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Performance of Nakagami Fading Channel over Energy Detection Based Spectrum Sensing

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***Abstract*—**Spectrum sensing is the main feature of cognitive radio technology. Spectrum sensing gives an idea of detecting the presence of the primary users in a licensed spectrum. In this paper we compare the theoretical results of detection probability of different fading environments like Rayleigh, Rician, Nakagami-*m* fading channels with the simulation results using energy detection based spectrum sensing. The numerical results are plotted as *Pf* Vs *Pd* for different SNR values, fading parameters. It is observed that Nakagami fading channel performance is better than other fading channels by using energy detection in spectrum sensing. A MATLAB simulation test bench has been implemented to know the performance of energy detection in different fading channel environment.

***Keywords*—**Spectrum sensing, Energy detection, fading channels, Probability of detection, probability of false alarm.

I. Introduction

COGNITIVE radio (CR) technique has been initiated to exploit the spectrum holes when the spectrum is under utilization [1]. It allows the secondary users called CR users to access the spectrum along with primary users (PU) in opportunistic manner. The CR users can able to access the required spectrum only when CR users do not cause interference to the primary users. Therefore, Continuous spectrum sensing is important task in Cognitive Radio technology in order to know an existence of primary users accurately. In many communication applications, when the primary signal from primary transmitter is unknown to us, then it is important to know the active communication link between transmitter and receiver.

In such cases for the detection of primary signal that is coming from primary transmitter, we will use an energy detector. It measures the energy in the received signal over the fixed duration of time interval. This concept was earlier studied by Urkowitz [2] in his research paper, he assumed that, deterministic signals are passed through a band-limited Gaussian noise channel. Urkowitz derived theoretical expressions for the probability of detection (P_d) and the probability of false alarm (P_f). Spectrum sensing performance can be degraded by shadowing, multi path fading [3].

We have developed a simulation model using MATLAB for energy detection based spectrum sensing. The results are obtained from simulation test bench, we are comparing with the theoretical results. Our simulation results are perfectly matched with the theoretical results for different fading environments. We have considered different cases like SNR values, fading parameters, for all the cases of our simulation results are perfectly matched with the theoretical results and that can shown in section-IV. The analytical expressions for the probability of detection in various fading channels (Rayleigh, Rician, & Nakagami) was given in [4], [5].

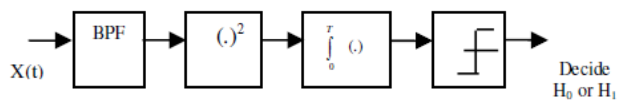

Fig. 1 Block diagram of Energy detection

Whenever the signal from the primary transmitter is unknown to us, we will use energy detector to detect that signal. Energy detector is non-coherent detector. The above Fig. 1 represents the block diagram of energy detector. It can be described as, the received signal is passed through the band pass filter and this filter selects the required frequency as f_c. Output of band pass filter is passed through the non-linear device called squaring device, it will measure the energy associated with the signal.

Non linear device output is passed through an integrator, it will measure energy over the fixed duration of time window. After this block the energy y(t) of the signal, is Compared with the threshold value (λ) to decide whether the primary signal is present or not. The sensing performance is degraded when the CR user effected by shadowing and fading even though it uses energy detector. The rest of the paper is organized as follows. In section II system model and important notations are described. Section III gives an idea about different fading channels and their calculations for detection probabilities. In Section IV we are given Results, its explanations and in Section V consist of conclusions.

II. System Model

Spectrum sensing is key task in cognitive radio technology. In order to assign CR users to the vacant bands in the spectrum, CR users should sense the spectrum continuously. Three different detection techniques have been introduced in Spectrum sensing to know the vacancy bands [6]: Energy detection, Matched filter detection and Cyclostationary detection. Out of all three, energy detection has been widely used for detecting a signal since it does not require any a priori idea of primary signals and has much lower complexity than the other two schemes.

