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Passivity Analysis of Stochastic Neural Networks With Multiple Time Delays

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Abstract—This paper deals with the problem of passivity analysis for stochastic neural networks with leakage, discrete and distributed delays. By using delay partitioning technique, free weighting matrix method and stochastic analysis technique, several sufficient conditions for the passivity of the addressed neural networks are established in terms of linear matrix inequalities (LMIs), in which both the time-delay and its time derivative can be fully considered. A numerical example is given to show the usefulness and effectiveness of the obtained results.

Keywords—Passivity, Stochastic neural networks, Multiple time delays, Linear matrix inequalities (LMIs).

I. Introduction

NEURAL networks have been extensively investigated due to its successful application in various areas such as pattern recognition, combinatorial optimization, smart antenna array and so on [1]-[5]. However, when the system is affected by external disturbances, deterministic neural networks will fail. Stochastic neural networks are usually introduced to describe this kind of practical system. The stochastic affects plays an important role in the analysis of neural networks. So, lots of attention is focused on the analysis the dynamical behavior on stochastic neural networks and many beautiful results have been proposed [6]-[9].

On the other hand, the passivity theory originated from circuit theory and is a important tool for the analysis of nonlinear system [10]. Neural networks are special nonlinear dynamic system, thus, there has been lots of research concerning the passivity analysis of neural networks [11]-[20]. Especially, in [14]-[20], the stochastic affects is considered in system. The passivity analysis for neural networks of neutral type with Markovian jumping parameters and leakage delay was discussed in [13]. In [14]-[15], authors investigated the passivity problem for delayed discrete-time stochastic neural networks, and several delay-dependent criteria for the passivity of delayed discrete-time stochastic neural networks were derived. The authors in [16]-[17] discussed the passivity problem for delayed stochastic neural networks, and given some sufficient conditions on the passivity of stochastic neural networks with time-varying delay. Moreover, the passivity issue for stochastic fuzzy BAM neural networks with time varying delays was considered in [18], by using delay partitioning technique and Lyapunov stability theory, a new set of sufficient conditions were established. In [19], the problem of passivity analysis was investigated for stochastic interval neural networks with interval time-varying delays and Markovian jumping parameters. New delay-dependent sufficient conditions were derived by utilizing the free-weighting matrix method and some stochastic analysis techniques. However, the problem of the passivity analysis for stochastic neural networks with leakage, discrete and distributed delays has not been studied. In general, while signal propagation is sometimes instantaneous and can be modeled as discrete delays. And it may also be distributed during a certain period of time so distributed delays are incorporated into the model [21]. Besides, time delay in the leakage delay has great effect on the dynamics of neural networks because time delay in the stabilizing negative feedback term has a tendency to destabilize a system [22]. So, it is necessary to discuss the passivity problem for stochastic neural networks with leakage, discrete and distributed delays.

Motivated by the above discussions, the passivity problem for stochastic neural networks with leakage, discrete and distributed delays is considered in this paper. Several sufficient criteria for stochastic neural networks with multiple time delays are derived by using delay partitioning technique, free weighting matrix method and stochastic analysis technique. A numerical example is given to illustrate the effectiveness and less conservation of the proposed method.

Notations: Throughout this paper, $P > 0$ means that the matrix P is symmetric positive definite; the superscripts $' - 1'$ and $'T'$ stand for the inverse and transpose of a matrix, respectively; R^n denotes n-dimensional Euclidean space; $R^{m\times n}$ is the set of $m \times n$ real matrices; I denotes the identity matrix with appropriate dimensions; $*$ denotes the symmetric block in symmetric matrix; $\lambda_{max}(\cdot)$ denotes the largest eigenvalue of a given matrix; $tr()$ denotes the trace of a given matrix; $\|\cdot\|$ is the Euclidean norm in R^n; $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

II. Problem statement and preliminaries

Consider the following neural networks with multiple time delays:

$$dx(t) =[-Cx(t-\sigma) + Af(x(t)) + B_1 f(x(t-h(t))$$

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$$+ B_2 \int_{t-d(t)}^{t} f(x(s))ds + u(t)]dt$$
$$+ \delta(t, x(t), x(t-h(t)))d\omega(t), \quad (1)$$

for all $\sigma\rho(t) < 1$, where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the state vector of the network with n neurons; $C = diag\{c_1, c_2, \ldots, c_n\}$ is a diagonal matrix with positive entries $c_i > 0$; the matrices A, B_1 and B_2 are connection weight matrix, discrete connection weight matrix and distributed connection weight matrix, respectively; $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$ denotes the neuron activation function; $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in R^n$ is the external input vector; $\delta \in R^{n\times q}$ is the diffusion coefficient vector; $\omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_q(t)]^T$ is a q-dimensional Brownian motion defined on a complete probability space $(\Omega, F, F_{t\{t\geq 0\}}, P)$ with a filtration $F_{t\{t\geq 0\}}$ satisfying the usual conditions (i.e., it is right continuous and F_0 contains all P-null sets); σ is the leakage delay; $h(t)$ and $d(t)$ is the discrete delays and distributed delays, respectively. Satisfying

$$h_1 \leq h(t) \leq h_2, \quad \dot{h}(t) \leq \mu, \quad 0 \leq d(t) \leq d, \quad (2)$$

where h_1, h_2, μ and d are constant scalars.

The initial condition associated with system (1) is given by

$$x(s) = \varphi(s), \quad s \in [-max\{\sigma, h_2\}, 0],$$

the state trajectory of system (1) from the above initial condition is denoted by $x(t, \varphi)$.

Assumption 1. The neuron activation function $f(\cdot)$ is continuous and bounded, and there exist constants K_i^- and K_i^+ such that

$$K_i^- \leq \frac{f_i(a) - f_i(b)}{a - b} \leq K_i^+, \qquad i = 1, 2, \ldots, n, \quad (3)$$

where $a,\, b \in R$, $a \neq b$.

