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Exp-Function Method for Finding Some Exact Solutions of Rosenau Kawahara and Rosenau Korteweg-de Vries Equations

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Abstract—In this paper, we apply the Exp-function method to Rosenau-Kawahara and Rosenau-KdV equations. Rosenau-Kawahara equation is the combination of the Rosenau and standard Kawahara equations and Rosenau-KdV equation is the combination of the Rosenau and standard KdV equations. These equations are nonlinear partial differential equations (NPDE) which play an important role in mathematical physics. Exp-function method is easy, succinct and powerful to implement to nonlinear partial differential equations arising in mathematical physics. We mainly try to present an application of Exp-function method and offer solutions for common errors wich occur during some of the recent works.

Keywords—Exp-function method, Rosenau Kawahara equation, Rosenau Korteweg-de Vries equation, nonlinear partial differential equation.

MSC Subject Classifications: 35D99, 65D25

I. Introduction

THE study of nonlinear partial differential equations (NPDE) plays an important role in mathematical physics, engineering and the other sciences. In the past several decades, various methods for obtaining solutions of NPDEs and ODEs have been presented, such as, tanh-function method [1], [2], [3], Adomian decomposition method [4], [5], Homotopy perturbation method [6], [7], [8], variational iteration method [9], [10], [11], spectral method [12], [13], [14], sine-cosine method [15], [16] and so on. Recently, Ji-Huan He and Xu-Hong Wu [17], [18] have proposed a novel method called Exp-function method, which is easy, succinct and powerful to implement to nonlinear partial differential equations arising in mathematical physics. The Exp-function method has been successfully applied to many kinds of NPDEs, such as, KdV equation with variable coefficients [19], Maccari's system [20], Boussinesq equations [21], Burger's equations [22], [23], [24], Double Sine-Gordon equation [25], [26], Schrödinger equations [27], Jaulent-Miodek equations [28] and other important nonlinear differential equations [29], [30], [31], [32], [33]. However, common errors have occurred during the application of Exp-function method in several recent papers. Seven common errors are formulated and classified by [34].
In this paper, we attempt to rectify these common errors in order to obtain the exact solutions of two nonlinear partial differential equations, namely, Rosenau-Kawahara equation and Rosenau Korteweg-de Vries (Rosenau-KdV) equation given by

$$u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ ,$$
$$u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0 \ ,$$

,respectively.

Evidently, the Rosenau-Kawahara equation can be considered as the combination of the following Rosenau equation and standard Kawahara equation

$$u_t + u_{xxxxt} + u_x + uu_x = 0 \ ,$$
$$u_t + uu_x + u_{xxx} - u_{xxxxx} = 0 \ ,$$

,respectively and the Rosenau-KdV equation can be considered as the combination of the following Rosenau equation and standard KdV equation

$$u_t + u_{xxxxt} + u_x + uu_x = 0 \ ,$$
$$u_t + uu_x + u_{xxx} = 0 \ ,$$

respectively. Rosenau equation was proposed by Rosenau [35], [36] to describe the dynamics of dense discrete systems. Also, Kawahara equation is a model equation for plasma waves, capillary-gravity water waves [37].
The rest of the paper is organized as follows: Section II describes the Exp-function method for finding some exact solutions for the NPDEs. Afterwards, the applications of the proposed analytical scheme are presented in Section III. The conclusions and findings are discussed in the Section IV.

II. Basic idea of Exp-function method

We consider a general nonlinear PDE in the following form

$$N(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, ...) = 0 \ , \tag{1}$$

where N is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformation. We introduce a complex variation as

$$u(x,t) = U(\eta) \ , \qquad \eta = kx + \omega t \ . \tag{2}$$

where k and ω are constants. We can rewrite (1) in the following nonlinear ordinary differential equations

$$N(U, kU', \omega U', k^2U'', ...) = 0 \ ,$$

where the prime denotes the derivation with respect to η. According to the Exp-function method [17], we assume that

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the solution can be expressed in the form

$$U(\eta)=\frac{\sum_{i=-d}^{c} a_i \exp(i\eta)}{\sum_{j=-q}^{p} b_j \exp(j\eta)}\ , \tag{3}$$

where c, d, p and q are positive integers which can be freely chosen and a_i and b_j are unknown constants to be determined. To determine the values of c and p, we balance the highest order linear term with the highest order nonlinear term in (3). Similarly to determine the values of d and q. So, by means of the Exp-function method, we obtain the generalized solitary solution and periodic solution for nonlinear evolution equations arising in mathematical physics.

