

Intuitionistic T-S Fuzzy Subalgebras and Ideals in BCI-algebras

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Abstract—The aim of this paper is to introduce the notions of intuitionistic T-S fuzzy subalgebras and intuitionistic T-S fuzzy ideals in BCI-algebras, and then to investigate their basic properties.

Keywords—BCI-algebra, intuitionistic T-S fuzzy subalgebra, intuitionistic T-S fuzzy ideal, norm intersection, direct product, epimorphism, isomorphism.

I. INTRODUCTION

A BCI-algebra is an important class of logical algebras, the theory of BCI-algebras introduced by Iséki [1] has been studied deep by several researchers so far. Xi [2] introduced the concepts of fuzzy subalgebras and ideas in BCI-algebras and discussed some properties of them. On the other hand, triangular norm is a powerful tool in the theory research and application development of fuzzy sets. On the basis of the definition of the intuitionistic fuzzy groups, Li [4], [5] generalized the operators “ \wedge ” and “ \vee ” to T-norm and S-norm and defined the intuitionistic fuzzy groups of T-S norms. In this paper, the concepts of intuitionistic T-S fuzzy subalgebras and intuitionistic T-S fuzzy ideas are introduced in BCI-algebras. Some properties are discussed. We prove that the norm intersection and direct product of two intuitionistic T-S fuzzy subalgebras (ideals) are also intuitionistic T-S fuzzy subalgebras (ideals) in BCI-algebras.

II. PRELIMINARIES

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following axioms:

$$(BCI-1) ((x * y) * (x * z)) * (z * y) = 0,$$

$$(BCI-2) (x * (x * y)) * y = 0,$$

$$(BCI-3) x * x = 0,$$

$$(BCI-4) x * y = 0 \text{ and } y * x = 0 \text{ imply } x = y,$$

for all $x, y, z \in X$. In a BCI-algebra X , we can define a partial ordering \leq by putting $x \leq y$ if and only if $x * y = 0$.

In this paper, X always means a BCI-algebra unless otherwise specified.

Definition 1 [3] Let S be any set. An intuitionistic fuzzy subset A of S is an object of the following form

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$$A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in S \right\}, \text{ where } \mu_A : S \rightarrow [0, 1]$$

and $\nu_A : S \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in S$ respectively and for every $x \in S$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

The all intuitionistic fuzzy subsets of S be denote $IFS[S]$.

Definition 2 [4] Mapping $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called as triangular norm, if T satisfies:

$$(T-1) T(0, 0) = 0, T(1, 1) = 1;$$

$$(T-2) T(a, b) \geq T(c, d) \text{ if } a \geq c, b \geq d;$$

$$(T-3) T(a, b) = T(b, a);$$

$$(T-4) T(a, T(b, c)) = T(T(a, b), c).$$

When the triangular T satisfies $T(a, 1) = a$, it is called as T-norm; When T satisfies $T(a, 0) = a$, it is called as S-norm.

Specifically, let $T_0 = x \wedge y$ and $S_0 = x \vee y$ for all $x, y \in [0, 1]$, we have $T \leq T_0 \leq S_0 \leq S$.

Definition 3 [4] Let $x^c = 1 - x$ for all $x \in [0, 1]$, we say that x^c is a complement of x . Given T-norm $T(x, y)$, let $S(x, y) = 1 - T(1 - x, 1 - y) = (T(x^c, y^c))^c$, obviously, $S(x, y)$ is an S-norm. We say that S and T are dual.

Definition 4 [7] Let X, Y be two nonempty classical sets and $A \in IFS[X], B \in IFS[Y]$. Let the mapping $f : X \rightarrow Y$, then the mapping f can induce a mapping F_f from $IFS[X]$ to $IFS[Y]$ and F_f^{-1} from $IFS[Y]$ to $IFS[X]$,

$$F_f(A) = \left\{ \langle y, \mu_{F_f(A)}(y), \nu_{F_f(A)}(y) \rangle : y \in Y \right\},$$

$$F_f^{-1}(B) = \left\{ \langle x, \mu_{F_f^{-1}(B)}(x), \nu_{F_f^{-1}(B)}(x) \rangle : x \in X \right\},$$

where

$$\mu_{F_f(A)}(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & f^{-1}(y) \neq \emptyset \\ 0, & f^{-1}(y) = \emptyset \end{cases},$$