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The received signal in narrowband energy detection follows a two hypothesis can be shown as below [7, (1)],

$$\mathrm{r(t)} = \begin{cases} n(t) & H_0 \\ h * s(t) + n(t) & H_1 \end{cases} \quad (1)$$

where *r(t)* is the signal received by secondary user and *s(t)* is primary user's transmitted signal, *n(t)* is the Additive White Gaussian Noise (AWGN), and h is the amplitude gain of the channel, h is different for different fading environments and it is equal to one when there is no fading occurs i.e. for AWGN fading channel.

The samples of *n(t)* are assumed to be zero-mean Gaussian random variables with variance N_0W where W and N_0 denoting the single-sided signal bandwidth and a single-sided noise power spectral density, respectively [7].

H0. Primary user is absent

H1. Primary user is present

The received signal is first pre-filtered by an ideal band pass filter with transfer function.

$$H(f) = \begin{cases} \frac{2}{\sqrt{N_{01}}} & |f - f_c| \le W \\ 0 & |f - f_c| > W \end{cases} \quad (2)$$

the received signal is passed through the following blocks as shown in the block diagram Fig. 1, filter, squaring and integrating over the time interval T which is given by [4, (2)]. The output of the integrator block measures the energy in the received waveform. The test statistic determines whether the received energy measure corresponds only to the noise energy (H_0) or to the energy of both the unknown deterministic signal and noise (H_1). These H_0/H_1 are the output of threshold detector.

$$Y \sim \begin{cases} {\chi^2}_{2u}, & H_0 \\ {\chi^2}_{2u}(2\gamma), & H_1 \end{cases} \quad (3)$$

The corresponding probability density function (PDF) in the presence of AWGN is expressed according to [4, (3)], namely,

$$f_Y(y) = \begin{cases} \frac{1}{2^u\Gamma(u)} y^{u-1} e^{-\frac{y}{2}}, & H_0 \\ \frac{1}{2}(\frac{y}{2\gamma})^{\frac{u-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{u-1}(\sqrt{2\gamma y}), & H_1 \end{cases} \quad (4)$$

where Γ (.) is the gamma function [8, section 8.31] and I_u (.) is the u^{th}- order modified Bessel function of the first kind [10, section 8.43].

An energy detector is mainly consisting of a predefined energy threshold, λ. This threshold value is important to measures three factors to evaluate the performance of the detector: i) the probability of false alarm, ii) the probability of detection and iii) the probability of missed detection, $P_m = 1 - P_d$. The first two factors are calculated by integrating (2) over the interval between the energy threshold to infinity, {λ, ∞}, yielding [4],

$$P_f = \Pr(\mathrm{y} > \lambda \,|H_0) \quad (5)$$

$$P_d = \Pr(\mathrm{y} > \lambda \mid H_1) \quad (6)$$

III. Average Detection Probability over Different Fading Channels

A. Non-Fading Channel (AWGN Channel)

When the signal is free from fading effect or shadowing effect, consider the fading parameter (*h*) is equal to one. For AWGN the detection probability and false alarm probability are given by the following formulae

$$P_d = P(Y > \lambda | H_1) = Q_m(\sqrt{2\gamma}, \lambda) \quad (7)$$

$$P_f = P(Y > \lambda | H_0) = \frac{\Gamma(m, \frac{\lambda}{2})}{\Gamma(m)} \quad (8)$$

where $\Gamma(a, b)$ is the incomplete gamma function [8] and $Q_m(a,\ b)$ is the generalized Marcum *Q*-function [9]. First we fix the false alarm probability P_f, then by using (8), the detection threshold λ value can be determined. When we are choosing the detection threshold λ value, the detection probability should be chosen as high enough to protect the PUs. Otherwise, a high probability of misdetection may result in intolerable interference to the PUs. The probability of false alarm is used to find the spectrum holes, always choose a low value of false alarm probability because a high value of false alarm will lead to low utilization of the spectrum holes. In this case, the average probability of detection (P_d) may be derived by averaging (7) over fading statistics [9].

$$P_d = \int Q_m(\sqrt{2\gamma}, \lambda) f_y(x)\, dx \quad (9)$$

where $f_y(x)$ is the probability distribution function (PDF) of SNR under fading.

