

# Spectral Broadening in an InGaAsP Optical Waveguide with $\chi^{(3)}$ Nonlinearity Including Two Photon Absorption

Keigo Matsuura, Isao Tomita

**Abstract**—We have studied a method to widen the spectrum of optical pulses that pass through an InGaAsP waveguide for application to broadband optical communication. In particular, we have investigated the competitive effect between spectral broadening arising from nonlinear refraction (optical Kerr effect) and shrinking due to two photon absorption in the InGaAsP waveguide with  $\chi^{(3)}$  nonlinearity. The shrunk spectrum recovers broadening by the enhancement effect of the nonlinear refractive index near the bandgap of InGaAsP with a bandgap wavelength of 1490 nm. The broadened spectral width at around 1525 nm (196.7 THz) becomes 10.7 times wider than that at around 1560 nm (192.3 THz) without the enhancement effect, where amplified optical pulses with a pulse width of  $\sim 2$  ps and a peak power of 10 W propagate through a 1-cm-long InGaAsP waveguide with a cross-section of  $4 (\mu\text{m})^2$ .

**Keywords**—InGaAsP Waveguide,  $\chi^{(3)}$  Nonlinearity, Spectral Broadening.

## I. INTRODUCTION

**T**HE rapid increase in data traffic in recent years has been demanding a broad bandwidth to perform multiplex communication in optical fibers, i.e., broadband optical communication [1], [2]. To implement this multiplex communication, several light sources that can generate multiple laser wavelengths, such as distributed-feedback (DFB) laser arrays [3], [4] and supercontinuum light sources [5], [6], have been developed. However, the low device yield of DFB laser arrays with equal-spacing wavelengths is not suitable for practical use, and supercontinuum light sources with precisely-spaced wavelengths are costly because of their dispersion-controlled, tapered optical fibers that are designed specially for the spectral broadening. Instead of these devices, we study an InGaAsP nonlinear optical waveguide to produce a broad spectrum using optical Kerr effect via  $\chi^{(3)}$  nonlinearity, where the InGaAsP waveguide possesses a four orders of magnitude larger nonlinear refractive index  $n_2$  than that of  $\text{SiO}_2$  in optical fibers, e.g., those used in the supercontinuum light sources. Several  $n_2$  values of typical nonlinear optical materials are shown in Table I [7], [8]. Here,  $\alpha_0$  is the intrinsic optical loss for those materials, and PU-STAD stands for Poly-Urethane containing Symmetrically substituted Tris-Azo Dye. Since semiconductor laser diodes (LDs) and semiconductor optical amplifiers

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TABLE I  
TYPICAL VALUES OF THE NONLINEAR REFRACTIVE INDEX

Material	$n_2$ [ $\text{m}^2/\text{W}$ ]	$\alpha_0$ [dB/m]
$\text{SiO}_2$	$3.0 \times 10^{-20}$	$2.0 \times 10^{-4}$
$\text{As}_3\text{S}_3$	$1.0 \times 10^{-18}$	$1.0 \times 10^{-1}$
GaAs	$2.1 \times 10^{-18}$	$2.0 \times 10^2$
AlGaAs	$1.0 \times 10^{-17}$	$2.0 \times 10^2$
PU-STAD	$3.0 \times 10^{-17}$	$3.0 \times 10^2$
InGaAsP	$1.0 \times 10^{-16}$	$1.0 \times 10^2$

(SOAs) for 1.55- $\mu\text{m}$  optical communication are made of InGaAsP [9], [10], the InGaAsP waveguide is compatible with them and is suitable for integrating all of them on the same chip [11], [12]. A broad spectrum in the InGaAsP waveguide seems to be easily obtained through the same principle of the supercontinuum light sources, but actually, two photon absorption in the waveguide impedes spectral broadening. In what follows, we investigate the competitive effect between the spectral broadening originating from the nonlinear refractive index and the shrinking due to the two photon absorption in the InGaAsP waveguide.