III. Applications of the Exp-function method

A. Rosenau-Kawahara equation

In this section, we detailed the steps of the Exp-function method to construct the exact solutions of Rosenau-Kawahara (RK) equation

$$u_t+u_{xxxxt}+u_x+uu_x+u_{xxx}-u_{xxxxx}=0\ . \tag{4}$$

Making the travelling wave transformation

$$u(x,t)=U(\eta),\qquad \eta=kx+\omega t\ , \tag{5}$$

and integrating with respect to η, (4) becomes an ordinary differential equation in the form

$$(\omega+k)U+\frac{k}{2}U^2+k^3U''+k^4(\omega-k)U''''=0\ , \tag{6}$$

where the prime denotes the derivative with respect to η and also where the integration constant is chosen as zero. In other words, we solved this problem for the case when integration constant is zero.

According to the Exp-function method [26], [17], [38], we assume that the solution of (6) can be expressed in the form

$$U(\eta)=\frac{a_c\exp(c\eta)+...+a_{-d}\exp(-d\eta)}{b_p\exp(p\eta)+...+b_{-q}\exp(-q\eta)}\ , \tag{7}$$

where c, d, p and q are positive integers which are unknown to be determined later. In order to determine the values of c and p, we balance the linear term of the highest order with the highest order nonlinear terms in (6), i.e. U'''' and U^2. we have

$$U''''=\frac{c_1\exp[(c+15p)\eta]+...}{c_2\exp[16p\eta]+...}\ , \tag{8}$$

and

$$U^2=\frac{c_3\exp[2c\eta]+...}{c_4\exp[2p\eta]+...}=\frac{c_3\exp[(2c+14p)\eta]+...}{c_4\exp[16p\eta]+...}\ , \tag{9}$$

where c_i are coefficients. By balancing the highest order of Exp-function in (8) and (9), we derive

$$c+15p=2c+14p\ ,$$

which leads to the following result

$$p=c\ .$$

Similarly to determine the values of d and q, we balance the linear term of lowest order in (6)

$$U''''=\frac{...+d_1\exp[-(15q+d)\eta]}{...+d_2\exp[-16q\eta]}\ , \tag{10}$$

and

$$U^2=\frac{...+d_3\exp[-2d\eta]}{...+d_4\exp[-2q\eta]}=\frac{...+d_3\exp[-(2d+14q)\eta]}{...+d_4\exp[-16q\eta]}\ , \tag{11}$$

where d_i are determined coefficients, we obtain

$$-(15q+d)=-(2d+14q)\ ,$$

which leads to the result $q=d$.

1) Case I: $p=c=1$, $q=d=1$: Based on this selection, (3) reduces to

$$U(\eta)=\frac{a_1\exp(\eta)+a_0+a_{-1}\exp(-\eta)}{\exp(\eta)+b_0+b_{-1}\exp(-\eta)}\ . \tag{12}$$

Substituting (12) into (6) and equating to zero the coefficients of each $exp(n\eta)$ yield a set of algebraic equations for a_0, b_0, a_{-1}, a_1, b_1, k and ω. By solving the system of algebraic equations with a professional mathematical software, we obtain

case 1.