B. Rayleigh Fading Channel

If the environment consists of some types of scattering, large number of plane waves then the signal amplitude follows a Rayleigh distribution, and then the SNR γ follows an exponential PDF given by [4].

$$f_y(x) = \frac{1}{\bar{\gamma}} \exp(-\frac{\gamma}{\bar{\gamma}}), \gamma \ge 0 \quad (10)$$

In this case, a closed-form formula for P_d may be obtained (after some manipulation) by substituting $f_\gamma(x)$ in (9), here $f_y(x) = f(\gamma)$.

$$\bar{P}_{dRay} = e^{\frac{-\lambda}{2}} \sum_0^{m-2} \frac{1}{k!}\left(\frac{\lambda}{2}\right)^k + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{m-1} \times \left(e^{\frac{-\lambda}{2(1+\bar{\gamma})}} - e^{\frac{-\lambda}{2}} \sum_0^{m-2} \frac{1}{k!}(\frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})})^k\right) \quad (11)$$

C. Rician Fading Channel

Some types of scattering environments have a specular or LoS (Line of Sight) component. In this case, the amplitude of received signals has a Rician distribution. The PDF of will be

$$f_y(x) = \frac{K+1}{\bar{\gamma}} \exp\left(-K \frac{(K+1)\gamma}{\bar{\gamma}}\right) * I_0\left(2\sqrt{\frac{K(K+1)}{\bar{\gamma}}}\right), \ \gamma \ge 0 \quad (12)$$

where K is the Rician factor. The average detection probability expression can be calculated by averaging (7) over (14) and substituting x for $\sqrt{2\gamma}$ in the case of a Rician channel, $\bar{P}_{dRic}$. The resulting expression can be solved for $u = 1$ using [11, (45)] to yield, here $f_{\gamma}(x) = f(\gamma)$.

$$\bar{P}_{dRic}|_{u=1} = Q\left(\sqrt{\frac{2K\bar{\gamma}}{k+1+\bar{\gamma}}}, \sqrt{\frac{\lambda(K+1)}{k+1+\bar{\gamma}}}\right) \quad (13)$$

if we substitute $K = 0$, in the above expression it reduces to the Rayleigh expression with $u = 1$.

D. Nakagami Fading Channel

If the signal amplitude follows a Nakagami distribution, then the PDF of γ follows a gamma PDF given by,

$$f(\gamma) = \frac{1}{\Gamma(M)}\left(\frac{M}{\bar{\gamma}}\right)^{M} \gamma^{M-1}\exp\left(-\frac{M}{\bar{\gamma}}\gamma\right), \gamma \geq 0 \quad (14)$$

where M is the Nakagami parameter. The average detection probability equation for Nakagami channels is given by

$$\bar{P}_{dNak} = \alpha\left[G_1 + \beta \sum_{n=1}^{m-1} \frac{\left(\frac{\lambda}{2}\right)^n}{2n!} {}_1F_1\left(M; n+1; \frac{\lambda}{2}\frac{\bar{\gamma}}{M+\bar{\gamma}}\right)\right] \quad (15)$$

where ${}_1F_1(.;.;.)$ is the confluent hypergeometric function [8, section 9.2].

$$\alpha = \frac{1}{\Gamma(M)2^{M-1}}\left(\frac{M}{\bar{\gamma}}\right)^{M} \quad (16)$$

$$\beta = \Gamma(M)\left(\frac{2\bar{\gamma}}{M+\bar{\gamma}}\right)^{M} e^{-\lambda/2} \quad (17)$$

$$G_1 = \frac{2^{M-1}(M-1)!}{\left(\frac{M}{\bar{\gamma}}\right)^{M}} \frac{\bar{\gamma}}{M+\bar{\gamma}} e^{\frac{-\lambda}{2}\frac{M}{M+\bar{\gamma}}}$$

$$\left[\left(1+\frac{M}{\bar{\gamma}}\right)\left(\frac{M}{M+\bar{\gamma}}\right)^{M-1} \times L_{M-1}\left(-\frac{\lambda}{2}\frac{\bar{\gamma}}{M+\bar{\gamma}}\right) + \sum_{0}^{M-2}\left(\frac{M}{M+\bar{\gamma}}\right)^{n} L_n\left(-\frac{\lambda}{2}\frac{\bar{\gamma}}{M+\bar{\gamma}}\right)\right] \quad (18)$$

where $L_n(.)$ is the Laguerre polynomial of degree (n) [6, 8.970].

