

Implementation of Intuitionistic Fuzzy Approach in Maximizing Net Present Value

Gaurav Kumar, Rakesh Kumar Bajaj

Abstract—The applicability of Net Present Value (NPV) in an investment project is becoming more and more popular in the field of engineering economics. The classical NPV methodology involves only the precise and accurate data of the investment project. In the present communication, we give a new mathematical model for NPV which uses the concept of intuitionistic fuzzy set theory. The proposed model is based on triangular intuitionistic fuzzy number, which may be known as Intuitionistic Fuzzy Net Present Value (IFNPV). The model has been applied to an example and the results are presented.

Keywords—Net Present Value, Intuitionistic Fuzzy Set, Investment Projects.

I. INTRODUCTION

ENGINEERING economics involves the evaluation of an investment project based on the major economic criteria such as the Net Present Value (NPV), future cash flows, discount rates, internal rate of return (IRR), Payback period. In recent years, a number of publications have dealt with the project scheduling problem under the npv objective is presented by Groenendaal et al. [1], Biezma et al. [2]. The majority of the contributions assume a completely deterministic project setting, in which all relevant problem data, including the various cash flows, are assumed known from the outset. An activity scheduling problem for a project where cash inflows and outflows are given and availability restrictions are imposed on capital and renewable resources is presented by C.S. Sung and S.K. Lim [12]. The multi-mode resource-constrained project scheduling problem with discounted cash flows is considered by M. Mika et al. [3]. Discrete continuous project scheduling problems with discounted cash flows are considered by G. Waligora [4]. Rafiee and Kianfar [5] also offered a scenario tree approach for multi-period project selection problems using the real option valuation method. Last but not least, Gutjahr et al. [6] proposed the multi-objective decision analysis for competence oriented project portfolio selection.

In real world application, the investment project may sometimes be associated with incomplete and imprecise/uncertain knowledge about future cash flows, rates of discount, discounted cash flows, profitability index etc. The reasons for uncertainty in investment projects are mainly due to the lack of precise information about the future

events which are not familiar to us. In order to deal with such uncertain values which may be abstract in nature, the theory of fuzzy sets has been applied in the literature Kahraman et al.[7], Sheen et al.[9] of engineering economics. A.H.Russell [19] was the First to introduce the objective of maximizing the NPV of cash flows in a network.

Several researchers have therefore proposed a series of excellent studies about the fuzzy techniques in order to assess the investment project. For example, Buckley [14] studied the fuzzy extension of the mathematics of finance to concentrate on the compound interest law. Kuchta [16] also generalized fuzzy equivalents for the methods of evaluating investment projects. Dourra and Siy [15] applied fuzzy information technologies to investments through technical analysis, and used it to examine various companies to achieve a substantial investment return. Majid Shahriari [8] proposed fuzzy Net Present Value (FNPV) by using the concept of triangular fuzzy numbers instead of a crisp number. The theory of fuzzy sets Zadeh [21] has capability to capture uncertain situations and provides us with a meaningful and powerful representation of such situations. In fuzzy set theory, a degree of membership is assigned to each element, where the degree of non-membership is just automatically equal to 1 minus the degree of membership.

In literature, Net Present Value (NPV) methodology is becoming more and more popular tool for the analysis of an investment project. The classical NPV model involves only the crisp and the certain data about the investment project. In real life applications, human being who expresses the degree of membership of a fuzzy set very often does not express the corresponding degree of non-membership as the complement to 1, i.e., there may be some hesitation degree. Atanassov [17] introduced the concept of intuitionistic fuzzy sets (IFS), as a generalization of fuzzy sets, which is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research. For example, in decision making problems, particularly in the case of sales analysis, new product marketing, financial services, etc. Atanassov et al. have widely applied theory of intuitionistic fuzzy sets in logic programming, Szmidt and Kacprzyk [18] in group decision making. Therefore, in various engineering applications, intuitionistic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years as the application of intuitionistic fuzzy sets introduces another degree of freedom into a set description.

The demonstration of the proper use of intuitionistic fuzzy sets in the evaluation of the investment projects is the main objective of the paper. In addition to this, we also explore its

Gaurav Kumar is with the Singhania University, Rajasthan, India as research scholar. (e-mail: gaurav.joshi8403@gmail.com).