$$\begin{cases} a_{-1}=-b_0^2, & b_{-1}=\frac{1}{4}b_0^2,\\ a_0=8b_0, & b_0=b_0,\\ a_1=-4, & k=\omega=\sqrt{2}. \end{cases} \tag{13}$$

inserting these results into (12), we obtain

$$U(\eta)=\frac{-4\exp(\eta)+8b_0-b_0^2\exp(-\eta)}{\exp(\eta)+b_0+\frac{1}{4}b_0^2\exp(-\eta)}\ . \tag{14}$$

where $\eta=\sqrt{2}(x+t)$ and b_0 is a free parameter which can be determined by initial or boundary conditions . We rewrite (14) by using (5)

$$u(x,t)=\frac{-4\exp(\sqrt{2}(x+t))+8b_0-b_0^2\exp(-\sqrt{2}(x+t))}{\exp(\sqrt{2}(x+t))+b_0+\frac{1}{4}b_0^2\exp(-\sqrt{2}(x+t))}\ . \tag{15}$$

These results cover some of the special solutions of (4) regarding to the initial value conditions. By considering $u(x,0)=-4+6\ csch^2(\frac{\sqrt{2}}{2}x)$ as a initial value condition, we have

$$\begin{cases} u_t+u_{xxxxt}+u_x+uu_x+u_{xxx}-u_{xxxxx}=0\ ,\\ u(x,0)=-4+6\operatorname{csc}h^2(\frac{\sqrt{2}}{2}x)\ . \end{cases} \tag{16}$$

From (16) and (15), we obtain

$$b_0=2\ , \tag{17}$$

Thus, from substituting (17) into (15), we obtain

$$\begin{aligned} u(x,t)&=\frac{-4\exp(\sqrt{2}(x+t))+16-4\exp(-\sqrt{2}(x+t))}{\exp(\sqrt{2}(x+t))+2+\exp(-\sqrt{2}(x+t))}\\ &=-4+6\times\frac{4}{\exp(\sqrt{2}(x+t))+2+\exp(-\sqrt{2}(x+t))}\\ &=-4+6\operatorname{csc}h^2\left[\frac{\sqrt{2}}{2}(x+t)\right]\ . \end{aligned}$$

which is the solution obtained by tanh method in [39].

If (4) be in the following form

$$\begin{cases} u_t+u_{xxxxt}+u_x+uu_x+u_{xxx}-u_{xxxxx}=0\ ,\\ u(x,0)=-4-6sech^2(\frac{\sqrt{2}}{2}x) \end{cases} \tag{18}$$

then, by considering, (15), we can obtain

$$b_0 = -2 \ , \tag{19}$$

Thus, substituting $b_0 = -2$ into (15), we have

$$\begin{aligned} u(x,t) &= \frac{-4\exp(\sqrt{2}(x+t)) - 16 - 4\exp(-\sqrt{2}(x+t))}{\exp(\sqrt{2}(x+t)) - 2 + \exp(-\sqrt{2}(x+t))} \\ &= -4 - 6 \times \frac{4}{\exp(\sqrt{2}(x+t)) - 2 + \exp(-\sqrt{2}(x+t))} \\ &= -4 - 6\sec h^2\left[\frac{\sqrt{2}}{2}(x+t)\right] . \end{aligned}$$

which is the solution obtained by tanh method in [39].

case 2.

$$\begin{cases} a_{-1} = 0, \quad b_{-1} = \frac{1}{4}b_0^2, \\ a_0 = -12b_0, \qquad b_0 = b_0, \\ a_1 = 0, \qquad k = \omega = i\sqrt{2}. \end{cases} \tag{20}$$

In this case, k and ω are imaginary numbers. Substituting these results into (12), we obtain the following solution

$$U(\eta) = \frac{-12b_0}{\exp(\eta) + b_0 + \frac{1}{4}b_0^2\exp(-\eta)} \ . \tag{21}$$

where $\eta = i\sqrt{2}(x+t)$ and b_0 is a free parameter which can be determined by initial or boundary conditions. We rewrite (21) by using (5)

$$u(x,t) = \frac{-12b_0}{\exp(i\sqrt{2}(x+t)) + b_0 + \frac{1}{4}b_0^2\exp(-i\sqrt{2}(x+t))} \ . \tag{22}$$

If (4) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ , \\ u(x,0) = -6\csc^2(\frac{\sqrt{2}}{2}x) \ . \end{cases} \tag{23}$$

then, by considering (22), we can obtain

$$b_0 = 2 \ , \tag{24}$$

Thus, substituting $b_0 = 2$ into (22), we have:

$$\begin{aligned} u(x,t) &= \frac{-24}{\exp(i\sqrt{2}(x+t)) + 2 + \exp(-i\sqrt{2}(x+t))} \\ &= -6\csc^2\left[\frac{\sqrt{2}}{2}(x+t)\right]. \end{aligned}$$

which is the same as Zuo's solutions [39].

