

Temperature Effect on Sound Propagation in an Elastic Pipe with Viscoelastic Liquid

S. Levitsky, R. Bergman

Abstract—Fluid rheology may have essential impact on sound propagation in a liquid-filled pipe, especially, in a low frequency range. Rheological parameters of liquid are temperature-sensitive, which ultimately results in a temperature dependence of the wave speed and attenuation in the waveguide. The study is devoted to modeling of this effect at sound propagation in an elastic pipe with polymeric liquid, described by generalized Maxwell model with non-zero high-frequency viscosity. It is assumed that relaxation spectrum is distributed according to the Spriggs law; temperature impact on the liquid rheology is described on the basis of the temperature-superposition principle and activation theory. The dispersion equation for the waveguide, considered as a thin-walled tube with polymeric solution, is obtained within a quasi-one-dimensional formulation. Results of the study illustrate the influence of temperature on sound propagation in the system.

Keywords—Elastic tube, sound propagation, temperature effect, viscoelastic liquid.

I. INTRODUCTION

WAVE dispersion in a pipe with flowing viscoelastic liquid has importance for polymer processing technology. Parameters of wave propagation can be used for flow diagnostics and pipe inspection [1], [2]; novel methods of acoustic monitoring of polymer dynamic modules by the use of guided waves were verified recently [3], [4]. Mathematical model of sound wave propagation in an elastic cylindrical waveguide with non-Newtonian liquid was developed in [5], [6]; it accounts for fluid viscoelasticity inherent to polymeric liquids (solutions and melts), suspensions of elastic particles in viscous liquid, etc. At a macroscopic level this kind of non-Newtonian behavior is described by relaxational models using time derivatives of the stress tensor, or by equivalent integral equations [7]. Parameters in such models can be estimated from empirical data obtained at relaxation measurements, or from molecular theories. Polymeric liquids have complex ramified structure, and different structure units are characterized by different mobility. It results in distribution of relaxation times and phenomenologically is accounted for within models with a spectrum of relaxation times θ . Temperature has essential impact on both relaxation spectrum of liquid and its viscosity, which ultimately leads to temperature dependence of losses at acoustic wave propagation. Because low-frequency dispersion of acoustic wave in a tube is governed mainly by the liquid rheology [8], it may yield also changes in the sound speed in the waveguide.

S. Levitsky and R. Bergman are with the Shamoon College of Engineering, 56 Bialik St., Beer Sheva, 84100 Israel (e-mail: levits@sce.ac.il, rudolf@sce.ac.il).

The target of the study is to describe this coupling and to estimate its manifestation quantitatively for the case of polymeric liquid flow in a thin-walled cylindrical tube.

II. MODEL FORMULATION

A. Rheological Model

It is supposed that the liquid follows generalized linear Maxwell model [7], which is used usually for description of polymer liquids dynamics:

$$\tau_{ij} = 2 \int_{-\infty}^t G(t-t_1) s_{ij}(t_1) dt_1 + 2\eta_s s_{ij} \quad (1)$$
$$s_{ij} = e_{ij} - \frac{1}{3}(\nabla \cdot \vec{v})I, \quad e_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

Here $G(t-t_1)$ is the relaxation function; τ_{ij} , s_{ij} - deviators of stress and rate-of-strain tensors, respectively; η_s - low-molecular solvent viscosity; \vec{v} - liquid velocity. Rheological formulation (1) means that only shear viscoelasticity of liquid is accounted for hereafter and relaxation features at bulk deformations of pure liquid in the wave are neglected. It was shown [9] that the input of liquid volume viscoelasticity is small in a wide range of conditions, as compared with other sources of dispersion and dissipation. In the case of acoustically induced oscillations in the system with frequency ω , all mechanical characteristics (flow velocity, deformations, pressure, etc.) are proportional to $\exp(i\omega t)$, and viscoelastic properties of liquid can be described by the complex dynamic viscosity coefficient $\eta^* = \eta'(\omega) - i\eta''(\omega)$, which must be used instead of the usual dynamic viscosity η . Complex dynamic viscosity of liquid is connected with the relaxation spectrum $F(\theta)$ by the equation:

$$\eta^* = \eta_s + (i\omega)^{-1}G^*, \quad G^* = \int_0^{\infty} \frac{(\omega\theta)F(\theta)(1+i\omega\theta)}{1+(\omega\theta)^2} d\theta \quad (2)$$

