World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:8, No:9, 2014

A Novel Stator Resistance Estimation Method and Control Design of Speed-Sensorless Induction Motor Drives

N. Ben Si Ali, N. Benalia, N. Zarzouri

***Abstract*—**Speed sensorless systems are intensively studied during recent years; this is mainly due to their economical benefit and fragility of mechanical sensors and also the difficulty of installing this type of sensor in many applications. These systems suffer from instability problems and sensitivity to parameter mismatch at low speed operation. In this paper an analysis of adaptive observer stability with stator resistance estimation is given.

***Keywords*—**Motor drive, sensorless control, adaptive observer, stator resistance estimation.

I. Introduction

THE use of induction machine without shaft encoder has known a great development. Modern control techniques avoid the use of sensors because they are unreliable and have an increased cost [1]. Sensorless systems require estimation of internal state variables of the machine such as speed and rotor flux from input variables like stator voltage and stator current. The estimation methods of speed and rotor flux for sensorless vector control drives are based on adaptive observer theory [2]-[4]. Machine parameters change during motor operation due to temperature rise. In this paper, adaptive observer stability and stator resistance estimation are analysed.

II. Development of Induction Motor Model

The induction motor is modelled from the equations below presented in the synchronous rotating reference frame

$$\begin{cases} \frac{d}{dt}\underline{\psi}_r = -(\frac{1}{T_r} + j\omega_{sl})\underline{\psi}_r + \frac{L_m}{T_r}\underline{i}_s \\ \frac{d}{dt}\underline{i}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\underline{\psi}_r - (a + j\omega_s)\underline{i}_s + \frac{1}{\sigma L_s}\underline{u}_s \end{cases} \quad (1)$$

where,

$\underline{u}_s = [u_{sd}, u_{sq}]^T$: Stator voltage vector

$\underline{i}_s = [i_{sd}, i_{sq}]^T$: Stator current vector

$\underline{\psi_r} = [\psi_{rd}, \psi_{rq}]^T$: Rotor flux vector

L_s, L_r, L_m : Stator, rotor and mutual inductance respectively.

R_s, R_r : Stator and rotor resistance

$\sigma = 1 - \frac{L_m^2}{L_s L_r}$: Leakage coefficient

$\omega_s, \omega, \omega_{sl}$ are the stator angular frequency, the motor angular velocity and the slip frequency, respectively.

$$b = \sigma L_s L_r,\ a = \frac{L_r^2 R_s + L_m^2 R_r}{\sigma L_s L_r^2} \text{ and } \omega_{sl} = \omega_s - \omega$$

III. Adaptive Observer

The conventional full order observer is defined by:

$$\begin{cases} \frac{d}{dt}\hat{\underline{\psi}}_r = -(\frac{1}{T_r} + j\hat{\omega}_{sl})\hat{\underline{\psi}}_r + \frac{L_m}{T_r}\hat{\underline{i}}_s + G_1(\underline{i}_s - \hat{\underline{i}}_s) \\ \frac{d}{dt}\hat{\underline{i}}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\hat{\omega})\hat{\underline{\psi}}_r - (\hat{a} + j\hat{\omega}_s)\hat{\underline{i}}_s + \frac{1}{\sigma L_s}\underline{u}_s + G_2(\underline{i}_s - \hat{\underline{i}}_s) \end{cases} \quad (2)$$

with: $\hat{a} = \frac{1}{\sigma L_s}(\hat{R}_s + \frac{L_m^2}{L_r^2}R_r)$, $G = [G_1 G_2]^T$

In order to derive the adaptive laws for rotor speed and stator resistance Lyapunov's theorem is used [4], [5].

Defining the state vector $\underline{x} = [\underline{\psi}_r\ \ \underline{i}_s]^T$ and its estimate $\hat{\underline{x}} = [\hat{\underline{\psi}}_r\ \ \hat{\underline{i}}_s]^T$.

From (1) and (2) the estimation error of rotor flux and stator current is described by the following equation:

$$\frac{d\underline{e}}{dt} = (A + GC)\underline{e} - \Delta A\hat{\underline{x}} \quad (3)$$

where

$$\underline{e} = \underline{x} - \hat{\underline{x}}$$

In observer (2) $\hat{\omega}$ and $\hat{R}_s$ are speed and stator resistance estimations respectively. They can be given by: $\hat{\omega} = \omega_s + \Delta\omega$, $\hat{R}_s = R_s + \Delta R_s$.