Remark 1. Assumption 1 on the activation function was firstly proposed in [23]. Obviously, K_i^- and K_i^+ can be positive, negative or zero, this type of activation function is more general than both the usual sigmoid activation function and the piecewise linear function. When $K_i^- = 0$ and $K_i^+ > 0$, assumption 1 describes the monotone nondecreasing activation function. Besides, monotone increasing activation function can be described when $0 < K_i^- < K_i^+$.

Assumption 2. There exist constant matrices M_1 and M_2 of appropriate dimensions such that the following inequality

$$tr(\delta^T(t, u, v)\delta(t, u, v)) \leq \|M_1 u\|^2 + \|M_2 v\|^2, \quad (4)$$

holds for all $(t, u, v) \in R \times R^n \times R^n$.

Before deriving our main results, the following definition and lemma are introduced.

Definition 1. [16]. System (1) is called globally passive in the sense of expectation if there exists a scalar $\gamma > 0$ such that

$$2\mathbf{E}\{\int_0^{t_p} f^T(x(s))u(s)ds\} \geq -\mathbf{E}\{\gamma \int_0^{t_p} u^T(s)u(s)ds\},$$

for all $t_p \geq 0$ and for all $x(t, 0)$.

Lemma 1. [22]. For any constant matrix $W \in R^{m\times m}$, $W > 0$, scalar $0 < h(t) < h$, vector function $\omega : [0, h] \to R^m$ such that the integrations concerned are well defined, then

$$\int_0^{h(t)} \omega^T(s)dsW \int_0^{h(t)} \omega(s)ds \leq h(t) \int_0^{h(t)} \omega^T(s)W\omega(s)ds.$$

Lemma 2. [24]. Let the functions $f_1(t), f_2(t), \ldots, f_N(t) : R^m \to R$ have the positive values in an open subset D of R^m and satisfy $\frac{1}{\alpha_1}f_1(t), \frac{1}{\alpha_2}f_2(t), \ldots, \frac{1}{\alpha_N}f_N(t) : D \to R$ with $\alpha_i > 0$ and $\sum_{i=1}^N \alpha_i = 1$, then the reciprocal technique of $f_i(t)$ over the set D satisfies

$$\sum_i \frac{1}{\alpha_i} f_i(t) \geq \sum_i f_i(t) + \sum_{i\neq j} g_{i,j}(t) \quad \forall g_{i,j}(t) : R^m \to R,$$
$$\begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}^T(t) & f_j(t) \end{bmatrix} \geq 0.$$

Lemma 3. [14]. Let $a, b \in R^n$, P be a positive definite matrix, then $2a^T b \leq a^T P^{-1} a + b^T P b$.

III. Main Results

For the sake of simplicity of matrix representation, $e_i (i = 1, \ldots, 11 + l)$ are defined as block entry matrices. (For example, $e_1 = [I, \underbrace{0, \ldots, 0}_{10+l}]$). The notations for some vectors and matrices are defined as following

$$K_1 = diag\{K_1^- K_1^+, K_2^- K_2^+, \ldots, K_n^- K_n^+\},$$
$$K_2 = diag\{\frac{K_1^- + K_1^+}{2}, \frac{K_2^- + K_2^+}{2}, \ldots, \frac{K_n^- + K_n^+}{2}\},$$
$$e_{R_1} = [\underbrace{I, \ldots, I}_{l}, 0, \ldots, 0], \quad e'_{R_1} = [0, \underbrace{I, \ldots, I}_{l}, 0, \ldots, 0],$$
$$\begin{aligned}
\Pi_1 = {} & e_1^T(-PC - CP + \lambda_{max}(P)M_1^T M_1)e_1 \\
& + e_1^T(PA + A^T P)e_{6+l} + e_1^T(PB_1 + B_1^T P)e_{7+l} \\
& + e_1^T(PB_2 + B_2^T P)e_{8+l} + 2e_1^T Pe_{10+l} + 2e_1^T CPCe_{5+l} \\
& - e_{5+l}^T(CPA + A^T PC)e_{6+l} - 2e_{5+l}^T CPe_{10+l} \\
& - e_{5+l}^T(CPB_1 + B_1^T PC)e_{7+l} \\
& - e_{5+l}^T(CPB_2 + B_2^T PC)e_{8+l} \\
& + e_{2+l}^T(\lambda_{max}(P)M_2^T M_2)e_{2+l},
\end{aligned}$$
$$\Pi_2 = e_1^T(Q_1 + \sigma^2 Q_2)e_1 - e_{4+l}^T Q_1 e_{4+l} - e_{5+l}^T Q_2 e_{5+l},$$
$$\begin{aligned}
\Pi_3 = {} & e_{R_1}^T R_1 e_{R_1} - {e'_{R_1}}^T R_1 e'_{R_1} + \begin{bmatrix} e_1 \\ e_{6+l} \end{bmatrix}^T R_2 \begin{bmatrix} e_1 \\ e_{6+l} \end{bmatrix} \\
& - (1-\mu) \begin{bmatrix} e_{2+l} \\ e_{7+l} \end{bmatrix}^T R_2 \begin{bmatrix} e_2 \\ e_{7+l} \end{bmatrix} + e_1^T R_3 e_1 - e_3^T R_3 e_3,
\end{aligned}$$
$$\begin{aligned}
\Pi_4 = {} & d^2 e_{6+l}^T Te_{6+l} - \begin{bmatrix} e_{8+l} \\ e_{9+l} \end{bmatrix}^T \begin{bmatrix} T & U \\ * & T \end{bmatrix} \begin{bmatrix} e_{8+l} \\ e_{9+l} \end{bmatrix} \\
& + h_1\lambda_{max}(S_1)e_1^T M_1^T M_1 e_1 + (h_2 - h_1)e_{11+l}^T S_4 e_{11+l} \\
& + h_1\lambda_{max}(S_1)e_{2+l}^T M_2^T M_2 e_{2+l} + h_1 e_{11+l}^T S_2 e_{11+l} \\
& + (h_2 - h_1)\lambda_{max}(S_3)e_1^T M_1^T M_1 e_1 \\
& + (h_2 - h_1)\lambda_{max}(S_3)e_{2+l}^T M_2^T M_2 e_{2+l},
\end{aligned}$$
$$\begin{aligned}
\Pi_5 = {} & e_1^T(-N_3 - N_3^T)e_1 + e_{1+l}^T(-N_4 - N_4^T)e_{1+l} \\
& + e_{2+l}^T(-N_5 - N_5^T)e_{2+l} + e_1^T N_3 e_{1+l} + e_{1+l}^T N_4 e_{2+l} \\
& + e_{2+l}^T N_5 e_{3+l},
\end{aligned}$$