For a large number of polymeric solutions the behavior of η^* can be closely approximated by a discrete spectrum with $\theta_k = \theta_1/k^\alpha$ ($k = 1, 2, \dots$), where $\alpha \geq 2$ is the distribution parameter (the Spriggs constant [7]). It leads to the following relation for dynamic liquid module G^* :

$$(i\omega)^{-1}G^* = \frac{\eta_p - \eta_s}{z(\alpha)} \sum_{k=1}^{\infty} \frac{k^\alpha - i\omega\theta_1}{k^{2\alpha} + (\omega\theta_1)^2} \quad (3)$$

$$z(\alpha) = \sum_{k=1}^{\infty} k^{-\alpha}$$

Here η_p , θ_1 are Newtonian viscosity of solution and the main relaxation time in the spectrum, respectively. Both η_p and θ_1 depend, besides the temperature T , from concentration ϕ and molecular mass M of the polymer, thermodynamic properties of the solvent, etc.

B. Parameter Temperature Dependence

The main relaxation time in the spectrum θ_1 can be estimated from the Rouse formula [10]:

$$\theta_1 = 0.608 \frac{(\eta_p - \eta_s)M}{\phi R_G T} \quad (4)$$

The dependency of Newtonian viscosity of the solution η_p on the polymer concentration ϕ at not too large values of ϕ is described usually by the Martin relation: $\eta_p = (1 + \beta \exp(k_M \beta)) \eta_s$. Here k_M is the Martin constant, $\beta = \phi[\eta]$ - reduced polymer concentration, $[\eta]$ - characteristic viscosity of solution, R_G is the universal gas constant and T the absolute temperature. The value of $[\eta]$ can be estimated from the Mark-Houwink relation

$$[\eta] = KM^a \quad (5)$$

where K and a are constants for a given polymer-solvent pair at a given temperature over a certain range of molecular mass variation. The parameter a (the Mark-Houwink exponent) lies in the range 0.5 to 0.6 for solutions of flexible chains polymers in thermodynamically bad solvents and in the range 0.7- 0.8 for good solvents [11]. For the former ones the constant $K \approx 10^{-2}$ (if the intrinsic viscosity $[\eta]$ is measured in cm^3/g), while for the latter $K \approx 10^{-3}$. Temperature dependence of the relaxation characteristics of polymeric liquids is described by the time-temperature superposition (TTS) principle [10], which states that with change in temperature the spectrum of relaxation times shifts as a whole in a self-similar manner along t axis, according to the value of the temperature-shift factor a_T , defined by the formula:

$$a_T = \frac{\rho(T_0)T_0(\eta_p(T) - \eta_s(T))}{\rho(T)T(\eta_p(T_0) - \eta_s(T_0))} \quad (6)$$

where ρ is the liquid density. With ωa_T for an argument it becomes possible to plot temperature-invariant curves $Re\{G_1^*(\omega a_T)\}$ and $Im\{G_1^*(\omega a_T)\}$. The dependence of

viscosity from the temperature can be described by the activation theory:

$$\eta_p = \eta_{p0} \exp[E_p (R_G T_0)^{-1} (T_0/T - 1)] \quad (7)$$

$$\eta_s = \eta_{s0} \exp[E_s (R_G T_0)^{-1} (T_0/T - 1)]$$

where E_p , E_s are activation energies of the solution and the solvent, respectively; $\eta_{p0} = \eta_p(T_0)$, $\eta_{s0} = \eta_s(T_0)$. The E_s value is usually about 10 to 20 kJ/mol . For low-concentrated solutions of polymers with moderate molecular masses, the difference between these two activation energies, $\Delta E = E_p - E_s$, does not exceed usually 10 kJ/mol [11]. For low-concentrated solutions of certain polymers in thermodynamically bad solvents negative ΔE values were reported [12].

