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Function Approximation with Radial Basis Function Neural Networks via FIR Filter

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Abstract—Recent experimental evidences have shown that because of a fast convergence and a nice accuracy, neural networks training via extended kalman filter (EKF) method is widely applied. However, as to an uncertainty of the system dynamics or modeling error, the performance of the method is unreliable. In order to overcome this problem in this paper, a new finite impulse response (FIR) filter based learning algorithm is proposed to train radial basis function neural networks (RBFN) for nonlinear function approximation. Compared to the EKF training method, the proposed FIR filter training method is more robust to those environmental conditions. Furthermore , the number of centers will be considered since it affects the performance of approximation.

Keywords—Extended kalmin filter (EKF), classification problem, radial basis function networks (RBFN), finite impulse response (FIR) filter.

I. Introduction

RADIAL basis function neural networks (RBFN) have originated from multidimensional interpolation models and have been proposed by D. S. Broomhead in the literature since 1985 [1]-[3]. Those have many uses, including system identification, classification, and system control. Additionally this the output of is weighted by trained weight. Special issues on RBFN are submitted [4], [5]. RBFN consists of m dimensional input x directly connected to hidden layer, c neurons which is prototype vector in the hidden layer, n dimensional output y, and $w_{ij} \in \Re^{c\times n}$ weight parameter to output of c vectors. This is the $m-n$ mapping. Fig. 1 shows a structure of an RBFN. a weight and a prototype vector called a center should be trained. Since the value of the prototype vector and a function which will be used as an activation function are out of question, it is assumed to be fixed. But because the number of centers is important to compare the performance between training methods, we control this value. Many researchers have trained RBFN with various methods and algorithms [6]-[9]. Because it is important to consider the number of centers, we simulate relation between center number and training method which are FIR filter training method and EKF training method. we have used method which picks random centers from the data set [10].

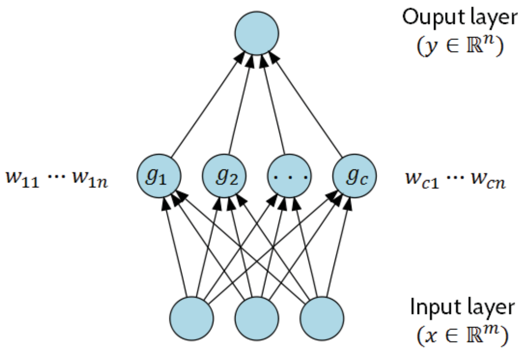

Fig. 1. Radial basis function network structure.

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In this article, we apply FIR filter [16]-[18] to the training of reformulated RBFN. EKF based learning method have been used extensively to many neural networks training such as multilayer perceptrons [11]-[13] and recurrent networks [14], [15], whereas there are a few FIR filter articles apllied to neural networks training method. But FIR filter is not so although it has a strong point which is robust to the uncertainty and the unmodeled system [19], [20]. Furthermore the guaranteed stability, temporary uncertainties, and perfect signal reconstruction such as a linear phase properties are well known desirable properties of the FIR structure. Also, it has the potential to be apllied to diverse fields of research [21], [22]. So using FIR filter training will confirm that the robust to uncertain situation is guaranteed. The following section presents the derivation of EKF [7] approach to the recursive training, and Section III presents the derivation of FIR filter training. Next Section will be simulation results on a comparison of EKF and FIR filter. Final section contains an analysis and a suggestion for a future research.

II. Training with FIR filter

A. A Mathmatical Model of RBFN

The fundamental model of the RBFN of Fig. 1 can be written as follows

$$\hat{y} = \begin{bmatrix} w_{10} & w_{11} & \dots & w_{1c} \\ w_{20} & w_{21} & \dots & w_{2c} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n0} & w_{n1} & \dots & w_{nc} \end{bmatrix} \begin{bmatrix} 1 \\ g(\|x-v_1\|^2) \\ \vdots \\ g(\|x-v_c\|^2) \end{bmatrix}, \quad (1)$$

where the function $g(\cdot)$ is an activation function of RBFN as

$$g(v) = \frac{1}{\sqrt{1+\|x-v\|^2}}, \qquad \forall v \in (0,\infty), \quad (2)$$

when v is a distance between a center and an input data. But various types of functions can be used [21]. $\hat{y}$ will be used with the simplified weight and activation function as the following notation (3-4):

$$\begin{bmatrix} w_{10} & w_{11} & \dots & w_{1c} \\ w_{20} & w_{21} & \dots & w_{2c} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n0} & w_{n1} & \dots & w_{nc} \end{bmatrix} = \begin{bmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_n^T \end{bmatrix} = W, \tag{3}$$

$$\begin{bmatrix} 1 \\ g(\|x - v_1\|^2) \\ \vdots \\ g(\|x - v_c\|^2) \end{bmatrix} = \boldsymbol{h}. \tag{4}$$

