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# Second-Order Slip Flow and Heat Transfer in a Long Isoflux Microchannel

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***Abstract*—**This paper presents a study on the effect of second-order slip on forced convection through a long isoflux heated or cooled planar microchannel. The fully developed solutions of flow and thermal fields are analytically obtained on the basis of the second-order Maxwell-Burnett slip and local heat flux boundary conditions. Results reveal that when the average flow velocity increases or the wall heat flux amount decreases, the role of thermal creep becomes more insignificant, while the effect of second-order slip becomes larger. The second-order term in the Deissler slip boundary condition is found to contribute a positive velocity slip and then to lead to a lower pressure drop as well as a lower temperature rise for the heated-wall case or to a higher temperature rise for the cooled-wall case. These findings are contrary to predictions made by the Karniadakis slip model.

***Keywords*—**Microfluidics, forced convection, thermal creep, second-order boundary conditions.

## I. Introduction

NOWADAYS, microelectromechanical systems (MEMS) have developed a large number of microfluidic devices and their applications, such as micropumps, microvalves, micromixers, microheat exchangers, microchannel heat sinks, microfuel cells, etc. A fundamental understanding of physical aspects of microfluidics, which may deviate from those at the macroscale, is required for the technological demands.

Numerous theoretical investigations have been carried out on microscale slip flow and heat transfer in the past two decades. Tunc & Bayazitoglu [1] performed an analytical study of fully developed forced convection in an isoflux rectangular microchannel by solving the Navier-Stokes and energy equations subject to the first-order Maxwell slip and local heat flux boundary conditions. Avci & Aydin [2] analytically investigated the role of buoyancy in fully developed forced convection through a vertical planar microchannel with asymmetric wall heat fluxes. Sadeghi & Saidi [3] placed emphasis on the importance of viscous dissipation in fully developed convection by considering planar and annular microchannels with asymmetric wall heat fluxes. Recently, Çetin [4] analytically studied the fully developed forced convection with thermal creep in isoflux planar and circular microchannels by considering the second-order Deissler and Karniadak is slip boundary conditions. Using the Maxwell-Burnett slip boundary conditions, however, has been shown to be an adequate way to model second-order slip flow [5].

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In this paper, a study on forced convection in a long heated or cooled planar microchannel with symmetric wall heat fluxes is conducted. The Navier-Stokes and energy equations subject to the second-order Maxwell-Burnett slip and local heat flux boundary conditions are analytically solved for the fully developed flow. The calculated results are presented for air at the standard reference state with complete accommodation. The Deissler and Karniadakis slip models are then tested via the comparisons of predictions made by them with those obtain by the present slip model, so as to see how well these two slip models describe the flow and heat transfer behavior.

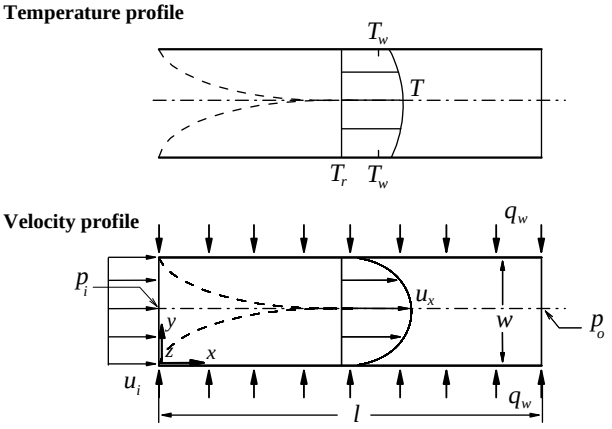



Fig. 1 Model geometry

## II. Problem Formulations

Consider a long symmetrically heated or cooled stationary horizontal planar microchannel of length $l$ and width $w$, whose heat flux is $q_w$, as shown in Fig. 1. The rarefied gas flow in the microchannel originates from a reservoir at a reference state and terminates in a discharge area of lower pressure. In the system considered, the flow enters the channel with a uniform velocity $u_i$. Let $x$ and $y$ denote the usual rectangular coordinates, let $u_x$ and $u_y$, denote the components of velocity in the $x$ and $y$ directions, let $T$ denote the temperature, let the subscript $r$ denote the reference-state values, and let the subscripts $i$ and $o$ denote the inlet and outlet values, respectively. For a sufficiently long microchannel, we assume that a hydrodynamically and thermally fully developed flow prevails in the isoflux microchannel, obeying the limit: $\partial u_x / \partial x = 0$, $u_y = 0$, and $\partial T/\partial x = c_1$ (a constant) [6]. The simplified field equations for steady two-dimensional