$$\begin{aligned}\Pi_6 = {}& [e_{11+l}^T N_1 + e_1^T N_2][-e_{11+l} - Ce_{4+l} + Ae_{6+l} + B_1 e_{7+l} \\ & + B_2 e_{8+l} + e_{10+l}] + [-e_{11+l}^T - e_{4+l}^T C + e_{6+l}^T A^T \\ & + e_{7+l}^T B_1^T + e_{8+l}^T B_2^T + e_{10+l}^T][N_1^T e_{11+l} + N_2^T e_1] \\ & - e_1^T K_1 L_1 e_1 + e_1^T K_2 L_1 e_{6+l} - e_{6+l}^T L_1 e_{6+l} \\ & - e_{2+l}^T K_1 L_2 e_{2+l} + e_{2+l}^T K_2 L_2 e_{7+l} - e_{7+l}^T L_2 e_{7+l} \\ & - 2e_{6+l}^T e_{10+l} - \gamma e_{10+l}^T e_{10+l},\end{aligned}$$

$$\eta(t) = [x^T(t), x^T(t - \frac{h_1}{l}), \ldots, x^T(t - \frac{l-1}{l} h_1)]^T,$$

$$\zeta(t) = [x^T(t), f^T(x(t))]^T, \quad \Omega = \sum_{i=1}^{6} \Pi_i,$$

$$\begin{aligned}y(t) = {}& -Cx(t-\sigma) + Af(x(t)) + B_1 f(x(t-h(t))) \\ & + B_2 \int_{t-d(t)}^{t} f(x(s))ds + u(t),\end{aligned}$$

$$\alpha(t) = \delta(t, x(t), x(t-h(t))),$$

$$\begin{aligned}\xi(t) = {}& [x^T(t), \eta^T(t - \frac{h_1}{l}), x^T(t-h(t)), x^T(t-h_2), \\ & x^T(t-\sigma), \int_{t-\sigma}^{t} x^T(s)ds, f^T(x(t)), f^T(x(t-h(t))), \\ & \int_{t-d(t)}^{t} f^T(x(s))ds, \int_{t-d}^{t-d(t)} f^T(x(s))ds, u^T(t), y^T(t)]^T.\end{aligned}$$

Now, we have the following theorem.

Theorem 1. For given scalars $h_1 > 0$, $h_2 > 0$, $d > 0$, and $\mu > 0$, the stochastic neural network (1) is passive in the sense of expectation if there exist symmetric positive definite matrices P, Q_1, Q_2, R_i $(i = 1,2,3)$, T and S_j $(j = 1,2,\ldots,4)$, the positive diagonal matrices L_1 and L_2, any appropriately dimensional matrices U, N_k $(k = 1,2,\ldots,5)$, a positive constant $\gamma > 0$ such that the following LMIs (5) and (6) hold:

$$\begin{bmatrix} T & U \\ * & T \end{bmatrix} > 0, \tag{5}$$

$$\begin{bmatrix} \Omega & h_1 e_1^T N_3 & e_1^T N_3 & (h_2-h_1)e_{1+l}^T N_4 & e_{1+l}^T N_4 & (h_2-h_1)e_{2+l}^T N_5 & e_{2+l}^T N_5 \\ * & -h_1 S_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -S_1 & 0 & 0 & 0 & 0 \\ * & * & * & -(h_2-h_1)S_4 & 0 & 0 & 0 \\ * & * & * & * & -S_3 & 0 & 0 \\ * & * & * & * & * & -(h_2-h_1)S_4 & 0 \\ * & * & * & * & * & * & -S_3 \end{bmatrix} < 0, \tag{6}$$

Proof: Consider the following Lyapunov-Krasovskii functional:

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{7}$$

where

$$\begin{aligned}V_1(t) = {}& (x(t) - C\int_{t-\sigma}^{t} x(s)ds)^T P(x(t) - C\int_{t-\sigma}^{t} x(s)ds), \\ V_2(t) = {}& \int_{t-\sigma}^{t} x^T(s)Q_1 x(s)ds + \sigma \int_{-\sigma}^{0}\int_{t+\theta}^{t} x^T(s)Q_2 x(s)dsd\theta, \\ V_3(t) = {}& \int_{t-\frac{h_1}{l}}^{t} \eta^T(t)R_1\eta(s)ds + \int_{t-h(t)}^{t} \zeta^T(s)R_2\zeta(s)ds \\ & + \int_{t-h_2}^{t} x^T(s)R_3 x(s)ds, \\ V_4(t) = {}& d\int_{-d}^{0}\int_{t+\theta}^{t} f^T(x(s))Tf(x(s))dsd\theta \\ & + \int_{-h_1}^{0}\int_{t+\theta}^{t} (tr(\alpha^T(s)S_1\alpha(s)) + y^T(s)S_2 y(s))dsd\theta \\ & + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} (tr(\alpha^T(s)S_3\alpha(s)) + y^T(s)S_4 y(s))dsd\theta,\end{aligned}$$