B. Derivation of RBFN Optimization Algorithm with FIR Filter

FIR filter structure is consisted of linear combinations of input and output in the horizon $[k - N, k]$ as follows:

$$x_{k\|\hat{k}-1} = H_{k-1}Y_{k-1} + L_{k-1}U_{k-1}. \tag{5}$$

The process to derive H_{k-1} and L_{k-1} as to nonlinear system is defined as finding Jacobian matrix about state, input and noise as follows:

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}),\ w_k\ \sim (0, Q), \\ y_k &= h(x_{k-1}, u_{k-1}, v_{k-1}) + v_k,\ v_k\ \sim (0, R). \end{aligned} \tag{6}$$

If we define matrixes as follows

$$Y_{k-1} = [y_{k-N}^T y_{k-N+1}^T \dots y_{k-1}^T], \tag{7}$$

$$W_{k-1} = [w_{k-N}^T w_{k-N+1}^T \dots w_{k-1}^T], \tag{8}$$

$$V_{k-1} = [v_{k-N}^T v_{k-N+1}^T \dots v_{k-1}^T], \tag{9}$$

and $\tilde{E}_{N,k}$, $\tilde{C}_{N,k}$, and $\tilde{G}_{N,k}$ are obtained from

$$\tilde{E}_{N,k} = \begin{bmatrix} A_{k-1:k-N+1}G & A_{k-1:k-N+2}G & \dots & G \end{bmatrix}, \tag{10}$$

$$\tilde{C}_{N,k} = \begin{bmatrix} C_{k-N} \\ C_{k-N+1}A_{k-N} \\ C_{k-N+2}A_{k-N+1:k-N} \\ \vdots \\ C_{k-1}A_{k-2:k-N} \end{bmatrix}, \tag{11}$$

$$\tilde{G}_{N,k} = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ C_{k-N+1}G & 0 & \dots & 0 & 0 \\ C_{k-N+2}A_{k-N+1}G & C_{k-N+2}G & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{k-1}A_{k-2:k-N+1}G & C_{k-1}A_{k--2:k-N+2}G & \dots & C_{k-1}G & 0 \end{bmatrix}, \tag{12}$$

where $G = ones(n_ic + (c+1)n_o, 1)$. Mind that matrix A has different values in uncertain situation although it is usually identity matrix. The other way C_k is calculated with jacobian matrix as follows:

$$C_{k-1} = \frac{\partial h}{\partial x}|_{x=\hat{x}_{k-1}}. \tag{13}$$

When matrix A, C is observable and $N \geq n$, for a singular or non singular A, the minimum variance FIR filter with a batch form on the horizon $[k - N, k]$ is given as follows:

$$\begin{aligned} \hat{x}_{k|k-1} = [A_{k-1:k-N}\tilde{E}_{N,k}] \begin{bmatrix} W_{1,1} & W_{1,2} \\ W_{1,2}^T & W_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{C_{N,k}}^T \\ \tilde{G_{N,k}}^T \end{bmatrix} \\ \times R_N^{-1} Y_{k-1}, \end{aligned} \tag{14}$$

$$V_{k-1} = [v_{k-N}^T v_{k-N+1}^T \dots v_{k-1}^T], \tag{15}$$

where

$$\begin{aligned} W_{1,1} &= \tilde{C}_{N,k}^T\ R_N^{-1}\ \tilde{C}_{N,k}^T, \\ W_{1,2} &= \tilde{C}_{N,k}^T\ R_N^{-1}\ \tilde{G}_{N,k}^T, \\ W_{2,2} &= \tilde{G}_{N,k}^T\ R_N^{-1}\ \tilde{G}_{N,k}^T, \\ Q_N &= diag(\underbrace{Q, \quad Q, \quad \dots, \quad Q}_{N}), \\ R_N &= diag(\underbrace{R, \quad R, \quad \dots, \quad R}_{N}). \end{aligned} \tag{16}$$

III. Simulation Results

To compare the performance between EKF and FIR filter training method, a nonlinear discrete-time function is used via simulation. The systems is written as

$$\begin{aligned} y_1(k+1) = \frac{y_1(k+1)y_2(k+1)y_3(k+1)}{1 + y_1(k+1)^2 + y_2(k+1)^2 + y_3(k+1)^2} \\ + 2u(k) + \delta_k, \end{aligned} \tag{17}$$

where δ_k is an uncertain model parameter. $y_2(k+1) = y_1(k)$, $y_3(k+1) = y_2(k)$, $y_4(k+1) = y_3(k)$and $u(t) = 0.5cos(3\pi kTs) + 0.1sin(4\pi kTs)^2 + 0.4sin(\pi kTs) \in R$. $Ts = 0.034$. The system noise covariance Q_w with the EKF training is $eye(n_ic + (c+1)n_o)$, the measurment noise covariance R_v with the EKF training is $12eye(n_o)$, the system noise covariance Q_w with the FIR filter training is $eye(1)$, the measurment noise covariance R_v with the EKF training is $12eye(n_o)$ and P_0 is $eye(n_ic + (c+1)n_o)$. In this paper, in order to examine the effectiveness of the training, we use center numbers from 1 to 20 in the hidden node. The initial centers are randomly selected. The uncertain model parameter δ_k is considered as

$$\delta_k = \begin{cases} 2,\ 50 \leq k \leq 100, \\ 2,\ 150 \leq k \leq 200, \\ \quad 0,\ \text{otherwise}. \end{cases} \tag{18}$$