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incompressible flow of constant material properties with negligible gravitational field and internal heat generation are

$$0 = -\frac{dp}{dx} + \mu_r \frac{d^2 u_x}{dy^2}, \tag{1}$$

$$\rho_r c_{p,r}\left(u_x \frac{\partial T}{\partial x}\right) = k_r \frac{\partial^2 T}{\partial y^2} + \mu_r \left(\frac{du_x}{dy}\right)^2, \tag{2}$$

where $p$ is the pressure, $\rho$ is the density, $\mu$ is the viscosity, $c_p$ is the constant-pressure specific heat, and $k$ is the thermal conductivity. It should be noted that for a low-speed flow, the field equations could be simplified to incompressible ones. In addition, a small temperature difference between the wall and the reservoir supports the assumption of constant material properties [7].

The corresponding second-order Maxwell-Burnett slip [5] and local heat flux boundary conditions are

$$\left.\begin{aligned} u_x(0) &= a_1 \frac{2-\sigma_m}{\sigma_m}\lambda_r \frac{du_x(0)}{dy} + a_2 \lambda_r^2 \frac{d^2 u_x(0)}{dy^2} \\ &\quad + a_3 \frac{c_{p,r}\rho_r}{\mu_r}\lambda_r^2 \frac{\partial T(x,0)}{\partial x}, \\ u_x(w) &= -a_1 \frac{2-\sigma_m}{\sigma_m}\lambda_r \frac{du_x(w)}{dy} + a_2 \lambda_r^2 \frac{d^2 u_x(w)}{dy^2} \\ &\quad + a_3 \frac{c_{p,r}\rho_r}{\mu_r}\lambda_r^2 \frac{\partial T(x,w)}{\partial x}, \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \frac{\partial T(x,0)}{\partial y} &= -\frac{q_w}{k_r}, \\ \frac{\partial T(x,w)}{\partial y} &= \frac{q_w}{k_r}, \end{aligned}\right\} \tag{4}$$

where $\sigma_m$ is the tangential momentum accommodation coefficient, $\lambda$ is the molecular mean free path, related to $T$ and $p$ by

$$\lambda = \frac{\sqrt{\pi \hat{R} T/2}\,\mu}{p}, \tag{5}$$

and

$$a_1 = 1,\ a_2 = -\frac{9}{4\pi}\frac{\gamma_r - 1}{\gamma_r}\mathrm{Pr},\ a_3 = \frac{3}{2\pi}\frac{\gamma_r - 1}{\gamma_r}. \tag{6}$$

Here $\hat{R}$ is the specific gas constant, $\gamma$ is the ratio of specific heats, and Pr is the Prandtl number. Note that the values of the second-order slip coefficient $a_2$ used by Çetin [4] are $0.5$, on the basis of the Karniadakis slip model [8], and $-1.125$, on the basis of the Deissler slip model [9]. The comparisons of predictions made by the Karniadakis and Deissler slip models with those obtain by the present slip model, which can describe the actual slip flow behavior, could be done to verify the validation of the two second-order boundary conditions.

Equations (1)−(4) can be non-dimensionalized by using the following parameters:

$$\left.\begin{aligned} X &= \frac{x}{l_c \mathrm{Re}},\ Y = \frac{y}{l_c},\ U = \frac{u_x}{u_c}, \\ \Theta &= \frac{T - T_c}{q_w l_c / k_r},\ P = \frac{p}{p_c},\ \mathrm{Re} = \frac{\rho_r u_c l_c}{\mu_r}, \\ \mathrm{Br} &= \frac{\mu_r u_c^2}{q_w l_c},\ \mathrm{Pr} = \frac{c_{p,r}\mu_r}{k_r},\ \mathrm{Kn} = \frac{\lambda_r}{l_c}, \end{aligned}\right\} \tag{7}$$

where Re is the Reynolds number, Br is the Brinkman number, Pr is the Prandtl number, and Kn is the Knudsen number. Here $l_c$, $u_c$, $T_c$, and $p_c$ are the characteristic length, velocity, temperature, and pressure, respectively, and defined as follows:

$$l_c = w,\ u_c = u_i,\ T_c = T_r,\ p_c = \rho_r u_c^2. \tag{8}$$

Thus, the dimensionless field equations can be written as

$$\frac{d^2 U}{dY^2} = \frac{dP}{dX}, \tag{9}$$

$$\frac{\partial^2 \Theta}{\partial Y^2} = \mathrm{Pr}\,U\frac{\partial \Theta}{\partial X} - \mathrm{Br}\left(\frac{dU}{dY}\right)^2, \tag{10}$$

and the corresponding dimensionless boundary conditions are given by

$$\left.\begin{aligned} U(0) &= a_1 \frac{2-\sigma_m}{\sigma_m}\mathrm{Kn}\frac{dU(0)}{dY} + a_2 \mathrm{Kn}^2 \frac{d^2 U(0)}{dY^2} \\ &\quad + a_3 \frac{\mathrm{Pr}}{\mathrm{Br}}\mathrm{Kn}^2 \frac{\partial \Theta(X,0)}{\partial X}, \\ U(1) &= -a_1 \frac{2-\sigma_m}{\sigma_m}\mathrm{Kn}\frac{dU(1)}{dY} + a_2 \mathrm{Kn}^2 \frac{d^2 U(1)}{dY^2} \\ &\quad + a_3 \frac{\mathrm{Pr}}{\mathrm{Br}}\mathrm{Kn}^2 \frac{\partial \Theta(X,1)}{\partial X}, \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} \frac{\partial \Theta(X,0)}{\partial Y} &= -1, \\ \frac{\partial \Theta(X,1)}{\partial Y} &= 1. \end{aligned}\right\} \tag{12}$$

The velocity solution of (9) as a function of only $Y$ is possible only if the pressure gradient $dP/dX$ is a constant. In addition, a dimensionless conservation condition for the flow rate is given by

$$\int_0^1 U dY = 1. \tag{13}$$

Solving (9) and (10) subject to the boundary conditions (11) and (12) and flow-rate conservation condition (13) gives the following velocity, pressure gradient, and temperature gradient analytical solutions:

$$U(Y)=\frac{1}{2}\frac{dP}{dX}Y^2+A_1Y+A_0, \tag{14}$$

$$\frac{dP}{dX}=-6\left(\frac{C_0}{a_3\mathrm{Kn}^2}+\sqrt{\left(\frac{C_0}{a_3\mathrm{Kn}^2}\right)^2-\frac{1}{3}\left(\frac{2}{\mathrm{Br}}-\frac{1}{a_3\mathrm{Kn}^2}\right)}\right), \tag{15}$$

$$\frac{\partial\Theta}{\partial X}=\frac{\mathrm{Br}}{a_3\,\mathrm{Pr}\,\mathrm{Kn}^2}\left(1-C_0\frac{dP}{dX}\right), \tag{16}$$

where

$$\left.\begin{aligned}A_0&=-\frac{1}{2}\left(a_1\frac{2-\sigma_m}{\sigma_m}-2a_2\mathrm{Kn}\right)\mathrm{Kn}\frac{dP}{dX}\\&\quad+a_3\frac{\mathrm{Pr}}{\mathrm{Br}}\mathrm{Kn}^2\frac{\partial\Theta}{\partial X},\\A_1&=-\frac{1}{2}\frac{dP}{dX},\\C_0&=-\frac{1}{12}-\frac{1}{2}\left(a_1\frac{2-\sigma_m}{\sigma_m}-2a_2\mathrm{Kn}\right)\mathrm{Kn}.\end{aligned}\right\} \tag{17}$$