The mathematical expectation of the stochastic derivative of $V_1(t)$ along the trajectory of (1) can be calculated as

$$\begin{aligned}\mathbf{E}\{dV_1(t)\} = {}& \mathbf{E}\{[2(x(t) - C\int_{t-\sigma}^{t} x(s)ds)^T P(-Cx(t) \\ & + Af(x(t)) + B_1 f(x(t-h(t))) + B_2\int_{t-d(t)}^{t} f(x(s))ds \\ & + u(t)) + tr(\alpha^T(t)P\alpha(t))]dt\}.\end{aligned} \tag{8}$$

From assumption 2, we have

$$\begin{aligned}& tr(\alpha^T(t)P\alpha(t)) \le \lambda_{max}(P) \\ & \quad [x^T(t)M_1^T M_1 x(t) + x^T(t-h(t))M_2^T M_2 x(t-h(t))].\end{aligned} \tag{9}$$

From (8) and (9), we can obtain

$$\begin{aligned}\mathbf{E}\{dV_1(t)\} \le {}& \mathbf{E}\{[x^T(t)(-PC - CP + \lambda_{max}(P)M_1^T M_1)x(t) \\ & + 2x^T(t)PAf^T(x(t)) + 2x^T(t)PB_1 f^T(x(t-h(t)) \\ & + 2x^T(t)PB_2\int_{t-d(t)}^{t} f^T(x(s))ds + 2x^T(t)Pu(t) \\ & + 2x^T(t)CPC\int_{t-\sigma}^{t} x(s)ds - 2\int_{t-\sigma}^{t} x^T(s)dsCPAf(x(t)) \\ & - 2\int_{t-\sigma}^{t} x^T(s)dsCPB_1 f(x(t-h(t))) \\ & - 2\int_{t-\sigma}^{t} x^T(s)dsCPB_2\int_{t-d(t)}^{t} f(x(s))ds \\ & + \lambda_{max}(P)x^T(t-h(t))M_2^T M_2 x(t-h(t)) \\ & - 2\int_{t-\sigma}^{t} x^T(s)dsCPu(t)]dt\}.\end{aligned} \tag{10}$$

Calculating the time-derivative of $V_2(t)$, $V_3(t)$ and $V_4(t)$, and using Lemma 1, we get

$$dV_2(t) = [x^T(t)(Q_1 + \sigma^2 Q_2)x(t) - x^T(t-\sigma)Q_1 x(t-\sigma)$$

$$-\sigma\int_{t-\sigma}^{t}x^T(s)Q_2x(s)ds]dt, \quad (11)$$

$$-\sigma\int_{t-\sigma}^{t}x^T(s)Q_2x(s)ds \le -\int_{t-\sigma}^{t}x^T(s)dsQ_2\int_{t-\sigma}^{t}x(s)ds, \quad (12)$$

$$\begin{aligned}dV_3(t) \le{} & [\eta^T(t)R_1\eta(t)-\eta^T(t-\frac{h_1}{l})R_1\eta(t-\frac{h_1}{l})\\ &+\zeta^T(t)R_2\zeta(t)-(1-\mu)\zeta^T(t-h(t))R_2\zeta(t-h(t))\\ &+x^T(t)R_3x(t)-x^T(t-h_2)R_3x(t-h_2)]dt, \end{aligned}\quad (13)$$

$$\begin{aligned}dV_4(t) ={} & [d^2f^T(x(t))Tf(x(t))-d\int_{t-d}^{t}f^T(x(s))Tf(x(s))ds\\ &+h_1tr(\alpha^T(t)S_1\alpha(t))+(h_2-h_1)tr(\alpha^T(t)S_3\alpha(t))\\ &-\int_{t-h_1}^{t}tr(\alpha^T(s)S_1\alpha(s))ds-\int_{t-h_2}^{t-h_1}tr(\alpha^T(s)S_3\alpha(s))ds\\ &+h_1y^T(t)S_2y(t)+(h_2-h_1)y^T(t)S_4y(t)\\ &-\int_{t-h_1}^{t}y^T(s)S_2y(s)ds-\int_{t-h_2}^{t-h_1}y^T(s)S_4y(s)ds]dt. \end{aligned}\quad (14)$$

By using Lemma 2 and Assumption 2, we have

$$-d\int_{t-d}^{t}f^T(x(s))Tf(x(s))ds \le -\begin{bmatrix}\int_{t-d(t)}^{t}f(x(s))ds\\ \int_{t-d}^{t-d(t)}f(x(s))ds\end{bmatrix}^T \begin{bmatrix}T & U\\ * & T\end{bmatrix}\begin{bmatrix}\int_{t-d(t)}^{t}f(x(s))ds\\ \int_{t-d}^{t-d(t)}f(x(s))ds\end{bmatrix}, \quad (15)$$

$$\begin{aligned}h_1tr(\alpha^T(t)S_1\alpha(t)) \le{} & h_1\lambda_{max}(S_1)\\ &[x^T(t)M_1^TM_1x(t)+x^T(t-h(t))M_2^TM_2x(t-h(t))],\end{aligned} \quad (16)$$

$$\begin{aligned}(h_2-h_1)tr(\alpha^T(t)S_3\alpha(t)) \le{} & (h_2-h_1)\lambda_{max}(S_3)\\ &[x^T(t)M_1^TM_1x(t)+x^T(t-h(t))M_2^TM_2x(t-h(t))].\end{aligned} \quad (17)$$