Dr. Rakesh Kumar Bajaj is with the Department of Mathematics, Jaypee Institute of Technology, Waknaghat, H.P. India. (e-mail: rakesh.bajaj@gmail.com).

capacity to provide relevant information towards the decision making process under uncertain and imprecise conditions. In Section II, we present the basic definitions and notations of the Net Present Value (NPV) and Intuitionistic fuzzy set theory. In Section III, the existing fuzzy NPV model along with the new proposed Intuitionistic Fuzzy Net Present Value (IFNPV) model have been explained. An illustrative example has been provided for the proposed model in Section IV. Finally, the conclusions and scope for the future work have been pointed out in Section V.

II. PRELIMINARIES

In this section, we describe the basic concepts of Net Present Value (NPV), Intuitionistic Fuzzy Sets (IFSs), Triangular Intuitionistic Fuzzy Number.

A. Evaluating Investment Project

"Investment" is a term which can be interpreted as real investment in a project as well as financial investment in equities, bonds or other securities. The decisions regarding the investment is one of the most important decisions taken by the management and the authorities. NPV is one of the most reliable measure used in capital budgeting for the evaluation of profitability for an investment project. NPV analysis is sensitive to the reliability of future cash inflows that an investment project will yield.

1) *Present Value and Net Present Value of an Investment Project*: Present value, also known as *present discounted value*, is the current worth of a future sum of money or stream of cash flows given a specified rate of return. Future cash flows are discounted at the discount rate and the higher the discount rate, the lower the present value of the future cash flows. In simple words, *present value* describes how much a future sum of money is worth today. The formula for evaluating the present value is given by

$$PV = \frac{CF}{(1+r)^n}, \quad (1)$$

where CF is cash flow in future period, r is the periodic rate of return or interest (also called the discount rate or the required rate of return) and n is the number of periods. The concept of present value is one of the most fundamental and pervasive in the world of finance. It is the basis for stock pricing, bond pricing, financial modeling, banking, insurance, pension fund valuation and even lottery pay-outs.

Net present value (NPV) is the present value of an investment's expected cash inflows minus the costs of acquiring the investment. Mathematically, the term Net Present Value (NPV) is given by

$$NPV = (\text{Cash inflows from investment}) - (\text{cash outflows or costs of investment})$$

,i.e.,

$$NPV = \sum_{t=1}^T \frac{C_t}{(1+r)^t} - C_0, \quad (2)$$

where C_0 is the initial cash outflow for the project at the beginning of the investment; C_t is the cash inflow at period t and r is the periodic rate of return or interest.

If the discount rates vary over time (i.e., r_1 is the rate in the first period, r_2 is the rate in the second period etc.) we would have to employ a more basic form of the calculation given by

$$NPV = \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_1)(1+r_2)} + \frac{C_3}{(1+r_1)(1+r_2)(1+r_3)} + \dots - C_0, \quad (3)$$

which would be tedious to calculate by hand but is fairly easy to implement in a spreadsheet.

The NPV calculation is very sensitive to the discount rate: a small change in the discount rates causes a large change in the NPV. NPV measures the overall profit of the project expressed in the beginning of the project. It may be noted that if the NPV of a prospective project is positive, it should be accepted. On the other hand, if NPV is negative, the project should probably be rejected because cash flows will also be negative. Basically, it compares the present value of money today to the present value of money in future, taking inflation and returns into account.

B. Intuitionistic Fuzzy Environment

Intuitionistic Fuzzy set theory Atanassov, deals with the rationality of uncertainty due to imprecision and vagueness in human thoughts and perceptions along with a degree of hesitation/indeterminacy in the decisions. In literature, it has been found that none has studied the net present value in a intuitionistic fuzzy environment, however there are studies under a fuzzy setup by various researchers Chiu et al.[10], Gutierrez [11], Montevechi et al.[20]. Here, we provide basics and fundamentals related to Intuitionistic Fuzzy setup.

Atanassov's intuitionistic fuzzy set (IFS) over a finite non empty fixed set X , is a set \tilde{A} defined as

$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle \mid x \in X \}$ which assigns to each element $x \in X$ to the set \tilde{A} , which is subset of X having the degree of membership $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$ and degree of non-membership $\nu_{\tilde{A}}(x) : X \rightarrow [0, 1]$, satisfying $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$, for all $x \in X$.