A. Rotor Speed Adaptation Law

In (3) for speed estimation we have:

$$\Delta A = \begin{bmatrix} 0_{2\times2} & -\frac{L_m}{b}\Delta\omega J \\ 0_{2\times2} & \Delta\omega J \end{bmatrix} \text{ with } J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

N. Ben Si Ali is with the Electrical Engineering Department Badji Mokhtar Annaba University, (e-mail: bensialin@ yahoo.fr).

N. Benalia and N. Zarzouri are with the Electrical Engineering Department Badji Mokhtar Annaba University.

We define the following Lyapunov function

$$V = e^T e + \frac{\Delta\omega^2}{\lambda} \tag{4}$$

with V, energy of signal e and λ a positive scalar.

Stability of observer is obtained for:

$$\frac{dV}{dt} \langle 0$$

This leads to

$$\frac{dV}{dt} = 2\underline{e}^T (A + GC)\underline{e} - 2\frac{\Delta\omega L_m}{b_s}(e_{id}\hat{\psi}_{rq} - e_{iq}\hat{\psi}_{rd}) + 2\frac{\Delta\omega}{\lambda}\frac{d\hat{\omega}}{dt} \tag{5}$$

The first term of (5) is negative semi definite [2]. Speed adaptation law is found by equalizing the second term to the third term.

$$\frac{d\hat{\omega}}{dt} = \frac{\lambda L_m}{b}(e_{id}\hat{\psi}_{rq} + e_{iq}\hat{\psi}_{rd}) \tag{6}$$

B. Stator Resistance Adaptation Law

In this case

$$\Delta A = \begin{bmatrix} -\frac{\Delta R_s}{\sigma L_s} I & 0_{2\times 2} \\ 0_{2\times 2} & 0_{2\times 2} \end{bmatrix} \text{ with } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Lyapunov function is defined by

$$V = e^T e + \frac{\Delta R_s^2}{\lambda} \tag{7}$$

Time derivative of V becomes

$$\frac{dV}{dt} = 2\underline{e}^T (A + GC)\underline{e} + 2\frac{\Delta R_s}{\sigma L_s}(e_{id}\hat{i}_{sd} + e_{iq}\hat{i}_{sq}) + 2\frac{\Delta R_s}{\lambda}\frac{d\hat{R}_s}{dt} \tag{8}$$

Stator adaptation law is

$$\frac{d\hat{R}_s}{dt} = -\frac{\lambda}{\sigma L_s}(e_{id}\hat{i}_{sd} + e_{iq}\hat{i}_{sq}) \tag{9}$$

Defining $e_\omega = \omega - \hat{\omega}$ and $e_{R_s} = R_s - \hat{R}_s$

$\hat{a}$ can be defined by $\hat{a} = a - \frac{1}{\sigma L_s} e_{R_s}$.

System describing the observer is then:

$$\begin{cases} \frac{d}{dt}\hat{\underline{\psi}}_r = -(\frac{1}{T_r} + j\omega_{sl})\hat{\underline{\psi}}_r + \frac{L_m}{T_r}\hat{\underline{i}}_s - je_\omega \hat{\underline{\psi}}_r + G_1(\underline{i}_s - \hat{\underline{i}}_s) \\ \frac{d}{dt}\hat{\underline{i}}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\hat{\underline{\psi}}_r - (a + j\omega_s)\hat{\underline{i}}_s + \frac{1}{\sigma L_s}\underline{u}_s + j\frac{L_m}{b}e_\omega \hat{\underline{\psi}}_r + \frac{1}{\sigma L_s}e_{R_s}\hat{\underline{i}}_s + G_2(\underline{i}_s - \hat{\underline{i}}_s) \\ \frac{d}{dt}\hat{\omega} = -K_i \Im\{\underline{e}_i \hat{\underline{\psi}}_r^*\} \\ \frac{d}{dt}\hat{R}_s = -K_i' \Re\{\underline{e}_i \hat{\underline{i}}_s^*\} \end{cases} \tag{10}$$