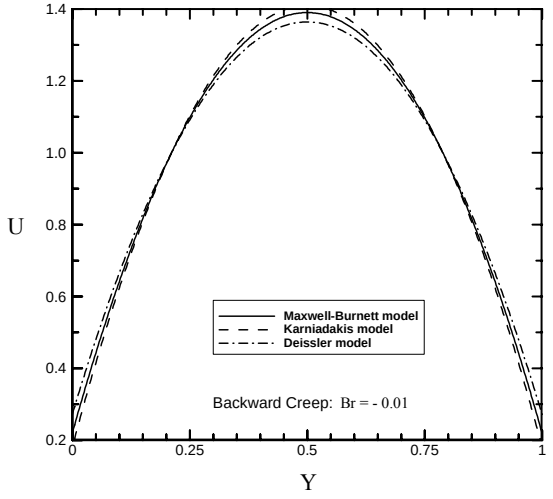



Fig. 2 Velocity distribution for different second-order slip models with Kn=0.1

## III. Results and Discussion

Air is used in many engineering application fields. We now pay attention to the influence of second-order slip on the forced convection of air at the standard reference state ($T_r = 25^{\circ}\mathrm{C}$ and $p_r = 1$ atm) with complete accommodation ($\sigma_m = 1$). The physical properties at this state can be found in Weng & Chen [7]. The parametric analysis of this problem is performed over the range $-0.5 \le \mathrm{Br} \le 0.5$, and the chosen reference value of Kn (or $w$) for the analysis is 0.1 (or 0.667 $\mu$m).

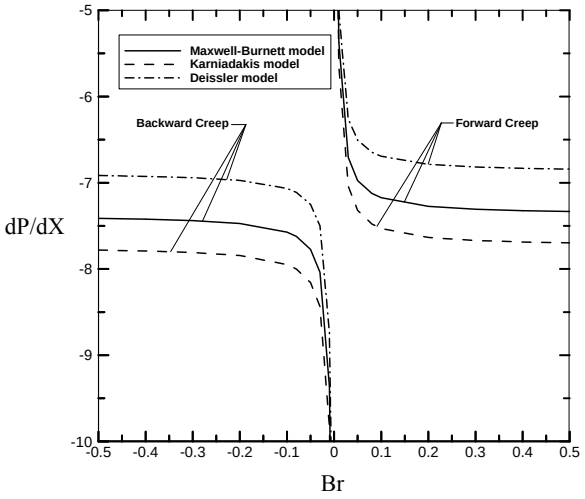



Fig. 3 Pressure gradient versus Br with Kn=0.1.

In Figs. 2–4, we investigate the influence of second-order slip on the velocity, pressure gradient, and temperature gradient relative to the forward-creep (heated-wall) case $\mathrm{Br} > 0$ and the backward-creep (cooled-wall) case $\mathrm{Br} < 0$ at a microscale level ($\mathrm{Kn} = 0.1$).Fig. 2 illustrates the velocity profiles for the Maxwell-Burnett slip model ($a_2 = -0.145$), the Karniadakis slip model ($a_2 = 0.5$), and the Deissler slip model ($a_2 = -1.125$). A comparison of cases $\mathrm{Br} = 0.01$ and $\mathrm{Br} = -0.01$ shows that a forward creep contributes a positive slip to the velocity along the wall surface and then leads to a more gradual velocity distribution; however, a backward creep contributes a negative slip and then results in a more extreme distribution. In both the two cases, the Deissler slip model predicts a significantly relatively large velocity slip while the Karniadakis slip model predicts a significantly relatively small slip. Figs. 3 and 4 illustrate the variations of the pressure gradient $dP/dX$ and the temperature gradient $\partial T/\partial X$ with the Brinkman number Br. It is found that thermal creep could play an important role in the region $|\mathrm{Br}| < 0.2$ for $dP(\mathrm{Br})/dX$ and the region $|\mathrm{Br}| < 0.05$ for $\partial T(\mathrm{Br})/\partial X$. When the value of $|\mathrm{Br}|$ increases, the role of thermal creep decreases, while the effect of second-order slip increases. In Fig. 3, the Deissler slip model predicts relatively large pressure gradient values, while the Karniadakis slip model predicts relatively small values, no matter what the creep type is. The larger (smaller) pressure gradient means that the second-order slip flow displays a lower (higher) pressure drop. In Fig. 4, the Deissler slip model predicts relatively large temperature gradient values for the backward-creep case but relatively small values for the forward-creep case. Such a conclusion for the Karniadakis slip model were found to be contrary to the Deissler predictions. The larger (smaller) temperature gradient means that the heat transfer displays a higher (lower) temperature rise.

