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New Subband Adaptive IIR Filter Based On Polyphase Decomposition

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***Abstract*—**We present a subband adaptive infinite-impulse response (IIR) filtering method, which is based on a polyphase decomposition of IIR filter. Motivated by the fact that the polyphase structure has benefits in terms of convergence rate and stability, we introduce the polyphase decomposition to subband IIR filtering, i.e., in each subband high order IIR filter is decomposed into polyphase IIR filters with lower order. Computer simulations demonstrate that the proposed method has improved convergence rate over conventional IIR filters.

***Keywords*—**Subband adaptive filter, IIR filtering. Polyphase decomposition.

I. Introduction

ADAPTIVE filters can be implemented with two structures, the finite impulse response (FIR) structure and the infinite impulse response (IIR) structure. Traditionally, the FIR structure has been used due to its guaranteed stability and simplicity. It could be more advantageous, however, to use IIR structure rather than FIR, since the desired filter can be modeled with fewer parameters using both poles and zeros [1].

The adaptive IIR filtering methods are commonly classified into two categories: the equation error (EE) approach and the output error (OE) approach. The EE approach generates biased coefficient estimates in the presence of noise. On the other hand, the OE approach does not produce biased estimates. However, instability can occur if the poles are located closed to the unit circle. Also, the adaptive OE IIR filtering converges slowly [2].

To overcome instability and slow convergence problem, the polyphase adaptive IIR filtering was proposed [3], [4]. Although it converges faster than the conventional OE IIR filtering, it still shows a slow convergence problem.

Recently, for the purpose of improving the convergence speed, the subband adaptive filtering methods have been proposed [5], [6]. The subband approaches are computational efficient than the usual full-band schemes, and converge faster due to the separate processing in subbands.

In this paper, we present a novel subband adaptive IIR filtering method which has improved convergence performance over conventional IIR filters. By incorporating subband approaches in polyphase IIR filtering, merits are inherited and shortcomings are overcome, i.e., stable and fast convergent IIR filtering is achieved.

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II. Subband Adaptive IIR Filtering employing Polyphase Decomposition

Let a zero-mean discrete-time signal $u(n)$ be the input to an unknown system $H(z)$. Suppose that the unknown system is stable and causal, represented by a rational system function.

$$H(z) = \sum_{n=0}^{\infty} h_n z^{-n} = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{K} b_k z^{-k}}{1+\sum_{l=1}^{L} a_l z^{-l}}. \tag{1}$$

Then the output $y(n)$ of the unknown system $H(z)$ is represented by a difference equation such that

$$y(n) = \sum_{k=0}^{K} b_k x(n-k) - \sum_{l=1}^{L} a_l y(n-l). \tag{2}$$

When the output $y(n)$ is corrupted by additive noise, the observed or the desired signal in the system identification problem is given by

$$d(n) = y(n) + v(n), \tag{3}$$

where $v(n)$ is white measurement noise with variance σ_v^2. The system identification problem is to estimate a_l and b_k given $u(n)$ and $d(n)$.

Then, an adaptive IIR filter estimates the unknown system parameters has a system function of the form

$$\hat{H}(z) = \sum_{n=0}^{\infty} \hat{h}_n z^{-n} = \frac{\hat{B}(z)}{\hat{A}(z)} = \frac{\sum_{k=0}^{K} \hat{b}_k z^{-k}}{1+\sum_{l=1}^{L} \hat{a}_l z^{-l}}, \tag{4}$$

where $\hat{a}_l$ and $\hat{b}_k$ are the adaptive filter coefficients for estimating a_l and b_k, respectively.