$$\nu_{F_f(A)}(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x), & f^{-1}(y) \neq \emptyset \\ 1, & f^{-1}(y) = \emptyset \end{cases},$$

$$\mu_{F_f^{-1}(B)}(x) = \mu_B(f(x)), \nu_{F_f^{-1}(B)}(x) = \nu_B(f(x)).$$

Proposition 1 [4] Let T be a T-norm and S be an S-norm, for all T and S , if $a, b, c, d \in [0, 1]$, then

$$(1) T(T(a, b), T(c, d)) = T(T(a, c), T(b, d)) \\ = T(T(a, d), T(b, c));$$

$$(2) S(S(a, b), S(c, d)) = S(S(a, c), S(b, d)) \\ = S(S(a, d), S(b, c)).$$

III. INTUITIONISTIC T-S FUZZY SUBALGEBRAS IN BCI-ALGEBRAS

Definition 5 Let T be a T-norm, S be an S-norm and S and T be dual norm. An intuitionistic fuzzy set A in X is called an intuitionistic T-S fuzzy subalgebra of X if the following are satisfied:

$$\mu_A(x * y) \geq T(\mu_A(x), \mu_A(y)),$$

$$\nu_A(x * y) \leq S(\nu_A(x), \nu_A(y)),$$

for all $x, y \in X$.

Proposition 2 Let T be a T-norm, S be an S-norm and S and T be dual norm. Let $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an intuitionistic T-S fuzzy subalgebra of X , then

$$\square A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \},$$

$$\diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle : x \in X \}$$

are both intuitionistic T-S fuzzy subalgebras of X .

Proof. Denote $\omega_A(x) = 1 - \mu_A(x)$, for all $x, y \in X$, we have

$$\omega_A(x * y) = 1 - \mu_A(x * y) \\ \leq 1 - T(\mu_A(x), \mu_A(y)) \\ = (T(\mu_A(x), \mu_A(y)))^c \\ = S((\mu_A(x))^c, (\mu_A(y))^c) \\ = S(1 - \mu_A(x), 1 - \mu_A(y)) \\ = S(\omega_A(x), \omega_A(y)).$$

Thus

$\square A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$ is an intuitionistic T-S fuzzy subalgebra of X .

Denote $\gamma_A(x) = 1 - \nu_A(x)$, for all $x, y \in X$, we have

$$\gamma_A(x * y) = 1 - \nu_A(x * y) \\ \geq 1 - S(\nu_A(x), \nu_A(y)) \\ = (S(\nu_A(x), \nu_A(y)))^c \\ = T((\nu_A(x))^c, (\nu_A(y))^c) \\ = T(1 - \nu_A(x), 1 - \nu_A(y))$$

$$= T(\gamma_A(x), \gamma_A(y)).$$

Thus

$\diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle : x \in X \}$ is also an intuitionistic T-S fuzzy subalgebra of X .

Definition 6 [8] Let X be any set, $A \in IFS[X]$, for all $\lambda \in [0, 1]$, $\lambda A = \{ \langle x, \lambda \mu_A(x), \lambda \nu_A(x) \rangle : x \in X \}$,

where

$$\lambda \mu_A(x) = \begin{cases} \mu_A(x), \lambda \geq \mu_A(x) \\ \lambda, \lambda < \mu_A(x) \end{cases}, \quad \lambda \nu_A(x) = \begin{cases} \nu_A(x), \lambda \geq \nu_A(x) \\ \lambda, \lambda < \nu_A(x) \end{cases},$$

λA is called as cutproduct of λ and A .

Proposition 3 Let A be an intuitionistic T-S fuzzy subalgebra of X , then for any $\lambda \in [0, 1]$, λA is also an intuitionistic T-S fuzzy subalgebra of X .

Proof. It is clear that $\lambda A \in IFS[X]$. In the following we need to verify that λA satisfies the conditions of Definition 5.

If $\lambda \geq \mu_A(x * y)$, then

$$\lambda \mu_A(x * y) = \mu_A(x * y) \\ \geq T(\mu_A(x), \mu_A(y)) \\ \geq T(\lambda \mu_A(x), \lambda \mu_A(y));$$

If $\lambda < \mu_A(x * y)$, then

$$\lambda \mu_A(x * y) = \lambda = T(\lambda, 1) \geq T(\lambda \mu_A(x), \lambda \mu_A(y));$$

hence we can obtain $\lambda \mu_A(x * y) \geq T(\lambda \mu_A(x), \lambda \mu_A(y))$ for any $\lambda \in [0, 1]$.

