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Instability of Electron Plasma Waves in an Electron-Hole Bounded Quantum Dusty Plasma

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Abstract**—**Using quantum hydrodynamical (QHD) model the linear dispersion relation for the electron plasma waves propagating in a cylindrical waveguide filled with a dense plasma containing streaming electron, hole and stationary charged dust particles has been derived. It is shown that the effect of finite boundary and stream velocity of electrons and holes make some of the possible modes of propagation linearly unstable. The growth rate of this instability is shown to depend significantly on different plasma parameters.

Keywords**—**Electron Plasma wave, Quantum plasma, Quantum Hydrodynamical model.

I. Introduction

In recent years there has been a great deal of interest in the investigation of various collective processes in quantum-dusty plasmas. Quantum effects may become important in a variety of environments when the plasma temperature is low and density is high. The dispersion caused by strong density correlation due to quantum fluctuations can play important role. Quantum effects become important when the thermal de Broglie wavelength becomes comparable to the interparticle distances. It has been found to be important in intense laser-solid interaction [1], in dense astrophysical and cosmological environments [2], [3], in micro-plasma systems [4], in nano-electronic devices with nano-electronic components [5], [6]. Consideration of the physical processes in dusty plasma is also interesting because of its importance in many laboratory and astrophysical situations [7]. For example, in the edge region of tokamaks the impurities coming off from the walls of the vessel can create a dusty plasma. Such a dusty plasma is created in several plasma technologies (e.g. high frequency plasma etching). It has been observed that the presence of dust particles can modify the linear and nonlinear properties of plasma waves. In many situations it becomes necessary to include quantum effects in dusty plasma [8], [9]. Most of the theoretical works on waves in quantum dusty plasma have been done with unbounded system. With reference to laboratory and space plasma, it is interesting to study the stability of a wave in a bounded system like plasma-filled waveguides. The boundaries in such a system may add additional effects. For example, a wave with no dispersion in unbounded system may become dispersive in bounded system; a stable wave of an unbounded system may become unstable in bounded geometry. Sayal and Sharma [10] have shown that ion-Langmuir oscillations (ILO) can propagate in cylindrical waveguide even when the ions are cold; the propagating ILO has dispersive characteristic and in the presence of nonlinearity it can excite solitary wave. Ghosh and Das [11] have investigated the effects of finite boundary on electron plasma waves and ion-acoustic KdV solitons using a planar waveguide geometry. Mondal et al. [12], 013] and Bhattacharya et al. [14], [15] have shown that the dimensions of the cylindrical system containing plasma have a positive influence on the stability of ion-acoustic waves. They have shown that due to the existence of multiple mode solution of the dispersion equation the usual conclusion of the analysis can get changed completely. Recently, the propagation of waves in a quantum and quantum dusty plasma has been investigated by a number of authors [16]-[22]. So it remains an interesting problem to investigate the stability of a quasi-monochromatic wave in a quantum dusty plasma including finite geometry effects.

In this paper, we consider an electron-hole quantum dusty plasma which may be relevant to semiconductor plasmas. We investigate the stability of an electron plasma wave propagating in a cylindrical waveguide filled with plasma consisting of streaming electrons, holes and stationary charged dust particles. Using quantum hydrodynamic (QHD) equations which include both the quantum statistical effect and the quantum diffraction effect, we have derived a dispersion relation from which it is shown that inclusion of streaming effects in finite geometry opens up the possibility of a number of wave modes. Some of these modes are found to be linearly unstable. It is shown that the finite boundary, stream velocity of plasma particles, quantum diffraction parameter, electron-hole mass ratio and the charge imbalance parameter all have significant effects on the instability growth rate.