using the following hypothesis

$$\frac{d}{dt}\omega = 0 \tag{11a}$$

$$\frac{d}{dt}R_s = 0 \tag{11b}$$

$$\hat{\underline{\psi}}_r \to \underline{\psi}_r \tag{11c}$$

According to (11a) (11b) (11c), motor model (1) may be written as:

$$\begin{cases} \frac{d}{dt}\underline{\psi}_r = -(\frac{1}{T_r} + j\omega_{sl})\underline{\psi}_r + \frac{L_m}{T_r}\underline{i}_s \\ \frac{d}{dt}\underline{i}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\underline{\psi}_r - (a + j\omega_s)\underline{i}_s + \frac{1}{\sigma L_s}\underline{u}_s \\ \frac{d}{dt}\omega = 0 \\ \frac{d}{dt}R_s = 0 \end{cases} \tag{12}$$

IV. Stability Analysis

Study of the open loop observer stability (G=0) by linearising motor model (12) and the conventional full order observer (10) around an equilibrium operating point is carried out.

The new state vectors are defined as follows:

$$\underline{x} = \underline{x}_0 + \delta\underline{x} \text{ and } \hat{\underline{x}} = \hat{\underline{x}}_0 + \delta\hat{\underline{x}}$$

with

$$\underline{x}_0 = \begin{bmatrix} \underline{\psi}_{ro} & \underline{i}_{so} & \omega_o & R_{so} \end{bmatrix}^T, \hat{\underline{x}}_o = \begin{bmatrix} \hat{\underline{\psi}}_{ro} & \hat{\underline{i}}_{so} & \hat{\omega}_o & \hat{R}_{so} \end{bmatrix}^T$$

$$\delta\underline{x} = \begin{bmatrix} \delta\underline{\psi}_r & \delta\underline{i}_s & \delta\omega & \delta R_s \end{bmatrix}^T, \delta\hat{\underline{x}} = \begin{bmatrix} \delta\hat{\underline{\psi}}_r & \delta\hat{\underline{i}}_s & \delta\hat{\omega} & \delta\hat{R}_s \end{bmatrix}^T$$

The reference frame is synchronized with estimated rotor flux ($\hat{\psi}_{rqo} = 0$), then its two components are $\hat{\psi}_{rd} = \hat{\psi}_o + \delta\hat{\psi}_{rd}$ and $\hat{\psi}_{rq} = \delta\hat{\psi}_{rq}$.

In these two systems, stator pulsations are regarded as identical: $\omega_s = \hat{\omega}_s$ [5].

Defining $\delta e = \begin{bmatrix} \delta e_\psi & \delta e_i & \delta e_\omega \, \delta e_{Rs} \end{bmatrix}^T$, the system describing the estimation error is:

$$\begin{cases} \dfrac{d}{dt}\delta e_{\psi} = -(\dfrac{1}{T_r} + j\omega_{sl})\delta e_{\psi} + \dfrac{L_m}{T_r}\delta e_i + je_{\omega}\delta\underline{\hat{\psi}}_r - j\delta e_w\hat{\psi}_o - j\delta\omega_{slo}e_{\psi o} \\ \dfrac{d}{dt}\delta e_i = \dfrac{L_m}{b}(\dfrac{1}{T_r} - j\omega)\delta e_{\psi} - (a + j\omega_s)\delta e_i - j\dfrac{L_m}{b}e_{\omega}\delta\underline{\hat{\psi}}_r \\ -\dfrac{1}{\sigma L_s}\delta e_{R_s}\underline{\hat{i}}_{so} - j\dfrac{L_m}{b}\hat{\psi}_o\delta e_{\omega} - j\dfrac{L_m}{b}e_{\psi o}\delta\omega - \dfrac{1}{\sigma L_s}e_{Rs}\delta i_s - j\delta\omega_s e_{is} - \dfrac{\delta R_s}{\sigma L_s}e\hat{i}_{so} \\ \dfrac{d}{dt}\delta e_{\omega} = K_i(-e_{ido}\delta\hat{\psi}_{rq} + e_{iqo}\delta\hat{\psi}_{rd} + \hat{\psi}_o\delta e_{iq}) \\ \dfrac{d}{dt}\delta e_{R_s} = K_i'(\delta e_{id}\hat{I}_{sdo} + \delta e_{iq}\hat{I}_{sqo} + e_{id}\delta\hat{I}_{sd} + e_{iq}\delta\hat{I}_{sq}) \end{cases} \tag{13}$$