IV. Results and Discussions

Simulation was performed on MATLAB over different fading channels and what interests us in this simulation, is the receiver performance. The receiver operating characteristic (ROC) curves ($P_f\, Vs\, P_d$) of the energy detection for one cognitive radio are plotted for different SNR values according to (14) and (15).

Figs. 2-4 show the ROC curves for different fading channels like AWGN, Rayleigh and Rician fading scenarios. From Fig. 3, we can tell that the Rayleigh fading performance degrades significantly when it uses energy detector under fading conditions for different SNRs. By observing the graphs from Figs. 2-5, it is clear that detection probability is less in Rayleigh fading when compared to AWGN and Rician fading. This performance indicates that, spectrum utilization is less when fading is considered. Theoretical results which are obtained from (14) are perfectly matched with our simulation results.

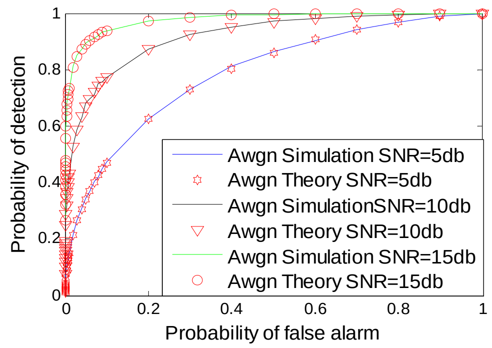

Fig. 2 ROC ($P_f\, Vs\, P_d$) under AWGN for different SNRs and (m=5)

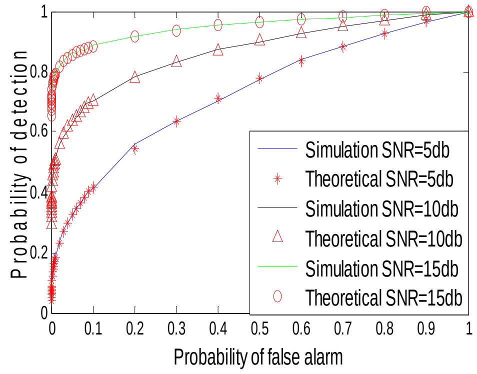

Fig. 3 ROC ($P_f\, Vs\, P_d$) under Rayleigh fading for different SNRs and (m=5)

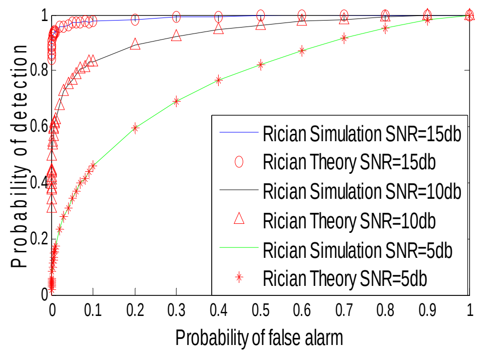

Fig. 4 ROC ($P_f\, Vs\, P_d$) under Rician fading for different SNRs and (m=5)

Figs. 5 and 6 show receiver operating curves ($P_f Vs\, P_d$) which are drawn for Rician and Nakagami fading channels. To know the performance of these two fading channels we have chosen different values of Rician factor, K=2, K=4 and K= 6 and Nakagami parameters [10], M=1, M=2, and M=3 are considered. For M=1, Nakagami fading channel performance is similar to the Rayleigh fading channel. Increase in Rician factor K=4, to K=6 and Nakagami parameters M=1 to M=3, significantly increase the probability of detection. From the Figs. 5 and 6, we can able to conclude that the performance of energy detector in Nakagami fading channel (particularly for

M=3) is better than the performance of Rayleigh fading channel (M=1) and Rician fading channel (K=4 to 6).