Also if (4) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ , \\ u(x,0) = -6\sec^2(\frac{\sqrt{2}}{2}x) \ . \end{cases} \tag{25}$$

then, by considering (22), we can obtain

$$b_0 = -2 \ , \tag{26}$$

Thus, substituting $b_0 = -2$ into (22), we have:

$$\begin{aligned} u(x,t) &= \frac{24}{\exp(i\sqrt{2}(x+t)) - 2 + \exp(-i\sqrt{2}(x+t))} \\ &= -6\sec^2\left[\frac{\sqrt{2}}{2}(x+t)\right]. \end{aligned}$$

which is the exact solution given by Zuo in[39].

2) Case II: $p = c = 2$, $q = d = 2$: In this case (7) becomes

$$U(\eta) = \frac{a_2\exp(2\eta) + a_1\exp(\eta) + a_0 + ...}{\exp(2\eta) + b_1\exp(\eta) + b_0 + ...} \frac{... + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{... + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)}, \tag{27}$$

There are some free parameters in this equation. For simplicity, we study the two different cases:

A. $a_{-1} = a_1 = 0$

In this case (27) can be expressed as

$$U(\eta) = \frac{a_2\exp(2\eta) + a_0 + ...}{\exp(2\eta) + b_1\exp(\eta) + b_0 + ...} \frac{... + a_{-2}\exp(-2\eta)}{... + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)} \tag{28}$$

Substituting (28) into (6) and equating to zero, the coefficients of each $\exp(n\eta)$ yield a set of algebraic equations for a_0, b_0, a_{-1}, a_1, b_1, k and ω. By solving the system of algebraic equations with a professional mathematical software, we obtain the coefficients

case A.1.

$$a_2 = a_{-2} = 0, \ a_0 = \frac{105}{13}b_1^2\frac{\sqrt{205}-13}{36}, \ b_0 = \frac{3}{8}b_1^2, \tag{29}$$
$$b_{-1} = \frac{1}{16}b_1^3, \quad b_{-2} = \frac{1}{256}b_1^4$$
$$k = \frac{1}{6}\sqrt{\sqrt{205}-13}, \quad \omega = -\frac{1}{6}\sqrt{\sqrt{205}-13} \times \frac{\sqrt{205}}{13}$$

where b_1 is a free parameter. Inserting (29) into (28), we have

$$u(x,t) = \frac{\frac{105}{13}b_1^2\frac{\sqrt{205}-13}{36}}{\exp(2\eta) + b_1\exp(\eta) + \frac{3}{8}b_1^2 + ...} \frac{...}{... + \frac{1}{16}b_1^3\exp(-\eta) + \frac{1}{256}b_1^4\exp(-2\eta)} \tag{30}$$

where $\eta = \frac{\sqrt{\sqrt{205}-13}}{6}\left[x - \frac{\sqrt{205}}{13}t\right]$. If (4) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ , \\ u(x,0) = \\ \left(-\frac{35}{12} + \frac{35}{156}\sqrt{205}\right)\csc h^4\left[\frac{1}{12}\sqrt{\sqrt{205}-13}\ x\right] . \end{cases} \tag{31}$$

then, by considering, (30), we can obtain

$$b_1 = 4 \ . \tag{32}$$

Thus, substituting $b_1 = 4$ into (36), we have:

$$\begin{aligned} u(x,t) &= \frac{\frac{140}{39}(\sqrt{205}-13)}{\exp(2\eta) + 4\exp(\eta) + 6 + 4\exp(-\eta) + \exp(-2\eta)} \\ &= \left(-\frac{35}{12} + \frac{35}{156}\sqrt{205}\right) * ... \\ &... * \csc h^4\left[\frac{1}{12}\sqrt{\sqrt{205}-13}\left(x - \frac{\sqrt{205}}{13}t\right)\right] . \end{aligned}$$