Splitting each state in d and q components, we define the state vector

$$\delta e = \lfloor \delta e_{\psi d} \quad \delta e_{\psi q} \quad \delta e_{id} \quad \delta e_{iq} \quad \delta e_{\omega} \quad \delta e_{Rs} \rfloor$$

The corresponding state matrix is:

$$\hat{A} = \begin{bmatrix} -\dfrac{1}{T_r} & \omega_{slo} & \dfrac{L_m}{T_r} & 0 & 0 & 0 \\ -\omega_{slo} & \dfrac{-1}{T_r} & 0 & \dfrac{L_m}{T_r} & \psi_o & 0 \\ \dfrac{L_m}{bT_r} & \dfrac{L_m}{b}\omega_o & -a & \omega_{so} & 0 & -\dfrac{\hat{I}_{sdo}}{\sigma L_s} \\ -\dfrac{L_m}{b}\omega_o & \dfrac{L_m}{bT_r} & -\omega_{so} & -a & -\dfrac{L_m}{b}\psi_o & -\dfrac{\hat{I}_{sqo}}{\sigma L_s} \\ 0 & 0 & 0 & K_i\psi_o & 0 & 0 \\ 0 & 0 & K_i'\hat{I}_{sdo} & K_i'\hat{I}_{sqo} & 0 & 0 \end{bmatrix} \tag{14}$$

V. Stator Resistance Estimation

First estimation of stator resistance with speed sensor is considered. State matrix becomes:

$$\hat{A} = \begin{bmatrix} -\dfrac{1}{T_r} & \omega_{slo} & \dfrac{L_m}{T_r} & 0 & 0 \\ -\omega_{slo} & -\dfrac{1}{T_r} & 0 & \dfrac{L_m}{T_r} & 0 \\ \dfrac{L_m}{bT_r} & \dfrac{L_m}{b}\omega_o & -a & \omega_{s0} & -\dfrac{\hat{I}_{sdo}}{\sigma L_s} \\ -\dfrac{L_m}{b}\omega_o & \dfrac{L_m}{bT_r} & \omega_{so} & -a & -\dfrac{\hat{I}_{sqo}}{\sigma L_s} \\ 0 & 0 & K_i'\hat{I}_{sdo} & K_i'\hat{I}_{sqo} & 0 \end{bmatrix} \tag{15}$$

Using the following property

$$\det(\hat{A}) = \prod_{i=1}^{5}\lambda_i \tag{16}$$

where λ_i are the Eigen values of matrix $\hat{A}$. The system stability implies that the five Eigen values must have a negative real part. Consequently, a condition of stability for system (15) is

$$\det(\hat{A}) \langle 0 \tag{17}$$

Stability limit is given by $\det(\hat{A}) = 0$.

Using Mapple/Matlab and without any simplification, we find:

$$\det(\hat{A}) = -\frac{k_i'\hat{I}_s^2}{\sigma L_s}[\frac{R_s}{T_r^2\sigma L_s} + \omega_{slo}^2\frac{R_s}{\sigma L_s} + \omega_{slo}\omega_s\frac{L_m^2 R_r}{\sigma L_s L_r^2}] \tag{18}$$

Condition $\det(\hat{A}) = 0$ leads to:

$$\omega_s = \frac{-R_s R_r(1 + \omega_{slo}^2 T_r^2)}{\omega_{slo}L_m^2} \tag{19}$$

Same result as in [2].

These stability conditions may be expressed in the torque/pulsation plane.

Under RFOC conditions and steady state ($\hat{\psi}_{rqo} = \psi_{rqo} = 0$), we obtain

$$i_{sqo} = \frac{L_r}{pL_m\hat{\psi}_o}T_{Lo} \tag{20}$$

From system (1) in the same conditions, we find

$$\omega_{slo} = \frac{L_m}{T_r\hat{\psi}_o}i_{sqo} \tag{21}$$

Finally using $\omega_{so} = \omega_{slo} + \omega_o$ (19) becomes:

$$T_{Lo}^2(\frac{R_s L_r^2}{L_m^2} + R_r) + p\psi_r^2\omega_o T_{Lo} + \frac{R_s}{L_m^2}p^2\psi_r^4) = 0 \tag{22}$$

Stability regions in the torque/pulsation plane are obtained by resolution of (22) Fig. 1.