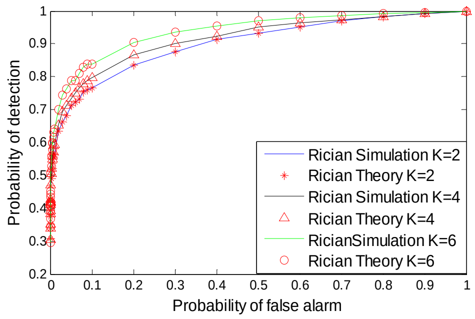

Fig. 5 ROC (P_f Vs P_d) under Rician fading for different Rician parameter (K) and (m=5)

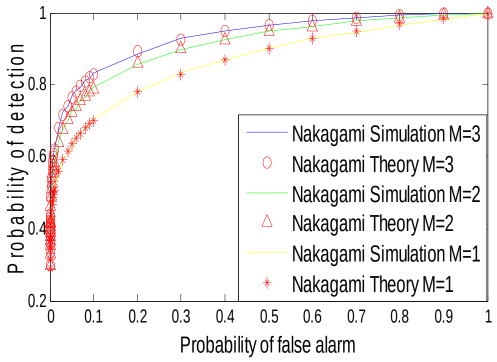

Fig. 6 ROC (P_f Vs P_d) under Nakagami fading for different Nakagami parameter (M) and (m=5)

Fig. 6, the effect of Nakagami parameter (M) on the P_d of single CR user based spectrum sensing is shown. As the Nakagami parameter m increases from 1 to 3, the probability of detection P_d increases which is intuitively true as higher M indicates a less sever.

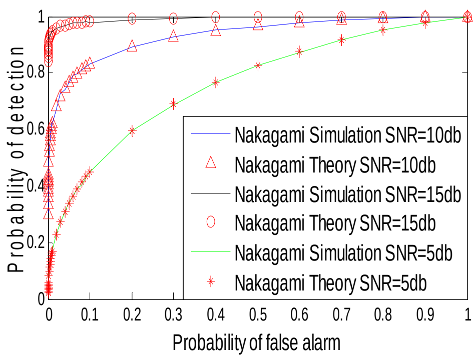

Fig. 7 ROC (P_f Vs P_d) under Nakagami fading for different SNRs and (m=5)

Fig. 7 describes the Nakagami ROC graph for different SNR values. By observing the graph the detection probability is more in Nakagami fading than other fading channels. The fading effect is more in Rician fading channel when compared to the Nakagami fading channel.

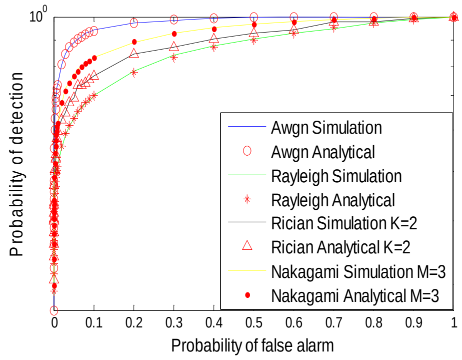

Fig. 8 ROC (P_f Vs P_d) under different fading channels for different fading parameters

Fig. 8 shows ROC curves for spectrum sensing with single CR user in the presence of Rayleigh, Rician and Nakagami-m fading channels. K = 2, and M = 3 are chosen as fading parameters of their respective fading channels called Rician, and Nakagami-m [12] respectively. A non fading channel (AWGN) curves also provided to compare with the other fading channels. From the above Fig. 8, we can state that spectrum sensing performance degrades in the presence of fading. Nakagami fading channel performance is good among all fading channels while detection is performed over energy detector.

V. Conclusion

In this paper we investigated the performance of energy detection over different fading channels. Among all these fading channels (Rayleigh, Rician, Nakagami) [12] it is observed that the detection probability performance of Nakagami fading channel is better under fading conditions. We have provided analytical expressions of detection probability for all above mentioned fading channels, also compared theoretical results with simulation test bed on using MATLAB. The overall performance of the detector is affected by the value of the involved parameters like fading parameters and SNR values. This is evident by the fact that even slight variations of the fading parameter results to substantial variation of the corresponding probability of detection.

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