A. Polyphase Adaptive IIR Filtering

We briefly review the polyphase decomposition of IIR filters. Consider the polynomial $P(z)$

$$P(z) = \prod_{i=1}^{N} \prod_{k=1}^{M-1} (1 - p_i e^{j(2\pi k/M)} z^{-1}), \tag{5}$$

where $\{p_i\}_{i=1}^{N}$ are roots of $A(z)$ and M is the polyphase expansion factor. Using $P(z)$, (1) is equivalently written as

$$H(z) = \frac{B(z)P(z)}{A(z)P(z)} = \frac{D(z)}{C(z^M)} = \frac{\sum_{k=0}^{NM} d_k z^{-k}}{1+\sum_{l=1}^{N} c_l z^{-lM}}. \tag{6}$$

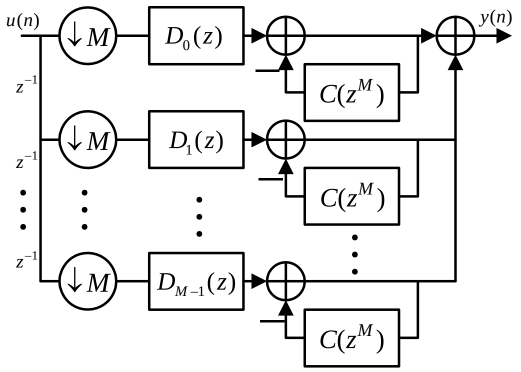

Fig. 1 Polyphase decomposition of IIR Filter

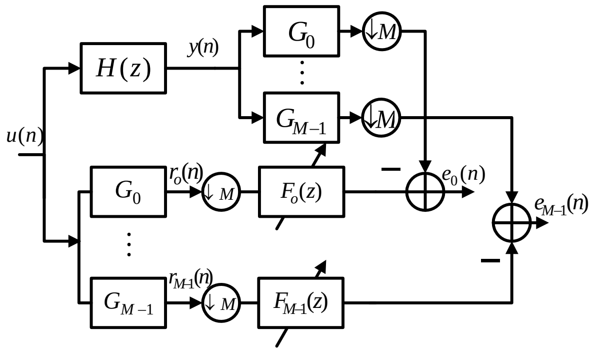

Fig. 2 Subband adaptive filtering structure

Note that it was shown that $C(z^M)$ and $D(z)$ are uniquely determined. In (6), the numerator can be decomposed into M polyphase components, i.e.,

$$\begin{aligned} D(z) &= \sum_{l=0}^{M-1}\Big(\sum_{k=0}^{N-1} d_{kM+l} z^{-(kM+l)}\Big) + d_{NM} z^{-NM} \\ &= \sum_{l=0}^{M-1} z^{-l} D_l(z^M), \end{aligned} \quad (7)$$

where

$$D_l(z^M) = \begin{cases} \sum_{k=0}^{N-1} d_{kM+l} z^{-kM} + d_{NM} z^{-NM} & if \quad l = 0 \\ \sum_{k=0}^{N-1} d_{kM+l} z^{-kM} & if \quad 1 \le l \le M-1. \end{cases}$$

Correspondingly, $H(z)$ is decomposed into M polyphase components,

$$\begin{aligned} H(z) &= \frac{D_0(z^M)}{C(z^M)} + z^{-1}\frac{D_1(z^M)}{C(z^M)} + \cdots + z^{-M+1}\frac{D_{M-1}(z^M)}{C(z^M)} \\ &= H_0(z) + z^{-1}H_1(z) + \cdots + z^{-M+1}H_{M-1}(z), \end{aligned} \quad (8)$$

where $H_i(z) = \dfrac{D_i(z^M)}{C(z^M)}$, $i = 0, 1, \cdots, M-1$.

Fig. 1 shows the implementation of the polyphase decomposition of $H(z)$. Thus, in the polyphase IIR filtering we need to estimate the polyphase components, c_l and d_k, instead of directly estimating the unknown system parameters, a_l and b_k.

It is known that the polyphase adaptive IIR filtering has improved convergence speed since its error surface is reduced and less flat than conventional adaptive IIR filtering. Also, the polyphase adaptive IIR filtering has been less likely unstable since the roots of $C(z^M)$ are not close to the unit circle as for $C(z)$.

B. Subband Adaptive Filtering With Polyphase decomposition

Fig. 2 shows subband adaptive filtering structure in system identification configuration. In subband adaptive filtering, the input signal $u(n)$ and the system output $y(n)$ are split into adjacent frequency subband by analysis filter banks G_i, $i = 0, 1, \ldots, M-1$. Each subband signal, $r_i(n), i = 0, 1, \cdots, M-1$, is subsampled and then is applied to adaptive filters, $F_i(z)$ in parallel.