If $\lambda \geq \nu_A(x * y)$ and $\lambda \geq \max\{\nu_A(x), \nu_A(y)\}$, then

$$\lambda \nu_A(x * y) = \nu_A(x * y) \\ \leq S(\nu_A(x), \nu_A(y)) \\ = S(\lambda \nu_A(x), \lambda \nu_A(y));$$

If $\lambda \geq \nu_A(x * y)$ and $\lambda \leq \max\{\nu_A(x), \nu_A(y)\}$, then

$$S(\lambda \nu_A(x), \lambda \nu_A(y)) \geq S(\lambda, 0) = S(0, \lambda) = \lambda, \\ \lambda \nu_A(x * y) = \nu_A(x * y) \leq \lambda \leq S(\lambda \nu_A(x), \lambda \nu_A(y));$$

If $\lambda < \nu_A(x * y)$ and $\lambda \geq \max\{\nu_A(x), \nu_A(y)\}$, then

$$\lambda \nu_A(x * y) = \lambda \\ < \nu_A(x * y) \\ \leq S(\nu_A(x), \nu_A(y)) \\ = S(\lambda \nu_A(x), \lambda \nu_A(y));$$

If $\lambda < \nu_A(x * y)$ and $\lambda \leq \max\{\nu_A(x), \nu_A(y)\}$, then

$$S(\lambda \nu_A(x), \lambda \nu_A(y)) \geq S(\lambda, 0) = S(0, \lambda) = \lambda, \\ \lambda \nu_A(x * y) = \lambda \leq S(\lambda \nu_A(x), \lambda \nu_A(y));$$

Summarizing the above arguments, λA is an intuitionistic T-S fuzzy subalgebra of X .

Definition 7 [6] If $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in S \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in S \}$ be any two intuitionistic fuzzy subsets of a set S , then

$$A \cap B = \{ \langle x, \mu_{A \cap B}(x), \nu_{A \cap B}(x) \rangle : x \in S \} \\ = \{ \langle x, T(\mu_A(x), \mu_B(x)), S(\nu_A(x), \nu_B(x)) \rangle : x \in S \}.$$

Proposition 4 Let A and B be two intuitionistic T-S fuzzy subalgebras of X , then $A \cap B$ is also an intuitionistic T-S fuzzy subalgebra of X .

Proof. For all $x, y \in X$. By Definition 5, Definition 7 and Proposition 1, we have

$$\mu_{A \cap B}(x * y) = T(\mu_A(x * y), \mu_B(x * y)) \\ \geq T(T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y))) \\ = T(T(\mu_A(x), \mu_B(x)), T(\mu_A(y), \mu_B(y))) \\ = T(\mu_{A \cap B}(x), \mu_{A \cap B}(y)),$$

$$\nu_{A \cap B}(x * y) = S(\nu_A(x * y), \nu_B(x * y)) \\ \leq S(S(\nu_A(x), \nu_A(y)), S(\nu_B(x), \nu_B(y))) \\ = S(S(\nu_A(x), \nu_B(x)), S(\nu_A(y), \nu_B(y))) \\ = S(\nu_{A \cap B}(x), \nu_{A \cap B}(y)).$$

Hence $A \cap B$ is an intuitionistic T-S fuzzy subalgebra of X .

Definition 8 [6] Let A and B be two intuitionistic fuzzy sets of a set X .

The Cartesian product of A and B is defined by $A \times B = \{ \langle x, \mu_{A \times B}(x), \nu_{A \times B}(x) \rangle : x = (x_1, x_2) \in X \times X \}$,

where

$$\mu_{A \times B}(x) = T(\mu_A(x_1), \mu_B(x_2)), \nu_{A \times B}(x) = S(\nu_A(x_1), \nu_B(x_2)).$$

Proposition 5 Let A and B be two intuitionistic T-S fuzzy subalgebras of X , then $A \times B$ is also an intuitionistic T-S fuzzy subalgebra of $X \times X$.