## II. NUMERICAL ANALYSIS PROCEDURE

### A. Basic Equation Describing the Phenomena

When amplified optical pulses (e.g., amplified pulses from an InGaAsP laser with an InGaAsP amplifier) start to pass through the InGaAsP waveguide, self-phase modulation via  $\chi^{(3)}$  nonlinearity [13] also starts to broaden the spectrum of the optical pulses. But, optical losses in the waveguide, such as the intrinsic optical loss and the two-photon-absorption loss, reduce the pulse power and restrict the size of the spectral broadening. These processes can be analyzed by the following nonlinear Schrödinger equation [13]:

$$\frac{\partial A}{\partial z} + \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial \tau^2} + \frac{\alpha_0}{2}A = i \frac{\omega_0 n_2}{c A_{\text{eff}}} |A|^2 A - \frac{\alpha_2}{2 A_{\text{eff}}} |A|^2 A, \quad (1)$$

where  $z$  is the coordinate along the waveguide,  $\tau = t - z/v_g$  is the time measured from the moving frame with a velocity of  $v_g$ ,  $A = A(z, \tau)$  is the amplitude of the optical pulses,  $\beta_2 = d^2\beta(\omega)/d\omega^2$  is the group velocity dispersion,  $\alpha_0$  is the intrinsic optical loss,  $n_2$  is the nonlinear refractive index,  $\alpha_2$  is the two-photon-absorption coefficient,  $A_{\text{eff}}$  is the effective cross-section of the waveguide,  $\omega_0$  is the center frequency of the optical pulses, and  $c$  is the velocity of light in vacuum.

### B. Simplification of the Basic Equation

Equation (1) is simplified for InGaAsP because the dispersion term  $(i/2)\beta_2\partial^2 A/\partial\tau^2$  is ignored when compared with the nonlinear term, e.g.,  $i(\omega_0 n_2/c A_{\text{eff}})|A|^2 A$ , by the following reason. Here, note that the term  $(\alpha_2/2A_{\text{eff}})|A|^2 A$  is on the same order of magnitude as  $i(\omega_0 n_2/c A_{\text{eff}})|A|^2 A$  and cannot be neglected. As an input pulse, if we employ

$$A(z, \tau) = A_0 e^{-\left(\frac{\tau}{\tau_0}\right)^2} e^{ikz - i\omega_0\tau}, \quad (2)$$

where  $A_0$  and  $\tau_0$  are constants and  $k$  is a wavenumber, then the ratio  $\eta$  between the nonlinear term and the dispersion term is of the form

$$\eta = \frac{\text{The nonlinear term}}{\text{The dispersion term}} = \frac{n_2 \omega_0 A_0^2 \tau_0^2}{c \beta_2 A_{\text{eff}}}. \quad (3)$$

Substituting typical values  $n_2 \approx 6 \times 10^{-16} \text{ m}^2/\text{W}$ ,  $A_0^2 \approx 1 \text{ W}$ ,  $A_{\text{eff}} \approx 4 (\mu\text{m})^2$ ,  $\beta_2 \approx 1 \times 10^{-24} \text{ s}^2/\text{m}$ ,  $\tau_0 \approx 1 \text{ ps}$ ,  $\omega_0 \approx 2\pi \times 193.5 \text{ THz}$ , and  $c = 3 \times 10^8 \text{ m/s}$  for (3), we obtain  $\eta \approx 6 \times 10^2$ , which means that the dispersion term can be omitted safely in (1).

### C. Calculation of the Output Spectrum

To calculate the spectrum of the output pulses in the waveguide, we insert  $A(z, \tau) = |A(z, \tau)|e^{i\theta(z, \tau)}$  into (1) excluding the dispersion term, where  $|A(z, \tau)|$  is the amplitude of the pulses and  $\theta(z, \tau)$  is their phase. We then obtain decoupled equations:

$$\frac{\partial I(z, \tau)}{\partial z} = -\alpha_0 I(z, \tau) - \alpha_2 I^2(z, \tau), \quad (4)$$

$$\frac{\partial \theta(z, \tau)}{\partial z} = \frac{\omega_0}{c} n_2 I(z, \tau), \quad (5)$$

where  $I(z, \tau) = |A(z, \tau)|^2/A_{\text{eff}}$ . Integrating (4) with respect to  $z$ , we obtain

$$I(z, \tau) = \frac{I_0(\tau)\alpha_0 e^{-\alpha_0 z}}{\alpha_0 + \alpha_2 I_0(\tau)(1 - e^{-\alpha_0 z})}, \quad (6)$$

where  $I_0(\tau) = I(z=0, \tau)$  is the pulse waveform at  $z=0$ . Substituting (6) for (5) and integrating (5) from  $z=0$  to  $L$  (the total waveguide length), we have

$$\theta(L, \tau) = \frac{\omega_0 n_2}{c \alpha_2} \ln \left( 1 + \alpha_2 I_0(\tau) \frac{1 - e^{-\alpha_0 L}}{\alpha_0} \right). \quad (7)$$

Finally, we calculate the spectrum  $S(\omega)$  of the output pulses by using the following Fourier transformation:

$$\begin{aligned} S(\omega) &= \left| \int_{-\infty}^{\infty} A(L, \tau) e^{-i(\omega - \omega_0)\tau} d\tau \right|^2 \\ &= \left| \int_{-\infty}^{\infty} |A(L, \tau)| e^{i\theta(L, \tau)} e^{-i(\omega - \omega_0)\tau} d\tau \right|^2. \end{aligned} \quad (8)$$

Here, FFT in C is used in computing (8) [14], and obtained numerical results are shown in the next section.