The identification result for y_2 is shown in Fig. 2 The result compares the robustness of two filters training given uncertainty. When the uncertain shift of the system occurs at

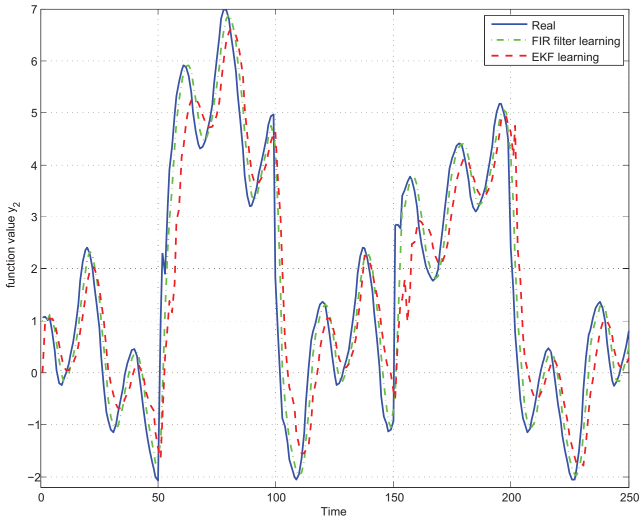

Fig. 2. Output function approximation

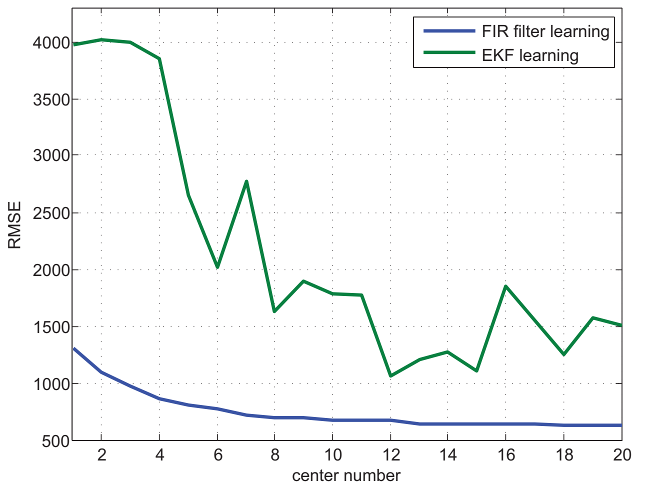

Fig. 4. Error related center number on FIR filter learning and EKF learning

$50 \leq k \leq 100$ and $150 \leq k \leq 200$, the difference between EKF training is maximized. This means that when EKF training method is applied to unstable system or unmodeled system, the approximation results will shows large deviation from the system value. On the other hand RBFN with FIR filter shows fast convergence to system function. If the system is moving plant, large deviation from real value is critical to plant processing. We apply squared error for finite time as $J(N) = \sum_{N}^{k=1} e^2(k)$. The results for J(N) are shown in Fig. 3 This visible result confirms the fact that FIR filter follows the system function well compared with the EKF. The final simulation is the comparison of squared error about center number on Fig. 4.

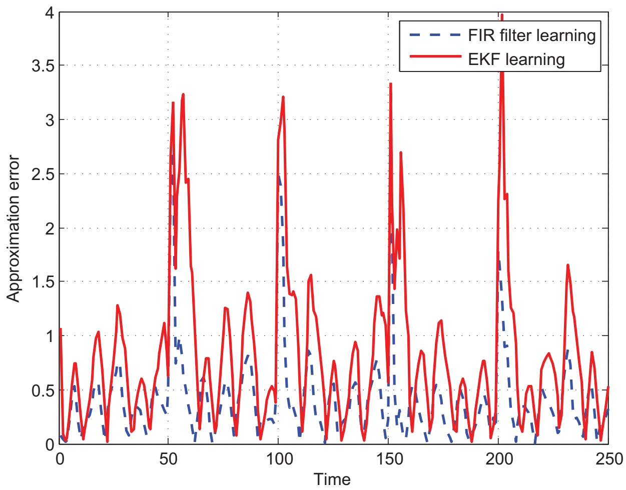

Fig. 3. Error comparison between FIR filter learning and EKF learning

When a data set is complex and large to be controlled, the number of center affect the computation load critically. Although the squared error of EKF training have a tendency to decrease, FIR filter training apparently excels EKF in a few number of center.

IV. Conclusion

A novel approach of neural networks approximation with the FIR filter training is proposed in this paper. FIR filter training method has a robust property to uncertain change of a system function. In a function approximation problem, the convergence performance is most important among properties. Moreover it is better to use robust training method because it is difficult to model majority of the plants in the world and most systems change dynamically. This learning method is well adapted to a weight and prototype vector training. Considered in a dynamic system perspective, such training can be applied usefully for all neural network applications. In future work, we will apply this training method to system control.

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