From the definition of $y(t)$ and $\alpha(t)$, one has

$$\begin{aligned}0 ={} & 2[y^T(t)N_1+x^T(t)N_2][-y(t)-Cx(t-\sigma)+Af(x(t))\\ &+B_1f(x(t-h(t)))+B_2\int_{t-d(t)}^{t}f(x(s))ds+u(t)],\end{aligned} \quad (18)$$

$$x(t)-x(t-h_1)-\int_{t-h_1}^{t}y(s)ds-\int_{t-h_1}^{t}\alpha(s)d\omega(s)=0. \quad (19)$$

By utilizing Lemma 3, one can deduce that

$$\begin{aligned}0 ={} & -2x^T(t)N_3[x(t)-x(t-h_1)-\int_{t-h_1}^{t}y(s)ds\\ &-\int_{t-h_1}^{t}\alpha(s)d\omega(s)]\\ \le{} & x^T(t)(-N_3-N_3^T+h_1N_3S_2^{-1}N_3^T+N_3S_1^{-1}N_3^T)x(t)\\ &+2x^T(t)N_3x(t-h_1)+\int_{t-h_1}^{t}y^T(s)S_2y(s)ds\\ &+\int_{t-h_1}^{t}\alpha^T(s)d\omega(s)S_1\int_{t-h_1}^{t}\alpha(s)d\omega(s).\end{aligned} \quad (20)$$

Similarly, we can get that

$$\begin{aligned}0 ={} & -2x^T(t-h_1)N_4[x(t-h_1)-x(t-h(t))\\ &-\int_{t-h(t)}^{t-h_1}y(s)ds-\int_{t-h(t)}^{t-h_1}\alpha(s)d\omega(s)]\\ \le{} & x^T(t-h_1)(-N_4-N_4^T+(h_2-h_1)N_4S_4^{-1}N_4^T\\ &+N_4S_3^{-1}N_4^T)x(t-h_1)+2x^T(t-h_1)N_4x(t-h(t))\\ &+\int_{t-h(t)}^{t-h_1}y^T(s)S_4y(s)ds\\ &+\int_{t-h(t)}^{t-h_1}\alpha^T(s)d\omega(s)S_3\int_{t-h(t)}^{t-h_1}\alpha(s)d\omega(s).\end{aligned} \quad (21)$$

$$\begin{aligned}0 ={} & -2x^T(t-h(t))N_5[x(t-h(t))-x(t-h_2)\\ &-\int_{t-h_2}^{t-h(t)}y(s)ds-\int_{t-h_2}^{t-h(t)}\alpha(s)d\omega(s)]\\ \le{} & x^T(t-h(t))(-N_5-N_5^T+(h_2-h_1)N_5S_4^{-1}N_5^T\\ &+N_5S_3^{-1}N_5^T)x(t-h(t))+2x^T(t-h(t))N_5x(t-h_2)\\ &+\int_{t-h_2}^{t-h(t)}y^T(s)S_4y(s)ds\\ &+\int_{t-h_2}^{t-h(t)}\alpha^T(s)d\omega(s)S_3\int_{t-h_2}^{t-h(t)}\alpha(s)d\omega(s).\end{aligned} \quad (22)$$

Similar to the proof of [22], we can obtain that

$$\begin{aligned}&\mathbf{E}\{\int_{t-h_1}^{t}\alpha^T(s)d\omega(s)S_1\int_{t-h_1}^{t}\alpha(s)d\omega(s)\}\\ &=\mathbf{E}\{\int_{t-h_1}^{t}tr\{\alpha^T(s)S_1\alpha(s)\}ds\},\end{aligned} \quad (23)$$

$$\begin{aligned}&\mathbf{E}\{\int_{t-h(t)}^{t-h_1}\alpha^T(s)d\omega(s)S_3\int_{t-h(t)}^{t-h_1}\alpha(s)d\omega(s)\}\\ &=\mathbf{E}\{\int_{t-h(t)}^{t-h_1}tr\{\alpha^T(s)S_3\alpha(s)\}ds\},\end{aligned} \quad (24)$$

$$\begin{aligned}&\mathbf{E}\{\int_{t-h_2}^{t-h(t)}\alpha^T(s)d\omega(s)S_3\int_{t-h_2}^{t-h(t)}\alpha(s)d\omega(s)\}\\ &=\mathbf{E}\{\int_{t-h_2}^{t-h(t)}tr\{\alpha^T(s)S_3\alpha(s)\}ds\}.\end{aligned} \quad (25)$$

For positive diagonal matrices L_1 and L_2, the following inequalities hold

$$\begin{bmatrix}x(t)\\ f(x(t))\end{bmatrix}^T\begin{bmatrix}K_1L_1 & -K_2L_1\\ -K_2L_1 & L_1\end{bmatrix}\begin{bmatrix}x(t)\\ f(x(t))\end{bmatrix}\le 0, \quad (26)$$

$$\begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix}^T \begin{bmatrix} K_1L_2 & -K_2L_2 \\ -K_2L_2 & L_2 \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix} \leq 0, \tag{27}$$

From (7) to (27), we can deduce that

$$\mathbf{E}\{dV(t) - 2f^T(x(t))u(t)dt - \gamma u^T(t)u(t)dt\} \leq \mathbf{E}\{\xi^T(t)\Omega^*\xi(t)\}, \tag{28}$$

where $\Omega^* = \Omega + e_1^T(h_1N_3S_2^{-1}N_3^T + N_3S_1^{-1}N_3^T)e_1 + e_{1+l}^T(h_1N_3S_2^{-1}N_3^T + N_3S_1^{-1}N_3^T)e_{1+l} + e_{2+l}^T(h_1N_3S_2^{-1}N_3^T + N_3S_1^{-1}N_3^T)e_{2+l}$.