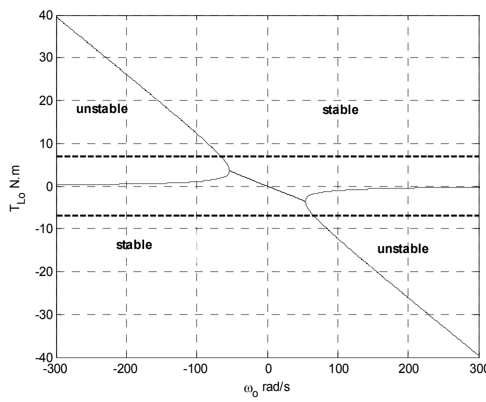


Fig. 1 Instability region in the pulsation/torque plan

Condition of stability (17) can be used only in regenerating mode. In order to validate this, we trace for each E.V, the locus, in the torque/speed plane, where, condition $\Re(\lambda_{i=1}^{5}) > 0$ is verified.

System stability implies that the five Eigen values must have a negative real part. Fig. 2 shows regions where different poles have a positive real part, respectively in regenerating and monitoring modes.

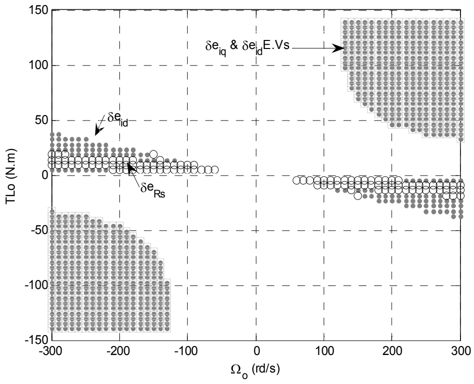
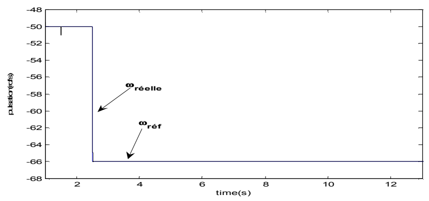


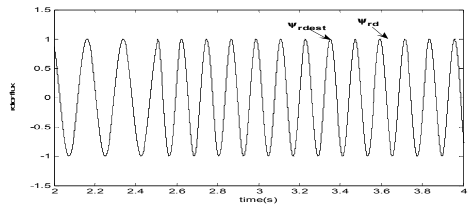
Fig. 2 Stator resistance estimation: instability regions (positive real parts)

VI. Simulation and Results Discussion

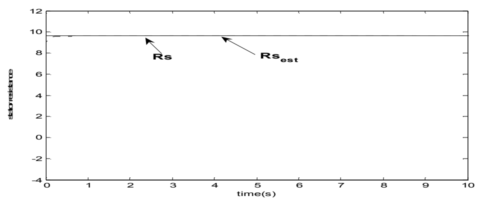
Figs. 3 and 4 illustrate simulation results for step change of pulsation reference under nominal torque.



(a)

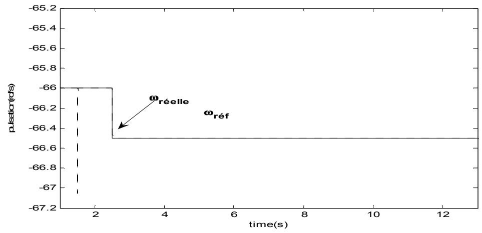


(b)

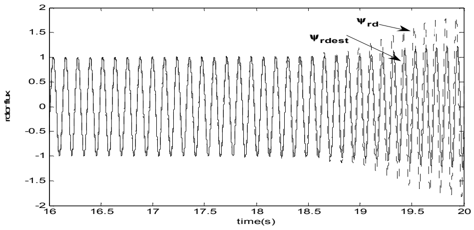


(c)