C. Dispersion Equation for the Waveguide

Wave propagation equations for elastic tube with viscoelastic liquid are formulated within quasi-one dimensional approach. The tube wall is considered as a thin-walled elastic cylindrical circular shell with the width $2h$ and the middle surface radius R (it is supposed that $\varepsilon = h/R \ll 1$); the Kirchhoff-Love model is used for description of axisymmetric shell dynamics in the wave:

$$\frac{E}{1-\nu^2} \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\nu}{R} \frac{\partial u_r}{\partial x} \right) = \rho_s \frac{\partial^2 u_x}{\partial t^2} \quad (8)$$

$$\frac{Eh}{(1-\nu^2)R} \left(\nu \frac{\partial u_x}{\partial x} + \frac{u_r}{R} \right) + \frac{Eh^3}{3(1-\nu^2)} \frac{\partial^4 u_r}{\partial x^4} - \frac{1}{2} \Delta p_c = -\rho_s h \frac{\partial^2 u_r}{\partial t^2}$$

Here x , r are axial and radial coordinates of cylindrical coordinate system with the origin on the tube axis; u_x , u_r - longitudinal and transverse displacements of the shell middle surface; ρ_s , E , ν - density, Young and Poisson module of the tube material; Δp_c is the contact pressure equal to normal stress in liquid at the pipe wall. Boundary conditions for (8) are formulated at the liquid-shell interface for $r = R - h \approx R$. They have the form:

$$v_r = \frac{\partial u_r}{\partial t}, \quad v_x = \frac{\partial u_x}{\partial t}, \quad \Delta p_c = \Delta p_f - \tau_{rr}, \quad \Delta p_f = p_f - p_0 \quad (9)$$

where τ_{rr} is the normal component of deviatoric stress in liquid at the interface; v_x , v_r - liquid velocity components in longitudinal and transverse directions; p_f , p_0 - pressure in liquid in the wave and equilibrium pressure in the waveguide, respectively.

The basic assumptions of liquid dynamics in the tube at acoustic excitation in a low frequency range imply:

$$v_r \ll v_x, \quad \frac{\partial^2 v_x}{\partial x^2} \ll \frac{1}{r} \frac{\partial v_x}{\partial r}, \quad \frac{\partial^2 v_x}{\partial x^2} \ll \frac{\partial^2 v_x}{\partial r^2} \quad (10)$$

$$\dot{u}_x \ll V, \quad \frac{\partial v_r}{\partial x} \ll \frac{\partial v_x}{\partial r}$$

As a result, the momentum and mass balance equations for liquid in the tube can be written in the form:

$$\rho_{f0} \frac{\partial v_x}{\partial t} = -\frac{\partial p_f}{\partial x} + \int_{-\infty}^t G(t-t_1) \left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right) dt_1 \quad (11)$$

$$+ \eta_s \left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right)$$

$$\frac{\partial \rho}{\partial t} + \rho_{f0} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_x}{\partial x} \right) = 0 \quad (12)$$

where ρ, ρ_{f0} are liquid density in the wave and its equilibrium value, respectively, related to the pressure in the wave by the state equation $p_f = p_0 + c_f^2(\rho - \rho_{f0})$. Equation (11) doesn't account for the cross effect of liquid's rheology and compressibility, which is small and can be neglected. The equations (1), (11), (12) are averaged along the tube cross-section and solved with account for (10). The obtained solution is coupled with solution of the shell dynamics equations through the boundary conditions (9). Then a standard procedure [13] leads to the following dispersion equation in a non-dimensional form:

$$Az^2 - Bz + C = 0, \quad z = c^{-2}, \quad c = \Omega / k \quad (13)$$

$$\Omega = \omega t_0, \quad t_0 = R(\rho_s / p_0)^{1/2}, \quad k = \bar{k}R$$

$$A = i\Omega \varepsilon \lambda Q \left(\frac{1 - \nu^2 Q^{-1}}{1 - \nu^2} \right) - \Omega^2 \bar{\eta}, \quad B = \frac{\varepsilon \lambda N Q}{\bar{c}_f^2} \left(\frac{1 - \nu^2 Q^{-1}}{1 - \nu^2} \right)$$

$$+ \kappa^{-1} N - \lambda^{-1} \Omega^2 \bar{\eta} (1 - \nu^2) + i\Omega \left[\varepsilon Q + \frac{\nu}{2\kappa} + \frac{\bar{\eta} N}{\bar{c}_f^2} \right]$$

$$C = \frac{(1 - \nu^2)}{\lambda} \left[\kappa^{-1} N + \frac{i\omega \bar{\eta} N}{\bar{c}_f^2} \right] + \frac{\varepsilon Q N}{\bar{c}_f^2}$$

$$Q = 1 - \Omega^2 \lambda^{-1} (1 - \nu^2)$$

$$N = i\Omega + 8\kappa \bar{\eta} D, \quad D = -\frac{1}{4} \frac{\mu T(\mu)}{1 - 2\mu^{-1} T(\mu)}, \quad T(\mu) = \frac{J_1(\mu)}{J_0(\mu)}$$

$$\mu = i(\Omega / \kappa \bar{\eta})^{1/2}, \quad \kappa = \rho_s / \rho_{f0}, \quad \lambda = E / p_0, \quad \bar{c}_f = (t_0 / R) c_f$$

Here \bar{k} is the dimensional wave number, t_0 - characteristic time, c_f - sound speed in liquid, $\bar{\eta} = \eta^* / (p_0 t_0)$, and J_0, J_1 are the Bessel functions of the first kind of the zero and first order, respectively. The details of calculations can be found elsewhere [6].

III. NUMERICAL RESULTS

Dispersion equation (13) was studied numerically with account for the temperature dependence of rheological

parameters, according to relations (3), (4), (6), (7), which were written in dimensionless form as follows:

$$\bar{\theta}_{10} = 0.608 \bar{\eta}_{s0} A \exp(k_M \beta), \quad A = ([\eta] p_0 M) / (R_G T_0) \quad (14)$$

$$\bar{\theta}_1(T^*) = a_T \bar{\theta}_{10}, \quad a_T = (\bar{\rho} T^*)^{-1} (\bar{\eta}_p - \bar{\eta}_s) / (\bar{\eta}_{p0} - \bar{\eta}_{s0})$$

$$\bar{\eta}_p = \bar{\eta}_{p0} \exp[\bar{E}_p (T^{*-1} - 1)], \quad \bar{\eta}_s = \bar{\eta}_{s0} \exp[\bar{E}_s (T^{*-1} - 1)]$$

$$\bar{\rho}_{f0} = [1 + \bar{\alpha}_1 (T^* - 1)], \quad \bar{\rho}_{f0} = \rho_{f0}(T) / \rho_{f0}(T_0), \quad \bar{\alpha}_1 = \alpha_1 T_0$$

$$\{\bar{\eta}_p, \bar{\eta}_s, \bar{\eta}_{p0}, \bar{\eta}_{s0}\} = \{\eta_p, \eta_s, \eta_{p0}, \eta_{s0}\} / (p_0 t_0)$$

$$\{\bar{E}_p, \bar{E}_s\} = \{E_p, E_s\} / (R_G T_0), \quad T^* = T / T_0, \quad \theta_{10} = \theta_1(T_0)$$

Plots of non-dimensional sound speed $c = \Omega / \text{Re}\{k\}$ and attenuation $\chi = -\text{Im}\{k\}$ versus frequency Ω and temperature T^* were obtained for different values of reduced polymer concentration. The values of the system parameters chosen for simulations are collected in the Table I; they correspond to aluminium thin-walled tube filled with solution of polymer with $M \sim 10^6$ [9]. The non-dimensional values of activation energies correspond to $E_s = 12 \text{ kJ/mol}$, $E_p = 16 \text{ kJ/mol}$; the studied temperature range is equal approximately to 60K. The Spriggs distribution parameter α was chosen equal to 2 (Rouse distribution).