By using Schur complement, it is easy to verify $\Omega^* < 0$ is equivalent to (6), so

$$\frac{\mathbf{E}\{dV(t)\}}{dt} - \mathbf{E}\{2f^T(x(t))u(t) + \gamma u^T(t)u(t)\} \leq 0. \tag{29}$$

It follows from (29) and the definition of $V(t)$ that

$$2\mathbf{E}\{\int_0^{t_p} f^T(x(s))u(s)ds\} \geq -\mathbf{E}\{\gamma\int_0^{t_p} u^T(s)u(s)ds\},$$

From Definition 1, it is obvious that if LMIs (5) and (6) hold. Consequently, the stochastic neural networks (1) is globally passive in the sense of expectation, and the proof of Theorem 1 is completed. ■

Remark 2. Different to [7], [8], [12], [16]-[20] and [22], in this paper, the distributed delays are incorporated into the model, we deal with the system (1) with discrete, distributed and leakage delays. By using Lemma 3, the term $-\int_{t-d}^{t-d(t)} f^T(x(s))Tf(x(s))ds$ is not omitted, and will leads to a superior result.

Remark 3. Compared to [22], this article the condition $P < \lambda_{max}(P)I$ replaced of $P < \lambda I$, which indicate that the tighter upper bound of P is utilized. Through the numerical example, the effectiveness of this method was demonstrated.

Remark 4. In this paper, the obtained conditions for checking the passivity of stochastic neural networks with multiple time delays are dependent on the size of distributed delay, leakage delay and the upper and lower bound of discrete delay, which implicate that the information on the size of both distributed delay, leakage delay and discrete delay is sufficiently used.

When there is no stochastic affects, the system (1) becomes

$$\frac{dx(t)}{dt} = -Cx(t-\sigma) + Af(x(t)) + B_1f(x(t-h(t)) + B_2\int_{t-d(t)}^{t} f(x(s))ds + u(t). \tag{30}$$

By a similar method to that employed in Theorem 1, we can get the following results.

Corollary 1. For given scalars $h_1 > 0$, $h_2 > 0$, $d > 0$ and $\mu > 0$, the stochastic neural network (30) is passive in the sense of expectation if there exist symmetric positive definite matrices P, Q_1, Q_2, R_i $(i = 1, 2, 3)$, T and S_j $(j = 1, 2)$, the positive diagonal matrices L_1 and L_2, any appropriately dimensional matrices U, N_k $(k = 1, 2, \ldots, 5)$, a positive constant $\gamma > 0$ such that the following LMIs (31) and (32) hold:

$$\begin{bmatrix} T & U \\ * & T \end{bmatrix} > 0, \tag{31}$$

$$\begin{bmatrix} \Xi & h_1e_1^TN_3 & (h_2-h_1)e_{1+l}^TN_4 & (h_2-h_1)e_{2+l}^TN_5 \\ * & -h_1S_1 & 0 & 0 \\ * & * & -(h_2-h_1)S_2 & 0 \\ * & * & * & -(h_2-h_1)S_2 \end{bmatrix} < 0, \tag{32}$$

where $\Xi = \Pi_1^* + \Pi_2 + \Pi_3 + \Pi_4^* + \Pi_5 + \Pi_6$,
$\Pi_1^* = e_1^T(-PC - CP)e_1 + e_1^T(PA + A^TP)e_{6+l} + e_1^T(PB_1 + B_1^TP)e_{7+l} + e_1^T(PB_2 + B_2^TP)e_{8+l} + 2e_1^TPe_{10+l} + 2e_1^TCPCe_{5+l} - e_{5+l}^T(CPA + A^TPC)e_{6+l} - 2e_{5+l}^TCPe_{10+l} - e_{5+l}^T(CPB_1 + B_1^TPC)e_{7+l} - e_{5+l}^T(CPB_2 + B_2^TPC)e_{8+l}$,
$\Pi_4^* = d^2e_{6+l}^TTe_{6+l} - \begin{bmatrix} e_{8+l} \\ e_{9+l} \end{bmatrix}^T \begin{bmatrix} T & U \\ * & T \end{bmatrix} \begin{bmatrix} e_{8+l} \\ e_{9+l} \end{bmatrix} + (h_2-h_1)e_{11+l}^TS_2e_{11+l}h_1e_{11+l}^TS_1e_{11+l}$.

Remark 5. Delay partitioning technique is a key method to reduce the possible conservation. It is shown that the value of l gets bigger which leads much less conservative results. In this paper, delay partitioning technique is employed in Theorem 1 and Corollary 1.

IV. Numerical Examples and Simulation

In this section, a numerical example is given to show the effectiveness of the results.

Example 1. Consider the stochastic neural networks (30) with the following parameters

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.3 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0.2 \\ -0.4 & 0.3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \end{bmatrix},$$
$$u(t) = \begin{bmatrix} -0.3cos(3.1t) \\ 0.7sin(1.4t) \end{bmatrix}, \quad f(x(t)) = tanh(x(t)),$$
$$h(t) = 0.2|sin(7t)| + 0.1, \quad \sigma = 0.01.$$

It is obvious that $h_1 = 0.1$, $h_2 = 0.3$ and $\mu = 0.2$. When Assumption 1 is satisfied, and then $K_i^- = 0$, $K_i^+ = 1$, thus, $K_1 = diag\{0, 0\}$, $K_2 = diag\{0.5, 0.5\}$, the distributed delay is chosen as $0 \leq d(t) \leq d = 0.5$. When $l = 2$, and by applying the MATLAB LMI Tool box to solve the problem, we find a solution to LMIs in (30) and (31) as follows:

$$P = \begin{bmatrix} 3.9009 & 0.4785 \\ 0.4785 & 3.7344 \end{bmatrix} \times 10^3,$$
$$Q_1 = \begin{bmatrix} 84.3605 & 24.5494 \\ 24.5494 & 150.3660 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 9.6803 & 0.4116 \\ 0.4116 & 5.8765 \end{bmatrix} * 10^4,$$
$$T = \begin{bmatrix} 2.1005 & 1.3707 \\ 1.3707 & 2.9146 \end{bmatrix} \times 10^3,$$
$$R_1 = \begin{bmatrix} 67.7765 & 18.3219 & 0 & 0 \\ * & 102.3192 & 0 & 0 \\ * & * & 34.3069 & 9.4428 \\ * & * & * & 52.2945 \end{bmatrix},$$