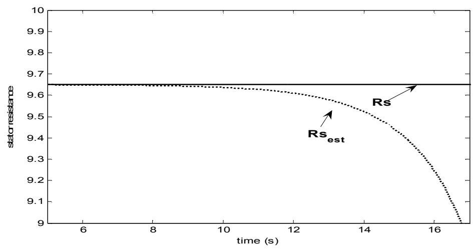
Fig. 3 Tests in regenerating mode, speed changes from $\omega_0 = -50\ rad/s$ to $\omega_0 = -66\ rad/s$ $T_{L0} = 7\ N.m$, $K_i = 300$, $Kp = 30$



(a)



(b)

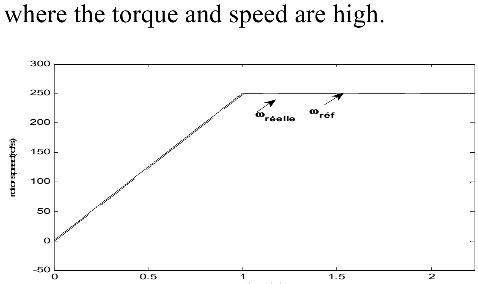


(c)

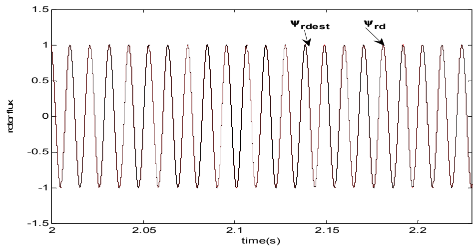
Fig. 4 Tests in regenerating mode. speed changes from $\omega_0 = -66\ rad/s$ to $\omega_0 = -66.5\ rad/s$, nominal torque $T_{L0} = 7\ N.m$, $K_i = 300$, $Kp = 30$

The load torque is kept at a nominal value $T_{Lo} = 7\ N.m$. Fig. 3 shows that system is stable. However, in Fig. 4 the reference speed has transited from stable to unstable region. Divergences on flux and resistance estimations are illustrated respectively in Figs. 4 (b) and (c).

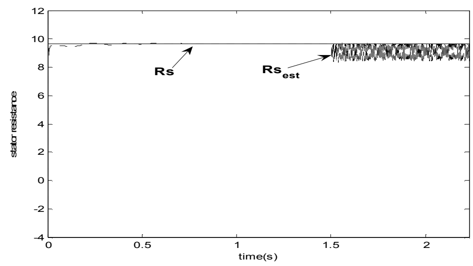
Fig. 5 shows simulation results for monitoring mode. In this case, as obtained on Fig. 2, instability regions are localized in the area where the torque and speed are high.



(a)



(b)



(c)

Fig. 5 Tests in monitoring mode (unstable region) $T_{L0} = 37{,}2\ N.m$,
$K_i = 300$, $Kp = 30$, $\Omega_0 = 250\ rad/s$

VII. Conclusion

In this paper it has been shown that stability of observers employed in sensorless control of induction motors is not ensured. Instability appears in both regenerating and monitoring modes. Dynamic performances and estimators tracking capability are strongly affected.

As perspective to continue this work it is important to study simultaneous speed and resistance estimation.

To minimize the instability regions, analysis of the three main approaches is required:

- Action on speed and stator resistance
- Action on feedback gain
- Simultaneous action on speed, stator resistance and feedback gain.

References

[1] G. c. Verghese, S. Sanders, “Observers for flux estimation in induction machines,” *IEEE Transactions on industrial electronics*, Vol. 35, n°. 1,1988.

[2] H. Kubota, K. Matsuse, and T. Nakano “DSP-based speed adaptive flux observer of induction motor”. *IEEE. Transaction on industry and application*. vol. 29, n°.2, March 1993.

[3] C. Shauder, “Adaptive speed identification for vector control of induction motors without rotational transducers”. *IEEE Transaction on industry application*, vol.28, n°.5, Sept 1992.

[4] C. Chaigne, E. Etien, S. Cauet, L. Rambault , “Commande vectorielle sans capteur des machines asynchrones” . *Hermes science* 2005.

[5] M. Hinkanen, “Analysis and design of full order flux observers for sensorless induction motor”. *IEEE transaction on industrial Electronics*, vol. 51,n°5. 2004.