TABLE I
 TUBE AND LIQUID PARAMETER VALUES AT $T_0 = 293\text{K}$

Symbol	Quantity	Value
E	Young module	$7 \cdot 10^{10} \text{ N/m}^2$
ε	relative shell half-width	0.05
ν	Poisson module	0.34
p_0	equilibrium pressure	10^5 Pa
η_s	solvent viscosity	$0.1 \text{ Pa}\cdot\text{s}$
ρ_{f0}	liquid density	10^3 kg/m^3
ρ_s	tube material density	$2.7 \cdot 10^3 \text{ kg/m}^3$
R	middle surface tube radius	0.01 m
c_f	sound speed in liquid	1500 m/s
k_m	Martin constant	0.4
\bar{E}_p	dimensionless activation energy of the solution	6.6
\bar{E}_s	dimensionless activation energy of the solvent	4.9
A	dimensionless molecular parameter	500
$\bar{\alpha}_1$	dimensionless thermal expansion coefficient	0.29

Results of simulations are presented on Figs. 1-4. The studied non-dimensional frequency range for the chosen parameter values corresponds approximately to dimensional frequency $1 < f < 10^3 \text{ Hz}$, $f = \omega / 2\pi$. The plots on Fig. 1 were calculated for three pure viscous liquids with Newtonian viscosity equal to viscosity of polymeric solution ($\eta = \eta_p$) with different reduced concentration of polymer and the same activation energy E_p (it means that the data presented on Fig. 1 don't account for the liquid viscoelasticity). The curves 1, 1'; 2, 2'; 3, 3' correspond to $\beta = 2, 4, 6$, respectively. One can see that sound speed in the waveguide for more

concentrated solution is less than for low concentrated one; the speed of the wave grows with temperature. This result relates to the viscosity-controlled dispersion range, and is explained by viscosity reduction with T^* . As it follows from Fig. 1, temperature effect on sound speed is more essential at low frequencies and for more viscous liquids.

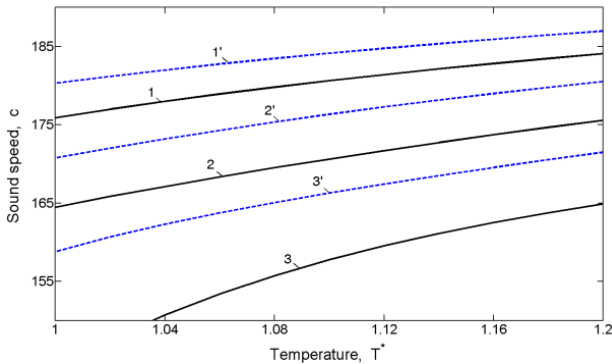


Fig. 1 Sound speed in the waveguide versus temperature - Newtonian liquid with viscosity of the solution. Solid line - $\Omega = 0.5$, dashed line - $\Omega = 1$

However, the temperature may have a different impact on the sound speed in the case, when the liquid possesses viscoelastic properties. The plots on the Fig. 2 correspond to the same variants as the plots on the Fig. 1 with only one difference – they account for the liquid viscoelasticity.

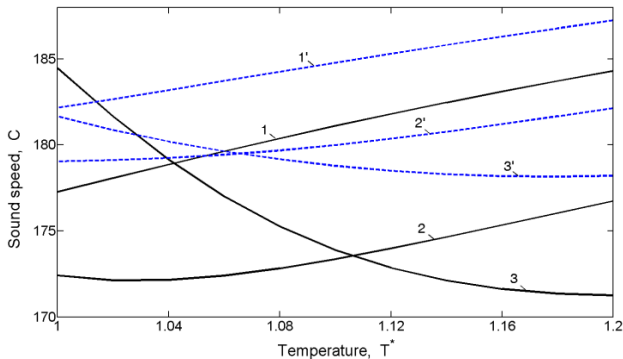


Fig. 2 Sound speed versus temperature - viscoelastic liquid. Solid line - $\Omega = 0.5$, dashed line - $\Omega = 1$