We denote $\mathcal{F}(X)$ the set of all the IFSs on X . For each intuitionistic fuzzy set in X , a hesitation margin $\pi_{\tilde{A}}(x)$, whether x belongs to \tilde{A} or not, which is the intuitionistic fuzzy index of element x in the IFS \tilde{A} , defined by $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$, denotes a measure of non-determinacy. It may be noted that the application of intuitionistic fuzzy sets instead of fuzzy sets introduces another degree of freedom into a set description (i.e. in addition to $\mu_{\tilde{A}}$, we also have $\nu_{\tilde{A}}$ or $\pi_{\tilde{A}}$).

A Triangular intuitionistic fuzzy number (TIFN) \tilde{A} is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}}(x)$ and non-membership function $\nu_{\tilde{A}}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_2-a_3}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1}, & \text{for } a'_1 \leq x \leq a_2 \\ \frac{x-a_2}{a'_3-a_2}, & \text{for } a_2 \leq x \leq a'_3 \\ 1, & \text{otherwise} \end{cases} \quad (5)$$

where $a'_1 < a_1 < a_2 < a_3 < a'_3$ and

$\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$, or $\mu_{\tilde{A}}(x) = \nu_{\tilde{A}}(x)$, for all $x \in R$.
This TIFN is denoted by $\tilde{A} = ((a_1, a_2, a_3)(a'_1, a_2, a'_3))$.

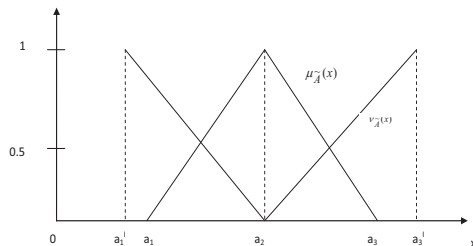


Fig. 1. Membership and non-membership functions of TIFN

C. Weighted Ranking of Triangular Intuitionistic Fuzzy Number

Let $\tilde{A} = ((a_1, a_2, a_3)(a'_1, a_2, a'_3))$ be a TIFN, then we define a preference for TIFN [13] i.e. for membership and non-membership values respectively as $P(\mu_{\tilde{A}}) = \frac{a_1 + a_2 + a_3}{3} + wa_2$ and $P(\nu_{\tilde{A}}) = \frac{a'_1 + a_2 + a'_3}{3} + wa_2$.

Then we define a ranking method based on membership and non-membership to de-fuzzify the number and which is given by

$$R(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a'_1 + a_2 + a'_3 + 6wa_2}{6}, \quad (6)$$

where w is the relative weight determined by nature and the magnitude of the most promising value.

Let $\tilde{A} = (a_1, a_2, a_3)(a'_1, a_2, a'_3)$ and $\tilde{B} = (b_1, b_2, b_3)(b'_1, b_2, b'_3)$ be two TIFNs and $R(\tilde{A})$, $R(\tilde{B})$ be average preference values of \tilde{A} and \tilde{B} , respectively. Then we can rank the TIFNs as follows:

- if $R(\tilde{A}) \geq R(\tilde{B})$ then $\tilde{A} \geq \tilde{B}$;
- if $R(\tilde{A}) \leq R(\tilde{B})$ then $\tilde{A} \leq \tilde{B}$;
- if $R(\tilde{A}) = R(\tilde{B})$ then $\tilde{A} = \tilde{B}$.

III. PROPOSED INTUITIONISTIC FUZZY NET PRESENT VALUE MODEL

General formulation of Intuitionistic fuzzy net present value for TIFN is generated given below:

$$(IFNPV) = \sum_{\substack{t=0 \\ ICF < 0}}^n \frac{(ICF)_{tl}}{(1+i_l)^t} + \sum_{\substack{t=0 \\ ICF > 0}}^n \frac{(ICF)_{tm}}{(1+i_m)^t} + \frac{(ICF)_{tr}}{(1+i_r)^t}, \quad (7)$$

where

$(IFNPV)$: Intuitionistic net present value of the project for

total time period t .

$(ICF)_t$: Net Intuitionistic fuzzy cash flow of the project estimated by the decision maker at t -th time period.

i : Required rate of return of the project estimated by the decision maker at t -th time period.

A. Intuitionistic Cash Flow Weight Algorithm

There are four steps on intuitionistic fuzzy cash flow weight algorithm. Suppose that the intuitionistic fuzzy cash flow weights of each activity are denoted by $ICFWs$ [22]. Step 1.