To improve convergence speed of the adaptive IIR filtering, we incorporate the subband approach to polyphase IIR filter.

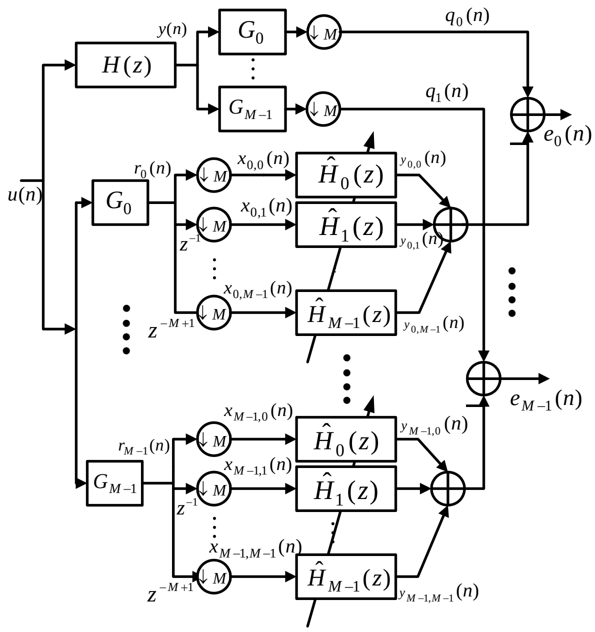

Fig. 3 Proposed subband IIR filtering structure

The proposed subband adaptive IIR filtering structure is presented in Fig. 3. In the proposed method, we use the M_{th} order polyphase structure for M-band structure.

From Fig. 3, we have computing each subband error $e_i(n)$, $i = 0, 1, \cdots, M-1$, as

$$\begin{aligned} e_0(n) &\equiv q_0(n) - [\mathbf{d}_0^T \mathbf{x}_{00}(n) + \mathbf{c}^T \mathbf{y}_{00}(n) + \mathbf{d}_1^T \mathbf{x}_{01}(n) + \mathbf{c}^T \mathbf{y}_{01}(n)] \\ e_1(n) &\equiv q_1(n) - [\mathbf{d}_0^T \mathbf{x}_{10}(n) + \mathbf{c}^T \mathbf{y}_{10}(n) + \mathbf{d}_1^T \mathbf{x}_{11}(n) + \mathbf{c}^T \mathbf{y}_{11}(n)], \end{aligned} \quad (9)$$

where $\hat{\mathbf{d}}_0^T = [d_{00}^T \ \ d_{01}^T \cdots d_{0N}^T]$, $\hat{\mathbf{d}}_1^T = [d_{10}^T \ \ d_{11}^T \ \cdots \ d_{1N}^T]$, $\hat{\mathbf{c}}^T = [c_M^T \ \ c_{2M}^T \ \cdots \ c_{NM}^T]$, $\mathbf{x}_{ij}(n) = [x_{ij}(n) \ \ x_{ij}(n-M) \ \cdots \ x_{ij}(n-NM)]$, and $\mathbf{y}_{ij}(n) = [y_{ij}(n) \ \ y_{ij}(n-M) \ \cdots \ y_{ij}(n-NM)]$

for $i,\ j = 0, 1$.

The adaptive filter in each band is adapted to minimize the power of the $e_i(n)$. Note that $\hat{H}_i(z)$, i_{th} polyphase component, is adapted at each subband, simultaneously. Then $\hat{H}_i(z)$ has a same coefficient at the end of adaptation. Now, we derive the adaptation algorithm to update $\hat{H}(z)$, efficiently.

To adapt $\hat{H}(z)$, we consider a stochastic recursive gradient-based iterative algorithm. Here, we represent two-band case for simplicity.

We define a cost function as

$$J(n) = E[\alpha_0 e_0^2(n) + \alpha_1 e_1^2(n)], \tag{10}$$

where α_0 and α_1 are proportional to the inverse of powers of $r_o(n)$ and $r_1(n)$, respectively, and $E(\cdot)$ denotes the expectation operator.