II. Basic Equation

We consider a cylindrical waveguide made of perfectly conducting material filled with a dense homogeneous plasma consisting of streaming electrons, holes and stationary charged dust particles. At equilibrium the charge neutrality condition requires,

$$n_{e0} + Z_d n_{d0} = n_{h0} \tag{1}$$

where, n_{e0}, n_{h0} and n_{d0} are respectively the equilibrium number density of electron, hole, and dust particles, Z_d is the number of electrons residing on the dust grain. The plasma

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particles are assumed to move only along the axis of the waveguide, which we choose as *x*-axis of an (r,θ,x) cylindrical coordinate system. So, there will be no interaction with the corresponding empty waveguide modes that have only transverse electric field components (H-modes) and we shall consider only the E-modes. We seek to examine the propagation of slow modes having the phase velocities much less than the phase velocity of light and therefore the quasi-static approximation can be used. Under the above conditions using one-dimensional QHD model the governing equations may be written as follows:

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}\left(n_j u_j\right)=0 \tag{2}$$

$$\frac{\partial u_j}{\partial t}+u_j\frac{\partial u_j}{\partial x}=\frac{q_j}{m_j}\frac{\partial \phi}{\partial x}-\frac{1}{m_j n_j}\frac{\partial p_j}{\partial x}+\frac{\hbar^2}{2m_j^2}\cdot \frac{\partial}{\partial x}[\frac{1}{\sqrt{n_j}}(\frac{\partial^2}{\partial x^2}\sqrt{n_j})] \tag{3}$$

$$[\nabla_\perp^2+\frac{\partial^2}{\partial x^2}]\phi=\frac{e}{\varepsilon_0}(n_e+Z_d n_{d0}-n_h) \tag{4}$$

where, n_j, u_j, p_j m_j, and q_j are respectively the perturbed number density, x-component of velocity, pressure, mass and charge of the j-th species ($j=e$ for electrons and h for holes), $q_e=-e$, $q_h=e$, ϕ is electrostatic potential, $\hbar$ is the Planck's constant divided by 2π and $\nabla_\perp^2$ is the transverse Laplacian in cylindrical co-ordinates given as

$$\nabla_\perp^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{5}$$

We assume that the plasma particles behave as a one-dimensional Fermi gas at zero temperature and therefore the pressure law [23]:

$$p_j=\frac{m_j V_{Fj}^2}{3n_{j0}^2}n_j^3 \tag{6}$$

where $V_{Fj}=\sqrt{2k_B T_{Fj}/m_j}$ is the Fermi thermal speed, T_{Fj} is the Fermi temperature and k_B is the Boltzmann constant.

The boundary condition to be used is that ϕ vanishes at the wall of the perfectly conducting cylinder i.e.

$$\phi=0 \text{ at } r=R \tag{7}$$

where 'R' is the radius of the cylinder.

We now introduce the following normalizations:

$$n_j=n_j/n_{j0},\ u_j=u_j/C_s,\ \phi=e\phi/(2K_B T_{Fe}),$$

$$x=\omega_{ph}x/C_s,\ r=r/R\ ,\ t=t.\omega_{ph} \tag{8}$$

where $\omega_{pj}=(n_{j0}e^2/\varepsilon_0 m_j)^{1/2}$ is the particle plasma frequency, T_{Fe} is the electron Fermi temperature, $C_s=(2K_B T_{Fe}/m_h)^{1/2}$ is the quantum acoustic speed and K_B is the Boltzmann constant.

The equations (2)-(4) and (7) can now be written in the following dimensionless form:

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}\left(n_j u_j\right)=0 \tag{9}$$

$$\mu(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x})=\frac{\partial\phi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}[\frac{1}{\sqrt{n_e}}(\frac{\partial^2}{\partial x^2}\sqrt{n_e})] \tag{10}$$

$$\frac{\partial u_h}{\partial t}+u_h\frac{\partial u_h}{\partial x}=-\frac{\partial\phi}{\partial x}-\sigma n_h\frac{\partial n_h}{\partial x}+\frac{\mu H^2}{2}\frac{\partial}{\partial x}[\frac{1}{\sqrt{n_h}}(\frac{\partial^2}{\partial x^2}\sqrt{n_h})] \tag{11}$$

$$[\frac{1}{d^2}\nabla_\perp^2+\frac{\partial^2}{\partial x^2}]\phi=\frac{n_e}{\delta}+\frac{Z_d n_{d0}}{n_{h0}}-n_h \tag{12}$$