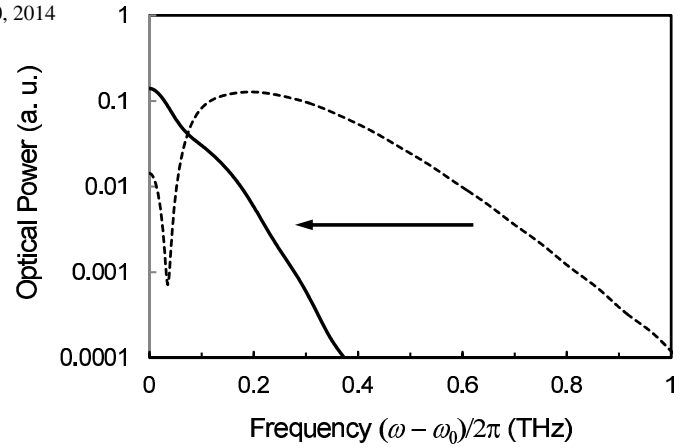


Fig. 1 Spectral shrinking when  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$  and  $\alpha_2 = 2.3 \times 10^{-10} \text{ m/W}$ , indicated by the solid line. The dashed line represents the case of no two photon absorption, i.e.,  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$  and  $\alpha_2 = 0 \text{ m/W}$ .

### III. CALCULATED RESULTS AND DISCUSSION

Numerical results for spectral broadening in an InGaAsP waveguide with a length of 1 cm and a cross-section of  $4 (\mu\text{m})^2$  are provided in this section. We have in mind using an InGaAsP waveguide made of  $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ , where the composition ratios  $x$  and  $y$  are set such that its bandgap wavelength is 1490 nm [8]. The waveform of the input optical pulse at  $z=0$  is assumed as

$$|A(z, \tau)|^2 = A_0^2 \text{sech} \left( \frac{\tau}{\tau_0} \right) \quad (9)$$

for one pulse. For multiple pulses with a time interval of  $T$ , it takes the form:

$$|A(z, \tau)|^2 = \sum_{n=-\infty}^{\infty} A_0^2 \text{sech} \left( \frac{\tau - nT}{\tau_0} \right). \quad (10)$$

When (10) is used as input pulses, the output spectrum is that made of discrete spectral lines with an equal spacing of  $2\pi/T$ . When (9) is used, the FFT result provides the envelope of the discrete spectrum. Here we use (9), which is sufficient to examine the spectral width and also considerably reduces the computation time when compared with that for (10).

To investigate the effect of the two-photon-absorption loss  $\alpha_2$  on spectral broadening associated with the nonlinear refractive index  $n_2$ , we compute (i) the spectral width for  $n_2 \neq 0$  and  $\alpha_2 = 0$  and (ii) that for  $n_2 \neq 0$  and  $\alpha_2 \neq 0$ , and compared their size.

The dashed line in Fig. 1 depicts the envelope of the output spectrum for Case (i) when a pulse with  $\omega_0 = 192.3 \text{ THz}$  (1560 nm),  $\tau_0 = 1.0 \text{ ps}$  (the pulse width  $\sim 2 \text{ ps}$ ), and  $A_0^2 = 10 \text{ W}$  is fed into the InGaAsP waveguide, where  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$  and  $\alpha_2 = 0 \text{ m/W}$ . The solid line in Fig. 1 illustrates that for Case (ii) when the same pulse is injected into the waveguide, where  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$  and  $\alpha_2 = 2.3 \times 10^{-10} \text{ m/W}$ . It is clear that the shrinking of the spectrum is observed due to non-zero two photon absorption.