which is the exact solution given by Zuo in [39] obtained via sine-cosine method.
Also, if (4) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ , \\ u(x,0) = \left(-\frac{35}{12} + \frac{35}{156}\sqrt{205} \right) * ... \\ \qquad ... * \sec h^4 \left[\frac{1}{12}\sqrt{\sqrt{205} - 13}x \right] \ . \end{cases} \tag{33}$$

then, by considering (36), we can obtain

$$b_1 = -4 \ . \tag{34}$$

Thus, substituting $b_1 = -4$ into (36), we have:

$$\begin{aligned} u(x,t) &= \frac{\frac{140}{39}(\sqrt{205}-13)}{\exp(2\eta) - 4\exp(\eta) + 6 - 4\exp(-\eta) + \exp(-2\eta)} \\ &= \left(-\frac{35}{12} + \frac{35}{156}\sqrt{205} \right) * ... \\ & ... * \sec h^4 \left[\frac{1}{12}\sqrt{\sqrt{205}-13}\left(x - \frac{\sqrt{205}}{13}t \right) \right] \ . \end{aligned}$$

Comparing our result and Zuo's result [39] show that the results are the same.
case A.2.

$$a_2 = a_{-2} = 0, \quad a_0 = -\frac{105}{13}b_1^2\frac{\sqrt{205}+13}{36}, \quad b_0 = \frac{3}{8}b_1^2, \tag{35}$$
$$b_{-1} = \frac{1}{16}b_1^3, \qquad b_{-2} = \frac{1}{256}b_1^4$$
$$k = \frac{1}{6}i\sqrt{\sqrt{205}+13}, \qquad \omega = \frac{1}{6}i\sqrt{\sqrt{205}+13} \times \frac{\sqrt{205}}{13}$$

where b_1 is a free parameter. Substituting these results into (28), we obtain the following exact solution

$$u(x,t) = -\frac{\frac{105}{13}b_1^2\frac{\sqrt{205}+13}{36}}{\exp(2\eta) + b_1\exp(\eta) + \frac{3}{8}b_1^2 + ...} \tag{36}$$
$$\frac{...}{... + \frac{1}{16}b_1^3\exp(-\eta) + \frac{1}{256}b_1^4\exp(-2\eta)}$$

where $\eta = i\frac{\sqrt{\sqrt{205}+13}}{6}\left[x + \frac{\sqrt{205}}{13}t \right]$. If (4) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ , \\ u(x,0) = \\ -\left(\frac{35}{12} + \frac{35}{156}\sqrt{205} \right) \csc^4 \left[\frac{1}{12}\sqrt{\sqrt{205}+13} \ x \right] \end{cases} \tag{37}$$

Then, by considering (36), we can obtain

$$b_1 = 4 \ . \tag{38}$$

Thus, substituting $b_1 = 4$ into (36), we would have the same solution obtained by sine-cosine method [39]. Also, if (4) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \ , \\ u(x,0) = \\ -\left(\frac{35}{12} + \frac{35}{156}\sqrt{205} \right) \sec^4 \left[\frac{1}{12}\sqrt{\sqrt{205}+13} \ x \right] \end{cases} \tag{39}$$

Then, by considering (36), we can obtain

$$b_1 = -4 \ . \tag{40}$$

Thus, substituting $b_1 = -4$ into (36), we would have the same solution obtained by sine-cosine method [39].
case A.3.