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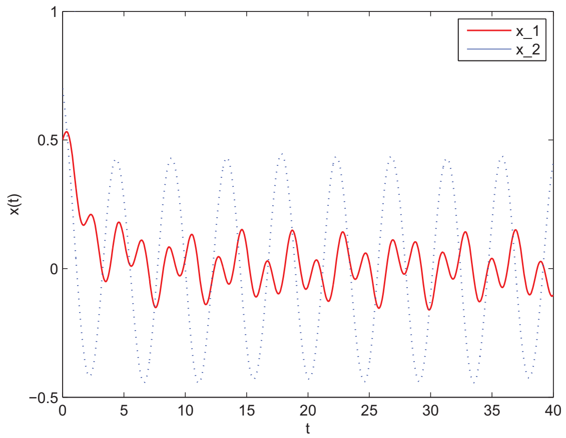

Fig. 1. State trajectories of system (30) with input $u(t)$.

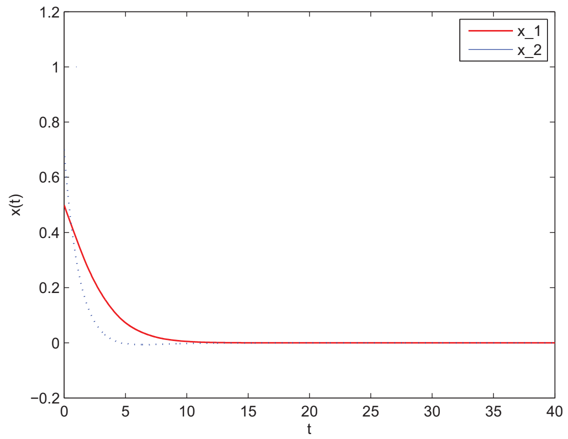

Fig. 2. State trajectories of system (30) without input $u(t)$.

$$R_2 = \begin{bmatrix} 2.8527 & 0.0131 & -2.8176 & -0.5789 \\ * & 2.2200 & 0.7543 & -2.2852 \\ * & * & 3.0714 & -0.2192 \\ * & * & * & 2.5060 \end{bmatrix} \times 10^3,$$

$$R_3 = \begin{bmatrix} 45.4658 & 12.4392 \\ 12.4392 & 69.3063 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 1.8388 & -0.0030 \\ -0.0030 & 1.8606 \end{bmatrix} \times 10^4,$$

$$S_2 = \begin{bmatrix} 9.3795 & 0.0209 \\ 0.0209 & 9.3384 \end{bmatrix} \times 10^3, U = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 13.7067 & -5.1567 \\ 14.2257 & 27.0227 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 3.8515 & 21.8974 \\ -15.8274 & 11.2541 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} 28.1292 & 21.2862 \\ 0.2574 & 67.8658 \end{bmatrix},$$

$$N_4 = \begin{bmatrix} 1.2025 & 0.1008 \\ -0.0185 & 1.0583 \end{bmatrix} \times 10^3,$$

$$N_5 = \begin{bmatrix} 40.5686 & 10.7713 \\ 10.9217 & 63.1691 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 5.1650 & 0 \\ 0 & 4.4375 \end{bmatrix} \times 10^3,$$

$$L_2 = \begin{bmatrix} 3.5674 & 0 \\ 0 & 2.1515 \end{bmatrix} \times 10^4, \gamma = 7.4564 \times 10^4.$$

Based on the Corollary 1, the neural networks (30) is passive.

Fig. 1 and 2 show that the states trajectories of $x_1(t)$ and $x_2(t)$ with input $u(t)$ and without input $u(t)$, respectively, where the initial condition is $[0.5, 0.7]^T$. When the initial condition is $[1.2, 0.6]^T$, the phase trajectory of system (30) with input $u(t)$ and without input $u(t)$ are depicted in Fig. 3 and 4, respectively.

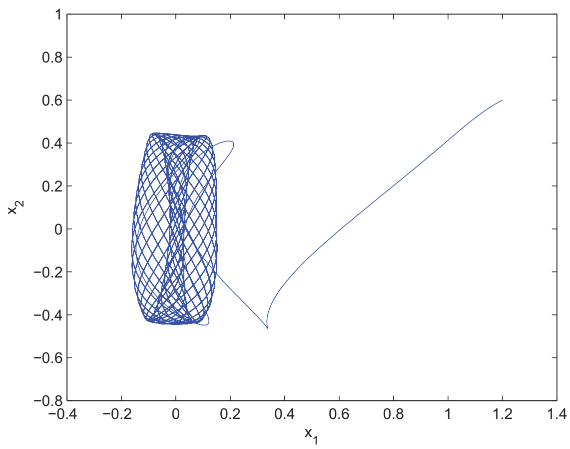

Fig. 3. Phase trajectories of system (30) with input $u(t)$.

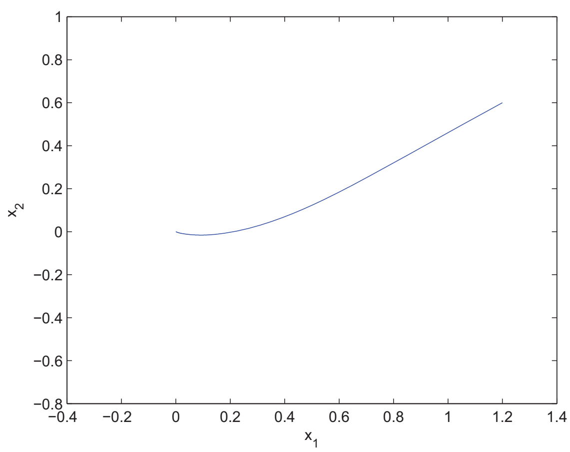

Fig. 4. Phase trajectories of system (30) without input $u(t)$.