- $ICFWs$ are determined and all activities are added without predecessors to the available list.
- $ICFWs$ values are ordered using propose ranking method in Section II.
- The activity with the highest $ICFWs$, is selected from the list of precedence available.

In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively. After assignment of the selected activity the resource available list is updated.

- If there is any unassigned activity the third step is repeated, otherwise the project schedule is completed.

B. Intuitionistic Discounted Cash Flow Weight Algorithm

Intuitionistic fuzzy discounted cash flow algorithm has the same procedure with fuzzy cash flow algorithm while it deals with intuitionistic fuzzy discounted cash flow weights $DICFWs$ instead of $ICFWs$. $DICFWs$ for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities.

IV. ILLUSTRATIVE EXAMPLE

In this section, we will illustrate the methodology given in the preceding section to evaluate the net present value with the following example.

A network diagram of a project is given in Fig.2 based on the Table III with the intuitionistic fuzzy cash flow, resource requirement and duration of the tasks. And the resource availability for the project is limited to 5.

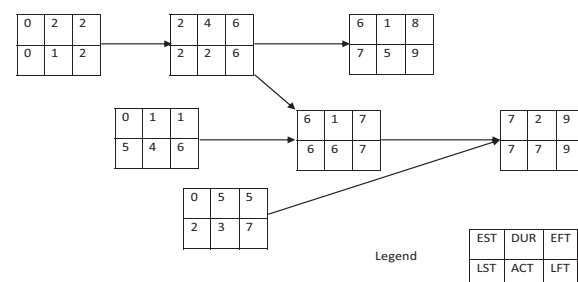


Fig. 2. Project Diagram

A. Intuitionistic Fuzzy Cash Weight Algorithm

By using the algorithm based on Intuitionistic fuzzy cash flow weighting given in the section III, the calculated values and the ranking of calculated *ICFW* for the tasks 1 to 7 are given in the Table IV.

From the Table IV, ranking of *ICFW* of each activity gives the priority for scheduling in the project. From the above proposed ranking method for TIFN calculated ranking, activity 1 is having highest ranking and scheduled first in period 1-2 and available resources are updated as 4. Activity 2 which has the second highest value is scheduled in the period 3-6 and available resources updated as 3. Activity 4 which has the third highest value is scheduled in the period 1 and available resources are updated as 2. Activity 3 which has the next highest value is scheduled in the period 3-7 and available resources are updated as 0 for the period 3-6 and as 2 for period 7. Activity 6 has next highest value is scheduled in the period 7 and available resources are updated as 1. Activity 5 has next highest value is scheduled in the period 8 and available resources are updated as 1. Activity 7 has next highest value is scheduled in the period 8-9 and available resources are updated as 3.

By using the ranking method for *ICFW*, the resulting project schedules is given in Fig.3

By using the schedule for *ICFW* given in Fig. 3, the intuitionistic fuzzy net present value (IFPV) with a intuitionistic fuzzy interest rate (FIR) is calculated as :

TABLE I
IFPV BY ICFW

FIR	IFPV
(0.06, 0.08, 0.10)(0.04, 0.08, 0.12)	(166.1, 218.6, 266.9)(142.2, 218.6, 317.6)

B. Intuitionistic Fuzzy Discounted Cash Weight Algorithm

By using the algorithm based on Intuitionistic fuzzy discounted cash flow weighting given in the section IV, the calculated values and the ranking of *DICFW* for the tasks 1 to 7 are given in the Table V. From the above proposed ranking method and from Table V, Activity 1 is having highest ranking and scheduled first in period 1-2 and available resources are updated as 4. Activity 2 which has the second highest value is scheduled in the period 3-6 and available resources updated as 3. Activity 4 which has the third highest value is scheduled in the period 1 and available resources are updated as 2. Activity 6 which has the next highest value is scheduled in the period 7 and available resources are updated as 4. Activity 3 has next highest value is scheduled in the period 2-6 and available resources are updated as 1 for the period 2 and as 0 for the period 3-6. Activity 5 has next highest value is scheduled in the period 7 and available resources are updated as 0. Activity 7 has next highest value is scheduled in the period 8-9 and available resources are updated as 3. By using the schedule from *DICFW* given in Fig.4, the intuitionistic fuzzy net present value (IFPV) with a intuitionistic fuzzy interest rate (FIR) is calculated as :