Proof. For all $x = (x_1, x_2), y = (y_1, y_2) \in X \times X$, by Definition 5, Definition 8 and Proposition 1, we get

$$\mu_{A \times B}(x * y) = \mu_{A \times B}((x_1, x_2) * (y_1, y_2)) \\ = \mu_{A \times B}((x_1 * y_1), (x_2 * y_2)) \\ = T(\mu_A(x_1 * y_1), \mu_B(x_2 * y_2)) \\ \geq T(T(\mu_A(x_1), \mu_A(y_1)), T(\mu_B(x_2), \mu_B(y_2))) \\ = T(T(\mu_A(x_1), \mu_B(x_2)), T(\mu_A(y_1), \mu_B(y_2))) \\ = T(\mu_{A \times B}(x), \mu_{A \times B}(y)),$$

$$\nu_{A \times B}(x * y) = \nu_{A \times B}((x_1, x_2) * (y_1, y_2)) \\ = \nu_{A \times B}((x_1 * y_1), (x_2 * y_2))$$

$$= S(\nu_A(x_1 * y_1), \nu_B(x_2 * y_2)) \\ \leq S(S(\nu_A(x_1), \nu_A(y_1)), S(\nu_B(x_2), \nu_B(y_2))) \\ = S(S(\nu_A(x_1), \nu_B(x_2)), S(\nu_A(y_1), \nu_B(y_2))) \\ = S(\nu_{A \times B}(x), \nu_{A \times B}(y)).$$

Hence $A \times B$ is an intuitionistic T-S fuzzy subalgebra of $X \times X$.

IV. INTUITIONISTIC T-S FUZZY IDEALS IN BCI-ALGEBRAS

Definition 9 Let T be a T-norm, S be an S-norm and S and T be dual norm. An intuitionistic fuzzy set A in X is called an intuitionistic T-S fuzzy ideal of X if the following are satisfied:

$$(F_1) \mu_A(0) \geq \mu_A(x), \\ (F_2) \mu_A(x) \geq T(\mu_A(x * y), \mu_A(y)), \\ (F_3) \nu_A(0) \leq \nu_A(x), \\ (F_4) \nu_A(x) \leq S(\nu_A(x * y), \nu_A(y)),$$

for all $x, y \in X$.

Proposition 6 Let A be an intuitionistic T-S fuzzy ideal of X and $\mu_A(0) = 1, \nu_A(0) = 0$. If $x \leq y$ holds in X , then

$$\mu_A(x) \geq \mu_A(y), \nu_A(x) \leq \nu_A(y).$$

Proof. For all $x, y \in X$, if $x \leq y$, then $x * y = 0$, so by Definition 9,

$$\mu_A(x) \geq T(\mu_A(x * y), \mu_A(y)) \\ = T(\mu_A(0), \mu_A(y)) \\ = \mu_A(y), \\ \nu_A(x) \leq S(\nu_A(x * y), \nu_A(y)) \\ = S(\nu_A(0), \nu_A(y)) \\ = \nu_A(y).$$

Proposition 7 Let A be an intuitionistic T-S fuzzy ideal of X , $\mu_A(0) = 1$ and $\nu_A(0) = 0$. If the inequality $x * y \leq z$ holds in X , then

$$\mu_A(x) \geq T(\mu_A(y), \mu_A(z)), \nu_A(x) \leq S(\nu_A(y), \nu_A(z)).$$

Proof. For all $x, y, z \in X$, if $x * y \leq z$, then

$$\mu_A(x) \geq T(\mu_A(x * y), \mu_A(y)) \\ \geq T(\mu_A(y), \mu_A(z)), \\ \nu_A(x) \leq S(\nu_A(x * y), \nu_A(y)) \\ \leq S(\nu_A(y), \nu_A(z)).$$

Proposition 8 Let $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an intuitionistic T-S fuzzy ideal of X , then

$$\square A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \},$$

$$\diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle : x \in X \}$$

are both intuitionistic T-S fuzzy ideals of X .

Proof. Denote $\omega_A(x) = 1 - \mu_A(x)$, for all $x, y \in X$, we have

$$\begin{aligned} \omega_A(0) &= 1 - \mu_A(0) \\ &\leq 1 - \mu_A(x) \\ &= \omega_A(x), \\ \omega_A(x) &= 1 - \mu_A(x) \\ &\leq 1 - T(\mu_A(x * y), \mu_A(y)) \\ &= (T(\mu_A(x * y), \mu_A(y)))^c \\ &= S((\mu_A(x * y))^c, (\mu_A(y))^c) \\ &= S(1 - \mu_A(x * y), 1 - \mu_A(y)) \\ &= S(\omega_A(x * y), \omega_A(y)). \end{aligned}$$

Thus

$\square A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$ is an intuitionistic T-S fuzzy ideal of X .