$$a_2 = -4, \quad a_{-2} = -\frac{1}{324}b_1^4, \quad a_0 = \frac{14}{9}b_1^2, \quad b_0 = \frac{5}{18}b_1^2, \tag{41}$$
$$b_{-1} = \frac{1}{36}b_1^3, \qquad b_{-2} = \frac{1}{1296}b_1^4, \qquad k = \omega = \pm\sqrt{2}$$

where b_1 is a free parameter. Substituting these results into (28), we obtain the following exact solution

$$u(x,t) =$$
$$\frac{-4\exp(2\eta) + \frac{14}{9}b_1^2 - ...}{\exp(2\eta) + b_1\exp(\eta) + \frac{5}{18}b_1^2 + ...}$$
$$\frac{... - \frac{1}{324}b_1^4\exp(-2\eta)}{... + \frac{1}{36}b_1^3\exp(-\eta) + \frac{1}{1296}b_1^4\exp(-2\eta)} \ ,$$

where $\eta = \pm\sqrt{2}(x+t)$.
B. $b_{-1} = b_1 = 0$
In this case (27) can be expressed as

$$U(\eta) = \frac{a_2\exp(2\eta) + a_1\exp(\eta) + a_0 + ...}{\exp(2\eta) + b_0 + ...} \tag{42}$$
$$\frac{... + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{... + b_{-2}\exp(-2\eta)} \ ,$$

Substituting (42) into (6) and equating to zero the coefficients of each $exp(n\eta)$ yield a set of algebraic equations for a_0, b_0, a_{-1}, a_1, b_1, k and ω. By solving the system of algebraic equations with a professional mathematical software, we obtain the coefficients

case B.1.

$$a_2 = -4, \qquad a_0 = -\frac{5}{72}a_1^2, \qquad b_0 = -\frac{1}{288}a_1^2, \tag{43}$$
$$b_{-2} = \frac{1}{331776}a_1^4, \qquad a_{-1} = \frac{1}{576}a_1^3$$
$$a_{-2} = -\frac{1}{82944}a_1^4, \qquad k = \omega = \pm\sqrt{2}$$

where a_1 is a free parameter. Inserting (43) into (42), we have

$$u(x,t) = \frac{-4\exp(2\eta) + a_1\exp(\eta) - \frac{5}{72}a_1^2 + ...}{\exp(2\eta) - \frac{1}{288}a_1^2 + ...}$$
$$\frac{... + \frac{1}{576}a_1^3\exp(-\eta) - \frac{1}{82944}a_1^4\exp(-2\eta)}{... + \frac{1}{331776}a_1^4\exp(-2\eta)} \ ,$$

where $\eta = \pm\sqrt{2}(x+t)$.

case B.2.

$$a_2 = a_{-2} = 0, \quad a_0 = \pm 48 b_{-1}^{1/2}, \quad b_0 = \mp 2 b_{-1}^{1/2}, \tag{44}$$
$$a_1 = \pm 24 b_{-1}^{1/4} \quad or \quad \pm 24 i b_{-1}^{1/4},$$
$$a_{-1} = \pm 24 b_{-1}^{3/4} \quad or \quad \pm 24 i b_{-1}^{3/4},$$
$$k = \omega = \pm i\sqrt{2}$$

where b_{-1} is a free parameter. Substituting (44) into (42), we get the exact solitary wave solutions of (4).

B. Rosenau-KdV equation

In this section, we consider a generalized form of the Rosenau-KdV equation, which reads

$$u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0 \ , \tag{45}$$

Using the transformation (5) and integrating with respect to η, then (45) becomes

$$(\omega + k)U + \frac{k}{2}U^2 + k^3 U'' + k^4 \omega U'''' = 0 \ , \tag{46}$$

where the prime denotes the derivative with respect to η and also where the integration constant is chosen as zero. In other words, we are solved this problem for the case when the integration constant is zero. Substituting (3) into (46) and then balancing the linear term of the highest order U'''' with the highest order nonlinear term U^2 in (46), we have

$$c + 15p = 2c + 14p \ , \qquad -(15q + d) = -(2d + 14q) \ ,$$

This leads to the result

$$c = p, \qquad q = d \ .$$

We can choose the values of c and d, we set $p = c = 2$ and $q = d = 2$, the trial function, (3) becomes

$$U(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + ...}{\exp(2\eta) + b_1 \exp(\eta) + b_0 + ...} \tag{47}$$
$$\frac{... + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{... + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)} \ ,$$