TABLE II
IFPV BY DICFW

FIR	IFPV
(0.06, 0.08, 0.10)(0.04, 0.08, 0.12)	(171.6, 224.9, 287.3)(158.2, 224.9, 302.5)

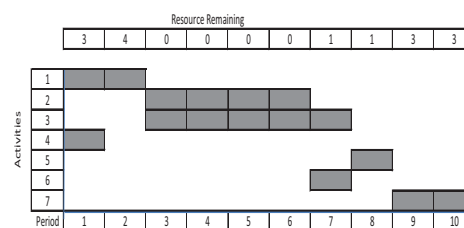


Fig. 3. Project Schedule Resulting from *ICFW*

TABLE III
PROJECT DATA

Task	IFCF	Predecessors	Duration	Resource Requirement
1	(40, 50, 60)(35, 40, 45)	-	2	1
2	(35, 40, 45)(30, 40, 50)	1	4	2
3	(43, 55, 67)(40, 55, 70)	-	5	3
4	(22, 24, 26)(20, 24, 30)	-	1	2
5	(37, 45, 53)(35, 45, 55)	2	1	4
6	(35, 50, 65)(30, 50, 70)	2, 4	1	1
7	(2, 10, 18)(1, 10, 20)	3, 6	2	2

TABLE IV
PROJECT DATA WITH CASH FLOW WEIGHTING

Task	IFCF	IFCW	Ranking
1	(40, 50, 60)(35, 40, 45)	(149, 195, 241)(131, 195, 260)	1
2	(35, 40, 45)(30, 40, 50)	(109, 145, 181)(96, 145, 195)	2
3	(43, 55, 67)(40, 55, 70)	(45, 65, 85)(41, 65, 90)	4
4	(22, 24, 26)(20, 24, 30)	(59, 84, 109)(51, 84, 120)	3
5	(37, 45, 53)(35, 45, 55)	(37, 45, 53)(35, 45, 55)	6
6	(35, 50, 65)(30, 50, 70)	(37, 60, 83)(31, 60, 90)	5
7	(2, 10, 18)(1, 10, 20)	(2, 10, 18)(1, 10, 20)	7

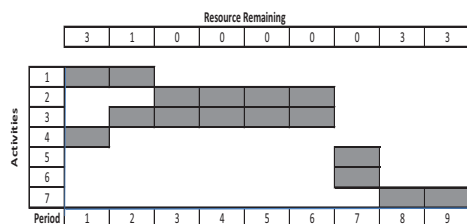


Fig. 4. Project Schedule Resulting from DICFW

TABLE V
PROJECT DATA WITH DISCOUNTED CASH FLOW WEIGHTING

T	ICF	DICFW	R
1	(40, 50, 60)(35, 40, 45)	(76.1, 101.7, 136.7)(62.4, 101.7, 170.9)	1
2	(35, 40, 45)(30, 40, 50)	(63.2, 87.1, 111.7)(51.1, 87.1, 145.5)	2
3	(43, 55, 67)(40, 55, 70)	(43.4, 58.1, 75.8)(40.1, 58.1, 81.1)	5
4	(22, 24, 26)(20, 24, 30)	(55.3, 78.2, 96.7)(47.4, 78.2, 115.1)	3
5	(37, 45, 53)(35, 45, 55)	(37, 45, 53)(35, 45, 55)	6
6	(35, 50, 65)(30, 50, 70)	(36.8, 59.9, 81.9)(30.8, 59.2, 89.2)	4
7	(2, 10, 18)(1, 10, 20)	(2, 10, 18)(1, 10, 20)	7

V. CONCLUSIONS

This paper has proposed a practicable notion through the conscientious mathematical derivation and proposed IFNPV technique offer the potential for flexibility beyond its classical interpretation. In practice, since the parameters in classical NPV formula such like real net cash flows and required rate of return in each year may vary with the shifting economy. Therefore, the IFNPV will more fit in with real situation to capture these uncertain parameters. In this paper, we have proposed a new model to calculate NPV under Intuitionistic fuzzy environment. We have also proposed two different methods for project scheduling to maximize IFNPV of investment project. Further, we conclude that resulting net present value from these two methods are different which make impact on investment project. We have also applied a ranking method on these two algorithm which changes the schedule, time and net present value.