To overcome the spectral shrinking, we utilize the enhancement effect of  $n_2$  caused by virtual carrier excitation

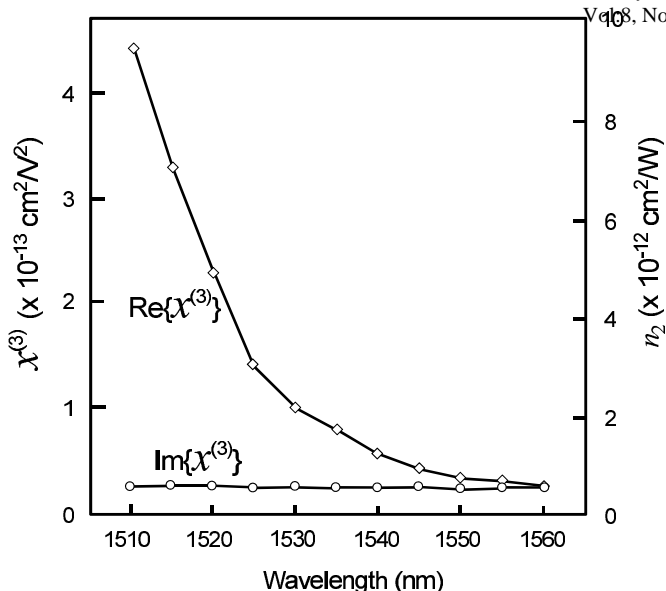


Fig. 2 Dependence of the real and imaginary parts of  $\chi^{(3)}$  on the input wavelength [8].  $n_2$  and  $\alpha_2$  are proportional to  $\text{Re}\{\chi^{(3)}\}$  and  $\text{Im}\{\chi^{(3)}\}$ , respectively.

near the band of InGaAsP, as shown in Fig. 2 [8]. This effect is also observed at other experiments and is utilized for nonlinear switching devices [15], [16]. According to the experiment by Darwish *et al.* [8], although  $\text{Im}\{\chi^{(3)}\} \propto \alpha_2$  is independent of the input wavelength at the measured region of 1510 – 1560 nm,  $\text{Re}\{\chi^{(3)}\} \propto n_2$  is dependent on the wavelength and is enhanced as the wavelength is shortened. Using the relations  $n_2 = (3/4cn^2\varepsilon_0)\text{Re}\{\chi^{(3)}\}$  and  $\alpha_2 = (3\omega_0/2c^2n^2\varepsilon_0)\text{Im}\{\chi^{(3)}\}$  ( $n$ : the InGaAsP refractive index,  $\varepsilon_0$ : the permittivity in vacuum), we can rewrite (7) as

$$\theta(L, \tau) = \frac{1}{2} \frac{\text{Re}\{\chi^{(3)}\}}{\text{Im}\{\chi^{(3)}\}} \ln \left( 1 + \alpha_2 I_0(\tau) \frac{1 - e^{-\alpha_0 L}}{\alpha_0} \right). \quad (11)$$

A calculated spectrum with (11) for the center frequency  $\omega_0$  shifted to 196.7 THz (1525 nm) is shown by the solid line in Fig. 3, where  $n_2$  is  $2.9 \times 10^{-16} \text{ m}^2/\text{W}$ , which is 5 times larger  $n_2$  at 192.3 THz (1560 nm). The dashed line in Fig. 3 is the same one as the solid line in Fig. 1, which is placed for comparison in the spectral width. We can see that owing to the  $n_2$ -enhancement effect, a 10.7 times wider spectral width is obtained when the solid and dashed lines in Fig. 3 are compared at the full width at half maximum (FWHM). If  $\omega_0$  is shifted to 198.7 THz (1510 nm),  $n_2$  becomes  $9.2 \times 10^{-16} \text{ m}^2/\text{W}$ . In this case, the spectral width seems to be much more widened, but actually, it is limited to  $< 10 \text{ nm}$  ( $< 1.3 \text{ THz}$ ) because of the narrow effective wavelength range ( $< 10 \text{ nm}$ ) for the large  $\text{Re}\{\chi^{(3)}\}$  or  $n_2$ , as seen in Fig. 2.

Finally, we examine the dependence of the spectral width on the input power. In laboratory experiments, achieving a pulse peak power of 10 W is possible by use of a mode-locked laser with an Er-doped fiber amplifier, but is not easily realized with LDs and SOAs.