It follows from the data that in this case the temperature dependence of the sound speed may be changed not only quantitatively but even qualitatively, which is revealed in a speed decrease with T^* for sufficiently high concentration of polymer. The effect is frequency dependent – as it follows from comparison of the lines 3 and 3', the speed reduction with temperature at the frequency $\Omega = 0.5$ is more pronounced than at $\Omega = 1$.

In order to explain this result, the plots, characterizing sound dispersion in a low frequency range, were calculated for the same values of T^* and β as the curves on the Fig. 2. They are presented on the Fig. 3; the lines 1, 2 correspond to $\beta = 6$,

1', 2' - to $\beta = 2$. For the plots 1, 1' $T^* = 1$, for 2, 2' - $T^* = 1.2$.

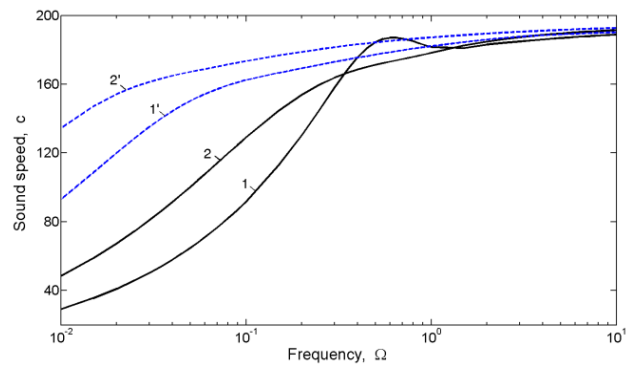


Fig. 3 Sound dispersion in the waveguide – temperature and concentration effect

As it follows from Fig. 3, sound dispersion in a waveguide with viscoelastic liquid can change its sign from positive to negative in a certain frequency range (curve 1), which results in an “overshoot”, characteristic for oscillatory flows of high-molecular liquids [7]. This frequency range is located near the frequency of viscoelastic transition $\omega_r \sim 1/\theta_1$; the effect takes place at sufficiently large values of molecular mass and concentration of polymer. The temperature increase yields decrease of both liquid viscosity and relaxation times; as a result, the “overshoot” disappears (curve 2). Because the curve 2 is located below the curve 1 in the vicinity of the “overshoot”, the temperature dependence of the sound speed on the Fig. 2 demonstrates unexpected reduction of c with T^* (plots 3, 3'). Note that at small polymer concentration (plots 1, 1') the “overshoot” on the dispersion curves is absent, and the temperature increase leads, as expected, to the sound speed increase.

Sound attenuation in the waveguide versus frequency is illustrated by the Fig. 4. The plots 1, 1', 3 correspond to $T^* = 1$, the plots 2, 2', 4 - to $T^* = 1.2$. For the lines 1, 1', 2, 2' $\beta = 6$; for 3, 4 - $\beta = 2$. The plots 1', 2' were calculated for the same parameter values as the graphs 1, 2, but without account for the liquid viscoelasticity – it means, for pure Newtonian liquid with $\eta = \eta_p$. One can see that attenuation of sound in a tube with viscoelastic liquid in the frequency range $\omega > \omega_r$ is less from that one for equivalent Newtonian liquid. It grows with polymer concentration, while the temperature increase yields the attenuation reduction. For all frequencies the sound attenuation in a tube with viscoelastic liquid stands below the attenuation in an equivalent waveguide with pure viscous liquid, possessing the same low-frequency viscosity.