Denote $\gamma_A(x) = 1 - \nu_A(x)$, for all $x, y \in X$, we have

$$\begin{aligned} \gamma_A(0) &= 1 - \nu_A(0) \\ &\geq 1 - \nu_A(x) \\ &= \gamma_A(x), \\ \gamma_A(x) &= 1 - \nu_A(x) \\ &\geq 1 - S(\nu_A(x * y), \nu_A(y)) \\ &= (S(\nu_A(x * y), \nu_A(y)))^c \\ &= T((\nu_A(x * y))^c, (\nu_A(y))^c) \\ &= T(1 - \nu_A(x * y), 1 - \nu_A(y)) \\ &= T(\gamma_A(x * y), \gamma_A(y)). \end{aligned}$$

Thus

$$\diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle : x \in X \}$$

is also an intuitionistic T-S fuzzy ideal of X .

Proposition 9 Let A and B be two intuitionistic T-S fuzzy ideals of X , then $A \cap B$ is also an intuitionistic T-S fuzzy ideal of X .

Proof. For all $x, y \in X$. By Definition 7, Definition 9 and Proposition 1, we have

$$\begin{aligned} \mu_{A \cap B}(0) &= T(\mu_A(0), \mu_B(0)) \\ &\geq T(\mu_A(x), \mu_B(x)) \\ &= \mu_{A \cap B}(x), \\ \nu_{A \cap B}(0) &= S(\nu_A(0), \nu_B(0)) \end{aligned}$$

$$\leq S(\nu_A(x), \nu_B(x))$$

$$= \nu_{A \cap B}(x),$$

$$\begin{aligned} \mu_{A \cap B}(x) &= T(\mu_A(x), \mu_B(x)) \\ &\geq T(T(\mu_A(x * y), \mu_A(y)), T(\mu_B(x * y), \mu_B(y))) \\ &= T(T(\mu_A(x * y), \mu_B(x * y)), T(\mu_A(y), \mu_B(y))) \\ &= T(\mu_{A \cap B}(x * y), \mu_{A \cap B}(y)), \end{aligned}$$

$$\begin{aligned} \nu_{A \cap B}(x) &= S(\nu_A(x), \nu_B(x)) \\ &\leq S(S(\nu_A(x * y), \nu_A(y)), S(\nu_B(x * y), \nu_B(y))) \\ &= S(S(\nu_A(x * y), \nu_B(x * y)), S(\nu_A(y), \nu_B(y))) \\ &= S(\nu_{A \cap B}(x * y), \nu_{A \cap B}(y)). \end{aligned}$$

Hence $A \cap B$ is also an intuitionistic T-S fuzzy ideal of X .

Proposition 10 Let A and B be two intuitionistic T-S fuzzy ideals of X , then $A \times B$ is also an intuitionistic T-S fuzzy ideal of $X \times X$.

Proof. For all $x = (x_1, x_2), y = (y_1, y_2) \in X \times X$,

by Definition 8, Definition 9 and Proposition 1, we get

$$\begin{aligned} \mu_{A \times B}(0) &= T(\mu_A(0), \mu_B(0)) \\ &\geq T(\mu_A(x_1), \mu_B(x_2)) \\ &= \mu_{A \times B}(x), \\ \nu_{A \times B}(0) &= S(\nu_A(0), \nu_B(0)) \\ &\leq S(\nu_A(x_1), \nu_B(x_2)) \\ &= \nu_{A \times B}(x), \\ \mu_{A \times B}(x) &= T(\mu_A(x_1), \mu_B(x_2)) \\ &\geq T(T(\mu_A(x_1 * y_1), \mu_A(y_1)), T(\mu_B(x_2 * y_2), \mu_B(y_2))) \\ &= T(T(\mu_A(x_1 * y_1), \mu_B(x_2 * y_2)), T(\mu_A(y_1), \mu_B(y_2))) \\ &= T(\mu_{A \times B}(x * y), \mu_{A \times B}(y)), \\ \nu_{A \times B}(x) &= S(\nu_A(x_1), \nu_B(x_2)) \\ &\leq S(S(\nu_A(x_1 * y_1), \nu_A(y_1)), S(\nu_B(x_2 * y_2), \nu_B(y_2))) \\ &= S(S(\nu_A(x_1 * y_1), \nu_B(x_2 * y_2)), S(\nu_A(y_1), \nu_B(y_2))) \\ &= S(\nu_{A \times B}(x * y), \nu_{A \times B}(y)). \end{aligned}$$

Hence $A \times B$ is an intuitionistic T-S fuzzy ideal of $X \times X$.