For decreased power, Fig. 4 depicts the dependence of the spectral width  $W$  (FWHM) on the peak power  $P_{\text{peak}}$  for  $\omega_0 =$

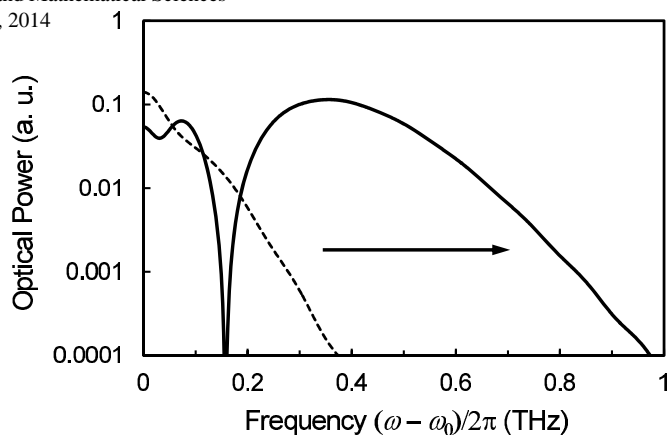


Fig. 3 Recovery of spectral broadening by the  $n_2$ -enhancement effect. The solid line represents the case of the center frequency  $\omega_0$  shifted to 196.7 THz (1525 nm), where  $n_2$  is  $2.9 \times 10^{-16} \text{ m}^2/\text{W}$ , which is 5 times larger than  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$  at 192.3 THz (1560 nm), indicated by the dashed line. Here,  $\alpha_2$  is  $2.3 \times 10^{-10} \text{ m/W}$  for both cases.

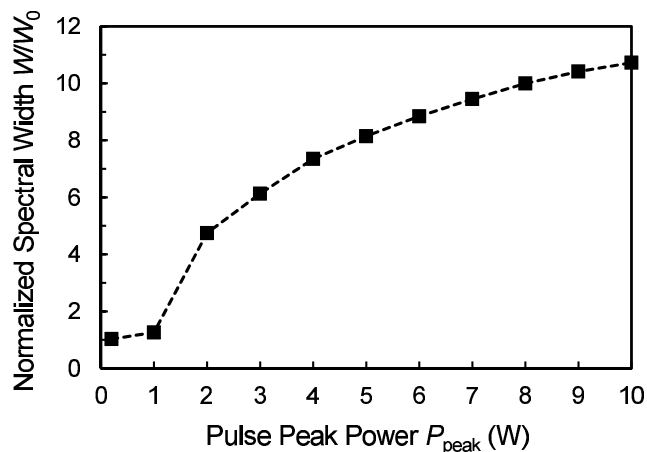


Fig. 4 Relation between the peak power  $P_{\text{peak}}$  of the input pulse and the spectral width  $W$  with  $n_2 = 2.9 \times 10^{-16} \text{ m}^2/\text{W}$  that is normalized by the spectral width  $W_0$  with  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$ , where  $\alpha_2$  is  $2.3 \times 10^{-10} \text{ m/W}$  for both cases.

196.7 THz (1525 nm) with  $n_2 = 2.9 \times 10^{-16} \text{ m}^2/\text{W}$  and  $\alpha_2 = 2.3 \times 10^{-10} \text{ m/W}$  that is normalized by the spectral width  $W_0$  (FWHM) for  $\omega_0 = 192.3 \text{ THz}$  (1560 nm) with  $n_2 = 0.58 \times 10^{-16} \text{ m}^2/\text{W}$  and  $\alpha_2 = 2.3 \times 10^{-10} \text{ m/W}$ . We can see that the magnification factor  $W/W_0$  is not linear with respect to the peak power  $P_{\text{peak}}$ , which is due to two photon absorption, and that, for instance, at a small peak power of  $P_{\text{peak}} \approx 2 \text{ W}$  [17], 4.7 times magnification of  $W/W_0$  is obtained.

#### IV. CONCLUSION

We have investigated spectral broadening for amplified optical pulses that propagate through a 1-cm-long InGaAsP waveguide with a cross-section of  $4 (\mu\text{m})^2$  via  $\chi^{(3)}$  nonlinearity. Although the InGaAsP waveguide has a four orders of magnitude larger nonlinear refractive index  $n_2$  than that of  $\text{SiO}_2$  used in optical fibers of supercontinuum light sources, we have seen that two photon absorption in the

InGaAsP waveguide considerably reduces the spectral width. But, using the enhancement effect of  $n_2$  near the bandgap of InGaAsP, we have observed that spectral broadening is recovered and that the obtained spectral width at 196.7 THz (1525 nm) is 10.7 times broader than that without the enhancement effect at 192.3 THz (1560 nm), where the input pulse has a width of  $\sim 2$  ps and a peak power of 10 W. But, since the 10-W peak power is fairly large in achieving with LDs and SOAs, we have examined the spectral-width dependence on reduced peak power, and for a peak power of 2 W, we have obtained a magnification factor of 4.7.

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