There are some free parameters in (47), we set $a_1 = a_{-1} = 0$ for simplicity, the trial function is simplified as follows

$$U(\eta) = \frac{a_2 \exp(2\eta) + a_0 + ...}{\exp(2\eta) + b_1 \exp(\eta) + b_0 + ...} \tag{48}$$
$$\frac{... + a_{-2}\exp(-2\eta)}{... + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)} \ ,$$

Substituting (48) into (46) and equating to zero the coefficients of each $exp(n\eta)$ yield a set of algebraic equations for a_0, b_0, a_{-1}, a_1, b_1, k and ω. By solving the system of algebraic equations with a professional mathematical software, we obtain the coefficients

case 1.

$$a_2 = a_{-2} = 0, \quad a_0 = \frac{105}{13} b_1^2 \frac{\sqrt{313} - 13}{72}, \tag{49}$$
$$b_0 = \frac{3}{8} b_1^2, b_{-1} = \frac{1}{16} b_1^3, \quad b_{-2} = \frac{1}{256} b_1^4$$
$$k = \frac{1}{12}\sqrt{2\sqrt{313} - 26},$$
$$\omega = -\frac{1}{12}\sqrt{2\sqrt{313} - 26} \times \left(\frac{1}{2} + \frac{\sqrt{313}}{26}\right)$$

where b_1 is free parameter which can be determined by the initial or boundary conditions. So, substituting (49) into (48) we have:

$$u(x,t) = \tag{50}$$
$$\frac{\frac{105}{13} b_1^2 \frac{\sqrt{313}-13}{72}}{\exp(2\eta) + b_1 \exp(\eta) + \frac{3}{8} b_1^2 + ...}$$
$$\frac{...}{... + \frac{1}{16} b_1^3 \exp(-\eta) + \frac{1}{256} b_1^4 \exp(-2\eta)} \ ,$$

where $\eta = \frac{1}{12}\sqrt{2\sqrt{313} - 26}\left[x - \left(\frac{1}{2} + \frac{\sqrt{313}}{26}\right)t\right]$. If (45) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0 \ , \\ u(x,0) = \left(-\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ... \\ \qquad ... * \csc h^4 \left[\frac{1}{24}\sqrt{2\sqrt{313} - 26}\ x\right] \end{cases} \tag{51}$$

then, by considering (50), we have

$$b_1 = 4 \ . \tag{52}$$

Thus, substituting $b_1 = 4$ into (50), we have:

$$u(x,t) = \frac{\frac{70}{39}(\sqrt{313} - 13)}{\exp(2\eta) + 4\exp(\eta) + 6 + 4\exp(-\eta) + \exp(-2\eta)}$$
$$= \left(-\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ...$$
$$... * \csc h^4 \left[\frac{1}{24}\sqrt{2\sqrt{313} - 26}\ \left(x - \left(\frac{1}{2} + \frac{\sqrt{313}}{26}\right)t\right)\right] \ .$$

which is the exact solution given by Zuo in [39] obtained via sine-cosine method.

Also, if (45) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0 \ , \\ u(x,0) = \left(-\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ... \\ \qquad ... * \sec h^4 \left[\frac{1}{24}\sqrt{2\sqrt{313} - 26}\ x\right] \ . \end{cases} \tag{53}$$

then taking into account (50), we obtain

$$b_1 = -4 \ . \tag{54}$$

Thus, substituting $b_1 = -4$ into (50), we have:

$$u(x,t) = \frac{\frac{70}{39}(\sqrt{313}-13)}{\exp(2\eta) - 4\exp(\eta) + 6 - 4\exp(-\eta) + \exp(-2\eta)}$$
$$= \left(-\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ...$$
$$... * \sec h^4 \left[\frac{1}{24}\sqrt{2\sqrt{313}-26}\ \left(x - \left(\frac{1}{2} + \frac{\sqrt{313}}{26}\right)t\right)\right].$$

which is the same as Zuo's solution[39].

case 2.