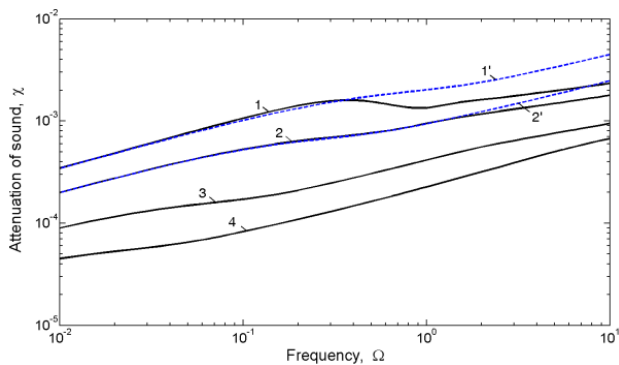


Fig. 4 Attenuation of sound in the waveguide – temperature and concentration effect

[11] S. P. Levitsky, and Z. P. Shulman, "Bubbles dynamics and boiling of polymeric solutions," in *Handbook of Solvents*, G. Wypych, Ed. Toronto: ChemTec Publishing, 2001, pp. 356-385.
 [12] V. Budtov, *Physical Chemistry of Polymer Solutions*. St Petersburg: Chemistry, 1992.
 [13] L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics*. 4th Ed, Wiley, 1999.

IV. CONCLUSION

Temperature has essential effect on sound propagation in a thin-walled tube with viscoelastic liquid. The study has revealed an unexpected sound speed reduction in the frequency range close to characteristic frequency of viscoelastic transition. The phenomenon is explained by existence of an "overshoot" on the sound dispersion curve for the waveguide with sufficiently concentrated high-molecular polymeric solution. Beyond this frequency range the rise of temperature leads to sound speed growth. Attenuation of sound decreases with temperature in the whole studied frequency range.

ACKNOWLEDGMENT

The study was supported by the Shamoon College of Engineering (SCE).

REFERENCES

[1] Y. Zhang, S. T. Lie, Z. Xiang, and Q. Lu, "A frequency shift curve based damage detection method for cylindrical shell structures," *J. Sound Vib.*, vol. 333, pp. 1671-1683, 2014.
 [2] R. Hou, A. Hunt, and R. A. Williams, "Acoustic monitoring of pipeline flows: particulate slurries," *Powder Technol.*, vol. 106, pp. 30-36, 1999.
 [3] R. Kazys, L. Mazeika, R. Sliteris, and R. Raisutis, "Measurement of viscosity of highly viscous non-Newtonian fluids by means of ultrasonic guided waves," *Ultrasonics*, vol. 54, pp. 1104-1112, 2014.
 [4] C. Glorieux, J. Descheemaeker, J. Vandenbroeck, J. P. Groby, L. Boeckx, P. Khurana, *et al.*, "Temperature and frequency dependence of the visco-elasticity of a poro-elastic layer," *Appl. Acoust.*, vol. 83, pp. 123-126, 2014.
 [5] S. Levitsky, R. Bergman, O. Levi, and J. Haddad, "Pressure waves in elastic tube with polymeric solution," *J. Appl. Mech. Eng.*, vol. 4, pp. 561-574, 1999.
 [6] S. P. Levitsky, R. M. Bergman, and J. Haddad, "Sound dispersion in a deformable tube with polymeric liquid and elastic central rod," *J. Sound Vib.*, vol. 275, pp. 267-281, 2004.
 [7] R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Liquids*. Vol. 1: *Fluid Mechanics*. John Wiley & Sons, 1987.
 [8] S. Levitsky, and R. Bergman, "Low frequency wave propagation in cylindrical elasto-viscoelastic trilayer in the presence of free gas," in *2013 Ultrasonics Symposium (IUS), 2013 IEEE International*, pp. 1610-1613.
 [9] S. P. Levitsky, and Z. P. Shulman, *Bubbles in Polymeric Liquids. Dynamics and Heat-Mass Transfer*. Lancaster, USA: Technomic Publishing Co., Inc., 1995.
 [10] A. Y. Malkin, "The state of the art in the rheology of polymers: Achievements and challenges," *Polym. Sci. Ser. A*, vol. 51, pp. 80-102, 2009.