REFERENCES

- [1] Van Groenendaal, W, Kleijnen, J (2002), Deterministic versus stochastic sensitivity analysis in investment problems: an environmental case study, *European Journal of Operational Research*, 2002, Vol. 141, 820.
- [2] Biezm, M.V, San Cristobal, J.R, Investment criteria for the selection of cogeneration plants a state of the art review, *Applied Thermal Engineering*, 2006, 583-588.
- [3] M. Mika, G. Waligora, and J. Weglarz, Simulated annealing and tabu search for multi-mode resource-constrained project scheduling with positive discounted cash flows and different payment models, *European Journal of Operational Research*, 2005, 639668.
- [4] G. Waligora, Discretecontinuous project scheduling with discounted cash flows A tabu search approach, *Computers and Operations Research*, 2008, 21412153.
- [5] Rafiee, M. and Kianfar, A scenario tree approach to multi-period project selection problem using real-option valuation method, *The International Journal of Advanced Manufacturing Technology*, 2007, 411.
- [6] Gutjahr, W.J, Katzensteiner, S. Reiter, P. Stummer, C. and Denk, Competence-driven project portfolio selection, scheduling and staff assignment, *Central European Journal of Operational Research*, 2008, 281-306.
- [7] Kahraman, C., Ruan, D. and Tolga, E., Capital budgeting techniques using discounted fuzzy versus probabilistic cash flows, *Information Sciences*, 2002, 57-76.

- [8] Majid Shahriari, Mapping Fuzzy Approach in Engineering Economics, *International Research Journal of Finance and Economics*, 2011, ISSN 1450-2887, Issue 81.
- [9] Sheen, J.N., Fuzzy evaluation of co-generation alternatives in a petrochemical industry, *Computers and Mathematics with Applications*, 2005, 741-755.
- [10] Chiu C.Y. and Park, C.S., Fuzzy cash flow analysis using present worth criterion, *The Engineering Economist*, 1994, 113-138.
- [11] Gutierrez, I., Fuzzy numbers and net present value, *Scandinavian Journal of Management*, 1989, 149-159.
- [12] C.S. Sung and S.K. Lim, A project activity scheduling problem with net present value measure, *Int. J. Production Economics*, Dec 1994, 177-187.
- [13] A. Nagoorgani and K. Ponnalagu, A New Approach on Solving Intuitionistic Fuzzy Linear Programming Problem, *Applied Mathematical Sciences*, 2012, 3467 - 3474.
- [14] J. Buckley, The fuzzy mathematics of finance, *Fuzzy Set. Syst.*, 1987, 257-273.
- [15] H. Dourra, and P. Siy, Investment using technical analysis and fuzzy logic, *Fuzzy Set. Syst.*, 2002, 221-240.
- [16] D. Kuchta, Fuzzy capital budgeting, *Fuzzy Set. Syst.*, 2000, vol. 111, no. 3, 367-385.
- [17] K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 1986, 87-96.
- [18] E. Szmidt, J. Kacprzyk, Distances between intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 2000, 505518.
- [19] A.H. Russel, Cash Flows in Networks, *Management Science*, 1970, 357-373.
- [20] Montevechi, J. A., Pamplona, E. O, Sanches, A. L., Capital Budgeting Using Triangular Fuzzy Numbers, *V Encuentro Internacional de Finanzas*, Santiago, Chile, 2005, 19 a 21 de janeiro.
- [21] Zadeh L., Fuzzy sets, *Inform. Control*, 1965, 8, 338-356.
- [22] Irem UCAL, Dorota KUCHTA, Project Scheduling to maximize Fuzzy Net Present Value, *Proceedings of the World Congress on Engineering*, London, 2011.

Gaurav Kumar Gaurav Kumar received his M.Sc. degree in Mathematics from Himachal Pradesh University, Shimla, India in 2007. He is pursuing Ph.D in Mathematics from Singhania University, Rajasthan, India.

Rakesh K. Bajaj Rakesh K. Bajaj received his M.Sc. from the Indian Institute of Technology, Kanpur, India in 2002. He received his Ph.D (Mathematics) from Jaypee University of Information Technology (JUIT), Warknaghat, Solan, INDIA in 2009. He is working as an Associate Professor in the Department of Mathematics, JUIT, Warknaghat since 2003. His interests include fuzzy statistics, information measures, pattern recognition, fuzzy clustering.