$$a_2 = a_{-2} = 0, \quad a_0 = -\frac{105}{13}b_1^2\frac{\sqrt{313}+13}{72}, \tag{55}$$
$$b_0 = \frac{3}{8}b_1^2, b_{-1} = \frac{1}{16}b_1^3, \quad b_{-2} = \frac{1}{256}b_1^4$$
$$k = i\frac{1}{12}\sqrt{2\sqrt{313}+26},$$
$$\omega = -i\frac{1}{12}\sqrt{2\sqrt{313}+26} \times \left(\frac{1}{2} - \frac{\sqrt{313}}{26}\right)$$

where b_1 is a free parameter. Substituting (55) into (48) we obtain the following solution:

$$u(x,t) = \frac{-\frac{105}{13}b_1^2\frac{\sqrt{313}+13}{72}}{\exp(2\eta) + b_1\exp(\eta) + \frac{3}{8}b_1^2 + ...} \tag{56}$$
$$\frac{...}{... + \frac{1}{16}b_1^3\exp(-\eta) + \frac{1}{256}b_1^4\exp(-2\eta)},$$

where $\eta = \frac{i}{12}\sqrt{2\sqrt{313}+26}\left[x - \left(\frac{1}{2} - \frac{\sqrt{313}}{26}\right)t\right]$. If (45) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0 \ , \\ u(x,0) = -\left(\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ... \\ \qquad ... * \csc^4\left[\frac{1}{24}\sqrt{2\sqrt{313}+26}\ x\right]. \end{cases}$$

then, taking into account (50), we obtain

$$b_1 = 4\ . \tag{57}$$

Thus, substituting $b_1 = 4$ into (50), we have:

$$u(x,t) = \frac{-\frac{70}{39}(\sqrt{313}+13)}{\exp(2\eta) + 4\exp(\eta) + 6 + 4\exp(-\eta) + \exp(-2\eta)}$$
$$= -\left(\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ...$$
$$... * \csc^4\left[\frac{1}{24}\sqrt{2\sqrt{313}+26}\ \left(x - \left(\frac{1}{2} - \frac{\sqrt{313}}{26}\right)t\right)\right].$$

which is the exact solution given by Zuo in [39] obtained via sine-cosine method.

Also, if (45) be in the following form

$$\begin{cases} u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0 \ , \\ u(x,0) = -\left(\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ... \\ \qquad ... * \sec^4\left[\frac{1}{24}\sqrt{2\sqrt{313}+26}\ x\right]. \end{cases} \tag{58}$$

then taking into account (50), we obtain

$$b_1 = -4\ . \tag{59}$$

Thus, substituting $b_1 = -4$ into (50), we have:

$$u(x,t) = \frac{-\frac{70}{39}(\sqrt{313}+13)}{\exp(2\eta) - 4\exp(\eta) + 6 - 4\exp(-\eta) + \exp(-2\eta)}$$
$$= -\left(\frac{35}{24} + \frac{35}{312}\sqrt{313}\right) * ...$$
$$... * \sec^4\left[\frac{1}{24}\sqrt{2\sqrt{313}+26}\ \left(x - \left(\frac{1}{2} - \frac{\sqrt{313}}{26}\right)t\right)\right].$$

which is the same as Zuo's solution[39].

In the applications of Exp-function method in the pervious decade, common errors in finding the exact solutions of nonlinear problems have been omitted [34]. In this paper, we present an application of this method by tackling these common errors

1) In all the cases, we present the initial condition to obtain the special solution.
2) We substitute the obtained solutions into the equations to check if they satisfy the equations..
3) We simplify the solutions of differential equations.
4) We solve these problems for the case when the integration constant is zero.
5) We use the known general solutions of the ordinary differential equations.
6) We use the new method for solving Rosenau-Kawahara and Rosenau-KdV equations.

IV. Conclusions

In this paper, we considered two combined mathematical physics problems Rosenau-Kawahara and Rosenau-KdV for presenting an application of the Exp-function method. In the applications of Exp-function method in previous decade, common errors in finding the exact solutions of nonlinear problems have been omitted [34]. In this paper, we have presented an application of this method by tackling these common errors. Also, we showed that this method is easy, succinct and powerful to implement for nonlinear partial differential equations arising in mathematical physics.

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