whole model parameters within the specified search space. Experiments conducted on benchmark modeling problems showed that the results obtained by the proposed ABC and ABC-LS based fuzzy modeling strategies outperform other existing advanced modeling methods. Enhanced fuzzy models are thus identified with superior approximation and generalization capabilities. Future works might investigate other aspects related to data-driven fuzzy modeling, namely the interpretability of the identified fuzzy models.

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