The recursive gradient-based algorithm for adaptation is given by

$$\begin{aligned} \hat{\mathbf{d}}_i(n+1) &= \hat{\mathbf{d}}_i(n) - \mu \frac{\partial J}{\partial \mathbf{d}_i} \qquad i = 0, 1 \\ \hat{\mathbf{c}}(n+1) &= \hat{\mathbf{c}}(n) - \mu \frac{\partial J}{\partial \mathbf{c}}, \end{aligned} \tag{11}$$

where μ is the step-size that controls converge of the adaptation process. Since each subband has the same polyphase structure of adaptive filter, $\hat{\mathbf{d}}_i(n)$ and $\hat{\mathbf{c}}(n)$ are adapted at each band, simultaneously.

From the cost function (10), we get

$$\begin{aligned} \frac{\partial J}{\partial \hat{\mathbf{d}}_i} &= 2\alpha_0 E[e_0(n)\frac{\partial e_0(n)}{\partial \hat{\mathbf{d}}_i}] + 2\alpha_1 E[e_1(n)\frac{\partial e_1(n)}{\partial \hat{\mathbf{d}}_i}] \\ \frac{\partial J}{\partial \mathbf{c}} &= 2\alpha_0 E[e_0(n)\frac{\partial e_0(n)}{\partial \mathbf{c}}] + 2\alpha_1 E[e_1(n)\frac{\partial e_1(n)}{\partial \mathbf{c}}]. \end{aligned} \tag{12}$$

Combining (9), (11) and (12) and replacing the expected value by the instantaneous values, we have the following stochastic recursive gradient-based update algorithm.

$$\begin{aligned} \hat{\mathbf{d}}_0(n+1) &= \hat{\mathbf{d}}_0(n) + 2\mu(\alpha_0 e_0(n)\mathbf{x}_{00}(n) + \alpha_1 e_1(n)\mathbf{x}_{10}(n)) \\ \hat{\mathbf{d}}_1(n+1) &= \hat{\mathbf{d}}_1(n) + 2\mu(\alpha_0 e_0(n)\mathbf{x}_{01}(n) + \alpha_1 e_1(n)\mathbf{x}_{11}(n)) \\ \hat{\mathbf{c}}(n+1) &= \hat{\mathbf{c}}(n) + 2\mu(\alpha_0 e_0(n)\mathbf{y}_{00}(n) + \alpha_1 e_1(n)\mathbf{y}_{10}(n)) \\ \hat{\mathbf{c}}(n+1) &= \hat{\mathbf{c}}(n) + 2\mu(\alpha_0 e_0(n)\mathbf{y}_{01}(n) + \alpha_1 e_1(n)\mathbf{y}_{11}(n)). \end{aligned} \tag{13}$$

When the subband order is expanded to M, we can obtain the adaptation algorithm as 2-band case.

As seen from Fig. 3, there are M filters to be adapted. The cost function is the extension of (10).

$$J(n) = E[\alpha_0 e_0^2(n) + \alpha_1 e_1^2(n) + \cdots + \alpha_{M-1} e_{M-1}^2(n)]. \tag{14}$$

Following the steps as in the two-subband case, we can obtain the adaptation equation for the filter coefficient as

$$\begin{aligned} \hat{\mathbf{d}}_i(n+1) &= \hat{\mathbf{d}}_i(n) + 2\mu \sum_{l=0}^{M-1} \alpha_l e_l(n)\mathbf{x}_{lk}(n) \\ \hat{\mathbf{c}}(n+1) &= \hat{\mathbf{c}}(n) + 2\mu \sum_{l=0}^{M-1} \alpha_l e_l(n)\mathbf{y}_{lk}(n) \\ k &= 0, 1, \cdots, M-1, \quad i = 0, 1, \cdots, M-1. \end{aligned} \tag{15}$$

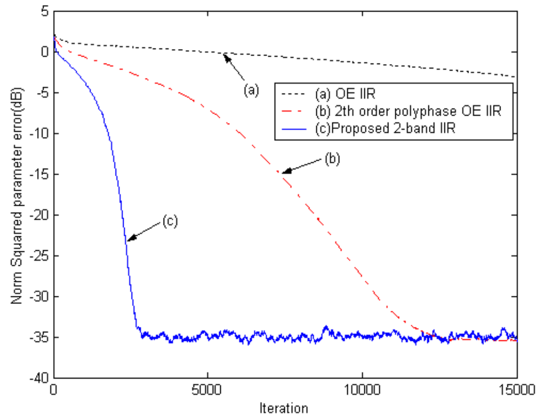

Fig. 4 Norm squared parameter error (M=2)