Proposition 11 If $f : X \rightarrow X'$ is an epimorphism from X to X' and B is an intuitionistic T-S fuzzy ideal of X' , then $F_f^{-1}(B)$ is an intuitionistic T-S fuzzy ideal of X .

Proof. Let $B = \{ \langle x', \mu_B(x'), \nu_B(x') \rangle : x' \in X' \}$ be an intuitionistic T-S fuzzy ideal of X' , then

$$F_f^{-1}(B) = \{ \langle x, \mu_{F_f^{-1}(B)}(x), \nu_{F_f^{-1}(B)}(x) \rangle : x \in X \}.$$

Since $f: X \rightarrow X'$ is an epimorphism from X to X' , therefore $F_f(0) = 0'$. Let $x', y' \in X'$, then exist $x, y \in X$, such that $x' = F_f(x), y' = F_f(y)$.

Let $F_f^{-1}(x') * F_f^{-1}(y') = \{x * y : x \in F_f^{-1}(x'), y \in F_f^{-1}(y')\}$,

then $F_f^{-1}(x') * F_f^{-1}(y') \subseteq F_f^{-1}(x' * y')$,

we get $F_f(x * y) = x' * y'$.

$$\begin{aligned} \mu_{F_f^{-1}(B)}(0) &= \mu_B(F_f(0)) \\ &= \mu_B(0') \\ &\geq \mu_B(x') \\ &= \mu_{F_f^{-1}(B)}(x), \end{aligned}$$

$$\begin{aligned} \nu_{F_f^{-1}(B)}(0) &= \nu_B(F_f(0)) \\ &= \nu_B(0') \\ &\leq \nu_B(x') \\ &= \nu_{F_f^{-1}(B)}(x), \end{aligned}$$

$$\begin{aligned} \mu_{F_f^{-1}(B)}(x) &= \mu_B(F_f(x)) \\ &= \mu_B(x') \\ &\geq T(\mu_B(x' * y'), \mu_B(y')) \\ &= T(\mu_{F_f^{-1}(B)}(x * y), \mu_{F_f^{-1}(B)}(y)), \end{aligned}$$

$$\begin{aligned} \nu_{F_f^{-1}(B)}(x) &= \nu_B(F_f(x)) \\ &= \nu_B(x') \\ &\leq S(\nu_B(x' * y'), \nu_B(y')) \\ &= S(\nu_{F_f^{-1}(B)}(x * y), \nu_{F_f^{-1}(B)}(y)). \end{aligned}$$

Hence $F_f^{-1}(B)$ is an intuitionistic T-S fuzzy ideal of X .

Proposition 12 If $f: X \rightarrow Y$ is an isomorphism mapping from X to Y and A, B are two intuitionistic T-S fuzzy ideals of X and Y respectively then

(1) $F_f^{-1}(F_f(A)) = A$;

(2) $F_f(F_f^{-1}(B)) = B$.

Proof. (1) Since f is an isomorphism mapping from X to Y , for all $x \in X$, let $y = f(x)$, then $f^{-1}(y) = \{x\}$ and

$$\begin{aligned} \mu_{F_f^{-1}(F_f(A))}(x) &= \mu_{F_f(A)}(f(x)) = \mu_{F_f(A)}(y) \\ &= \sup_{x \in f^{-1}(y)} \mu_A(x) = \mu_A(x). \end{aligned}$$

By the similar proof ways, we can have

$$\nu_{F_f^{-1}(F_f(A))}(x) = \nu_A(x), \text{ for every } x \in X.$$

This implies that $F_f^{-1}(F_f(A)) = A$.

(2) It is clear that $F_f(F_f^{-1}(B))$ is an intuitionistic T-S fuzzy ideal of Y , for all $y \in Y$, we have

$$\mu_{F_f(F_f^{-1}(B))}(y) = \sup_{x \in f^{-1}(y)} \mu_{F_f^{-1}(B)}(x) = \sup_{f(x)=y} \mu_B(f(x)) = \mu_B(y). \text{ By}$$

the similar proof ways, we can have

$$\nu_{F_f(F_f^{-1}(B))}(y) = \nu_B(y), \text{ for every } x \in X.$$

This implies that $F_f(F_f^{-1}(B)) = B$.

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