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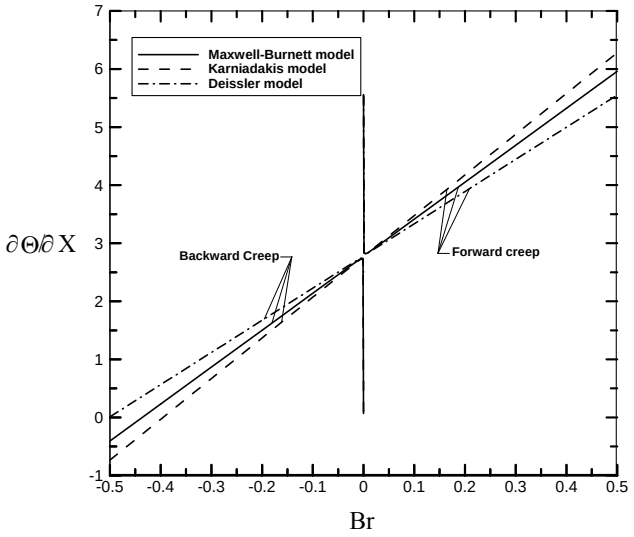



Fig. 4 Temperature gradient versus Br with Kn=0.1.

## IV. Conclusions

An analytical study on forced convection in a long heated or cooled planar microchannel with symmetric wall heat fluxes has been made by solving the Navier-Stokes and energy equations subject to the second-order Maxwell-Burnett slip and local heat flux boundary conditions. The fully developed solutions of velocity, pressure gradient, and temperature gradient were presented for air at the standard reference state with complete accommodation. Second-order slip was proven to have a significant effect, except for sufficiently small absolute Brinkman numbers (small average flow velocities or great wall heat flux amounts), at which thermal creep could play an important role. The Deissler and Karniadakis slip models were tested via the comparisons of predictions made by them with those obtain by the present slip model. For flow analysis, it was found that the second-order term in the Deissler slip boundary condition contributes a positive velocity slip and then leads to a lower pressure drop, while the Karniadakis slip model predicts a negative slip and then a higher drop. As for heat transfer analysis, it was observed that, for the heated-wall (forward-creep) case, the second-order term in the Deissler slip boundary condition results in a lower temperature rise, while the Karniadakis slip model predicts a higher rise. The conclusions for the cooled-wall (backward-creep) case were found to be contrary to the heated-wall predictions.

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**Huei Chu Weng** was born on July 11, 1976, in Tainan, Taiwan. He received his Ph.D. in Engineering Mechanics from National Cheng Kung University in 2006 and spent one year as a Postdoctoral Research Fellow at National Cheng Kung University. He is currently an Associate Professor in the Department of Mechanical Engineering at Chung Yuan Christian University and has taught a variety of undergraduate and graduate courses in all the major disciplines, including Thermodynamics, Fluid Mechanics, Heat Transfer, Numerical Solutions of Differential Equations, Theory and Applications of Nanofluids, and Application Topics Workshop.

His primary research interests are in the fields of thermal-fluid and energy sciences. Research topics include Smart Nanomaterial Science, Micro/Nanoscale Thermal-Fluid Science, Power and Energy Science. A summary of academic contributions is given below:

1. Smart nanomaterial science:

A magnetic fluid is a colloidal magnetic nanofluid, usually consisting of three components: magnetic nanoparticles, carrier liquid, and stabilizer (or dispersant). He proposed a modification in the magnetization equation (the WC model) in the dynamics of magnetic fluids, initiated the study of the influence of the spin of magnetic moments within particles on the non-Newtonian flow, and reviewed the arguments for different field equations in hydrodynamics of fluids with micro/nanostructure and stated the deterministic nature. Recently, He performed an analysis for the effects of particles and magnetic field on biomedical magnetic fluid flow to study the targeted magnetic-particle delivery in a blood vessel.

2. Micro/nanoscale thermal-fluid science:

Micro/nanotechnology develops a large number of microfluidic and nanofluidic systems in silicon, quartz, glass, plastics, etc. The importance of micro/nanoflow arises from new applications in these system devices. He initiated the studies of buoyancy-driven gas microflow and thermocreep-driven gas microflow. He also placed emphasis on the importance of thermal creep and examined the roles of variable physical properties and wall-surface curvature. Recently, he found that based on the Navier-Stokes (NS) equations subject to the second-order slip boundary conditions, continuum modeling can be valid for the Knudsen numbers up to 1.60. In addition, he provided a more detailed microscale theory of magnetogasdynamics (MGD) and developed a mathematical model of pressure-driven gas flow and heat transfer through a heated microchannel in the presence of an applied electric and magnetic field.

As a scientist and researcher Prof. Weng have published more than 23 peer reviewed journal papers and hold the following memberships:

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- CSME: Chinese Society of Mechanical Engineers
- STAM: Society of Theoretical and Applied Mechanics
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