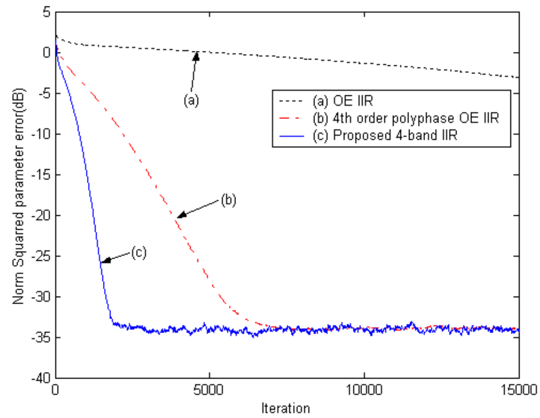

Fig. 5 Norm squared parameter error (M=4)

III. Simulation Results

Computer simulations are carried out in the system identification configuration to illustrate the convergence property of the proposed method. Conventional OE method and the polyphase OE method are compared with the proposed method. The signal is a white, zero-mean, Gaussian random sequence having unit-variance. The measurement noise sequence is Gaussian with variance σ_v^2. The squared norm of the parameter estimation errors, $\|\mathbf{a} - \hat{\mathbf{a}}(n)\|^2 + \|\mathbf{b} - \hat{\mathbf{b}}(n)\|^2$ and $\|\mathbf{c} - \hat{\mathbf{c}}(n)\|^2 + \|\mathbf{d} - \hat{\mathbf{d}}(n)\|^2$, are taken and averaged over 50 independent trials. The unknown system is given by

$$H(z) = \frac{0.2178 - 0.8402z^{-1} + 0.7971z^{-2}}{1 - 1.1314z^{-1} + 0.3591z^{-2}}.$$

The signal-noise ratio (SNR) is calculated by

$$SNR = 10\log(E[y^2(n)] / E[v^2(n)]).$$

For the simulation, SNR is set to 30dB. For subband decomposition, we used cosine modulated filter banks with length of 40. The step-sizes are chosen so that adaptive algorithms have the same steady-state performance.

Fig. 4 shows the learning curves of conventional OE IIR filtering, 2_{th} order polyphase OE IIR, and the proposed method when $M = 2$. As illustrated, the proposed method converges faster than other IIR filters. Fig. 5 shows the performance of the proposed method in case of $M = 4$. We observe that the convergence performance is improved as the order of band increases.

IV. Conclusion

By introducing polyphase decomposition to subband structure, we have present a novel adaptive IIR filtering scheme which provides improved convergence performance over conventional IIR filters in terms of convergence speed.

Reference

[1] J. J. Shynk, “Adaptive IIR filtering,” *IEEE Acoust., Speech, Signal Processing Mag.*, vol. 6, pp. 4-21, April 1989.

[2] N. Bershad and P. Feintuch, “The recursive adaptive LMS filter,” *IEEE trans. Acoust., Speech, Signal Processing*, vol, 28, pp. 650-660, Dec 1980.

[3] Phillip M. S. Burt and M. Gerken, “A Polyphase IIR Adaptive Filter,” *IEEE Trans. Circuits and System-II*, vol. 49, pp. 356-359, May 2002.

[4] R. Lopez-Valcarce, C. Mosquera and F. Perez-Gonzalez, “Hyperstable Adaptive IIR Algorithm with Polyphase structure,” *IEEE Trans. Signal Processing*, vol. 47, pp. 2043-2046, July 1999.

[5] A. Gilloire and M. Vetteli, ″Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experiments, and Application to Acoustic Echo Cancellation,” *IEEE Trans. Signal Processing*, vol. 40, pp. 1862-1875, Aug. 1992.

[6] S. S. Pradhan and V. U. Reddy, ″A New Approach to Subband Adaptive Filtering,” *IEEE Trans. Signal Processing*, vol. 47, pp. 655-664, March 1999.