V. Conclusion

In this paper, the problem of the passivity analysis for stochastic neural networks with leakage, discrete and distributed delays has been investigated. The presented sufficient criteria are based on delay partitioning technique, free weighting matrix method and stochastic analysis technique, in which the information on the size of both distributed delay, leakage delay and discrete delay is sufficiently used. A numerical example is given to illustrate the effectiveness of the obtained results. In future work, we will utilized the proposed method to deal with the system with parameter uncertainties or Markovian jumping parameters.

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References

[1] M. Galicki, H. Witte, J.Dörschel, M. Eiselt, G. Griessbach, Common optimization of adaptive preprocessing units and a neural network during learning period. Application in EEG pattern recognition, Neural Networks 10 (1997) 1153-1163.
[2] D. Zhou, J. Cao, Globally exponential stability conditions for cellular neural networks with time-varying delays, Appl. Math. Comput. 131 (2002) 487-496.
[3] J. Cao, J. Wang, Global asymptotic and robust stability of recurrent neural networks with time delays, IEEE Trans. Circuits Syst. I, Reg. Papers, 52 (2) (2005) 417-426.
[4] Z. Wu, P. Shi, H. Su, J. Chu, Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays, IEEE Trans. Neural Netw. 22 (2011) 1566-1575.
[5] A. Rawat, R.N. Yadav, S.C. Shrivastava, Neural network applications in smart antenna arrays: a review, Int. J. Electron. Commun. (AEÜ) 66 (2012) 903-912.
[6] H. Huang, G. Feng, Delay-dependent stability for uncertain stochastic neural networks with time-varying delay, Physica A 381 (15) (2007) 93C103.
[7] J. Zhang, P. Shi, J. Qiu, Novel robust stability criteria for uncertain stochastic Hopfield neural networks with time-varying delays, Nonlinear Anal. Real World Appl. 8 (4) (2007) 1349-1357.
[8] W. Chen, X. Lu, Mean square exponential stability of uncertain stochastic delayed neural networks, Phys. Lett. A. 372 (7) (2008) 1061-1069.
[9] H. Zhang, Y. Wang, Stability analysis of Markovian jumping stochastic CohenCGrossberg neural networks with mixed time delays, IEEE Trans. Neural Netw. 19 (2) (2008) 366-370.
[10] W. Yu, Passivity analysis for dynamic multilayer neuro identifier, IEEE Trans. Circuits Syst. I 50 (1) (2003) 173-178.
[11] H. Li, H. Gao, P. Shi, New passivity analysis for neural networks with discrete and distributed delays, IEEE Trans, Neural Netw. 21 (11) (2010) 1842-1847.
[12] R. Sakthivel, A. Arunkumar, K. Mathiyalagan, S. Marshal Anthoni, Robust passivity analysis of fuzzy Cohen-Grossberg BAM neural networks with time-varying delays, Appl. Math. Comput. 218 (7) (2011) 3799-3809.
[13] P. Balasubramaniam, G. Nagamani, R. Rakkiyappan, Passivity analysis for neural networks of neutral type with Markovian jumping parameters and time delay in the leakage term. Commun. Nonlinear SCI. Numer. Simulat. 16 (2011) 4422-4437.
[14] Q. Song, J. Liang, Z. Wang, Passivity analysis of discrete-time stochastic neural networks with time-varying delays, Neurocomputing 72 (7) (2009) 1782-1788.
[15] H. Li, C. Wang, P. Shi, New passivity results for uncertain discrete-time stochastic neural networks with mixed time delays, Neurocomputing 73 (16) (2010) 3291-3299.
[16] J. Fu, H. Zhang, T. Ma, On passivity analysis for stochastic neural networks with interval time-varying delay, Neurocomputing 73 (4) (2010) 795-801.
[17] Y. Chen, H. Wang, A. Xue, Passivity analysis of stochastic time-delay neural networks, Nonlinear Dynam. 61 (1-2) (2010) 71-82.
[18] K. Mathiyalagan, R. Sakthivel, S. Marshal Anthoni, New robust passivity criteria for stochastic fuzzy BAM neural networks with time-varying delays, Commun. Nonlinear SCI. Numer. Simulat. 17 (3) (2012) 1392-1407.
[19] P. Balasubramaniam, G. Nagamani, Global robust passivity analysis for stochastic interval neural networks with interval time-varying delays and Markovian jumping parameters, J. Optim. Theory Appl. 149 (1) (2011) 197-215.
[20] P. Balasubramaniam, G. Nagamani, Global robust passivity analysis for stochastic fuzzy interval neural networks with time-varying delays, Expert Syst. Appl. 39 (1) (2012) 732-742.
[21] X. Chen, W.X. Zheng, Stochastic state estimation for neural networks with distributed delays and Markovian jump, Neural Netw. 25 (2012) 14-20.
[22] Z. Zhao, Q. Song, S. He, Passivity analysis of stochastic neural networks with time-varying delays and leakage delay, Neurocomputing 125 (2014) 22-27.
[23] Z. Wang, H. Shu, Y. Liu, DWC. Ho, X. Liu, Robust stability analysis of generalized neural networks with discrete and distributed time delays, Chaos Solitons Fract 30 (4) (2006) 886-896.
[24] T. Li, X. Yang, P. Yang, S. Fei, New delay-variation-dependent stability for neural networks with time-varying delay, Neurocomputing 101 (2013) 361-169.