$$\phi=0 \text{ at } r=1 \tag{13}$$

where $\mu=m_e/m_h$ is the electron-hole mass ratio, $\delta=n_{h0}/n_{e0}$ is the charge imbalance parameter originating from dust particles, $\delta>1$ for negatively charged background dust grains and $\delta<1$ for positively charged dust grains; $\sigma=T_{Fh}/T_{Fe}$ is the ratio of the Fermi temperatures, $H=\hbar\omega_{pe}\delta^{1/2}/(2K_B T_{Fe})$ is a dimensionless parameter proportional to quantum diffraction, $d=R/\sqrt{2}.\lambda_h$ in which $\lambda_h=(\varepsilon_0 K_B T_{Fe}/e^2.n_{h0})^{1/2}$ is the quantum hole Debye- length. In many cases the electron-hole mass ratio μ is taken to be one but in semiconductors μ should be taken as the ratio of effective masses of electrons and holes. In that case μ can be different from one due to parabolicity of conduction band.

III. Derivation of Dispersion Relation

To derive the dispersion relation we make the following perturbation expansions of the field quantities

$$\begin{aligned} n_j &= 1+\varepsilon n_j^{(1)}+\varepsilon^2 n_j^{(2)}+\cdots \\ u_j &= u_j^{(0)}+\varepsilon u_j^{(1)}+\varepsilon^2 u_j^{(2)}+\cdots \\ \phi &= \varepsilon\phi^{(1)-}+\varepsilon^2\phi^{(2)}+\cdots \end{aligned} \tag{14}$$

We assume that the space and time dependence of the perturbed quantities to be of the form $\exp\ i(kx-\omega t-s\theta)$ in which k is the wave number, ω is the wave frequency, and s is a positive integer or zero.

Substituting these expansions in (9)-(12) we get under linear approximation the following equation satisfied by $\phi^{(1)}$:

$$\frac{d^2\phi^{(1)}}{dr^2}+\frac{1}{r}\frac{d\phi^{(1)}}{dr}+(p_{sn}^2-\frac{s^2}{r^2})\phi^{(1)}=0 \tag{15}$$

where,

$$\frac{1/\delta}{\{\mu(\omega-ku_{oe})^2-k^2(1+k^2H^2/4)\}}+\frac{1}{\{(\omega-ku_{oh})^2-k^2(\sigma+\mu k^2H^2/4)\}}=1+\left(\frac{p_{sn}}{kd}\right)^2 \tag{16}$$

The boundary condition satisfied by $\phi^{(1)}$ is

$$\phi^{(1)}=0 \text{ at } r=1 \tag{17}$$

Solution of (15) under the boundary condition (17) is

$$\phi^{(1)}=AJ_s(p_{sn}r) \tag{18}$$

where A is independent of r and p_{sn} is the n-th zero of $J_s(r)=0$ i.e., $J_s(p_{sn})=0$.

Thus the relation (16) is the desired dispersion relation with p_{sn} as the n-th zero of $J_s(r)=0$. It includes modifications due to the presence of quantum diffraction effect, charged dust grain, electron-hole streaming and finite boundary effects.

For a non-drifting ($u_j^{(0)}=0$) plasma, the dispersion relation (16) reduces to a quadratic equation in ω^2:

$$a_4\omega^4+a_2\omega^2+a_0=0 \tag{19}$$

in which

$$a_4=\gamma_0\mu\delta$$
$$a_2=-\gamma_0[1+\mu\delta+\delta k^2(1+\frac{H^2k^2}{4})+\mu\delta k^2\cdot(\sigma+\mu\frac{H^2k^2}{4})]$$
$$a_0=k^2(1+\frac{H^2k^2}{4})+k^2(\sigma+\mu\frac{H^2k^2}{4})+\gamma_0\delta k^4(1+\frac{H^2k^2}{4})\cdot(\sigma+\mu\frac{H^2k^2}{4}) \tag{20}$$

in which

$$\gamma_0=1+\left(\frac{p_{sn}}{dk}\right)^2 \tag{21}$$

Equation (19) has the following two roots for ω^2:

$$\omega_\pm^2=\frac{1}{2a_4}[-a_2\pm\sqrt{a_2{}^2-4a_4a_0}] \tag{22}$$

For non-streaming electron-hole dusty plasma we find that $a_2< 0$ and $a_2^2>4a_4a_0$. So in this case we have two positive roots ω_+,ω_- and hence two stable modes of electron plasma wave propagation. For large *k* we have approximately

$$\omega_-^2\approx k^2(1+\frac{H^2k^2}{4}) \tag{23}$$

$$\omega_+^2\approx k^2(\sigma+\frac{\mu H^2k^2}{4}) \tag{24}$$

The dispersion relations (23) and (24) show that the wave frequencies for these modes increase with increase in μ, σ and *H* for a fixed value of *k*.

In order to study the effects of finite geometry on the wave propagation we drop the contributions from holes, streaming of plasma particles and assume that electrons are globally neutralized by fixed background of ions then the dispersion relation (16) reduces to the form

$$\omega^2=\frac{1}{1+\left(\frac{p_{sn}}{kd}\right)^2}+k^2(1+k^2H^2/4) \tag{25}$$

It corresponds to simple electron plasma stable wave mode in a cylindrical waveguide filled with two component electron-ion plasma.

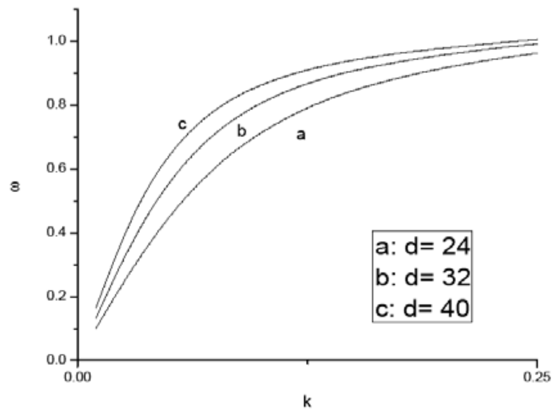

Fig 1 Dispersion relation for the lowest radial mode (*s=0, n=1*) for different transverse dimensions of the waveguide (d= 24, 32 and 40). H=1.25, P_{01}= 2.4048

In Fig. 1 we have plotted the dispersion relation for the lowest radial mode (*s=0, n=1*) for different transverse dimensions of the waveguide. As expected finite boundary effects become important in the long wavelength limit when transverse dimension of the waveguide is comparable to the wavelength.

IV. Instability of the Wave

In order to study the behaviour of the wave in presence of electron-hole streaming we note that (16) can be put, after a few algebraic steps, in the following form:

$$A_1k^{10} + A_2k^9 + A_3k^8 + A_4k^7 + A_5k^6 + A_6k^5 + A_7k^4 + A_8k^3 + A_9k^2 + A_{10}k + A_{11} = 0 \quad (26)$$

In which,

$$A_1 = -\frac{\alpha_1 d^2 H^4}{16}$$

$$A_2 = -\frac{\alpha_1 d P_{01} H^4}{8}$$

$$A_3 = \frac{H^2 d^2}{4}\{\delta\sigma - \mu\alpha_1 u_{0c}^2 - \delta u_{0h}^2\} + \frac{\alpha_1 P_{01}^2 H^2}{16}$$

$$A_4 = \frac{H^2 d^2 \omega}{2}\{\delta u_{0h} + \mu\alpha_1 u_{0c}\} + \frac{P_{01} H^2 d}{2}\{\delta\sigma - \delta u_{0h}^2 + \alpha_1 - \mu\alpha_1 u_{0c}^2\}$$

$$A_5 = H^2 d\omega P_{01}\{\delta u_{0h} + \mu\alpha_1 u_{0c}\} - \frac{H^2 d^2}{4}\{\delta\omega^2 + \delta - \mu\} + \alpha_1 d^2 u_{0c}^2\{\sigma - u_{0h}^2\} + \frac{P_{01}^2 H^2}{4}\{\delta u_{0h}^2 + \alpha_1 - \delta\sigma\} + d\,\delta\{u_{0h}^2 + d\sigma\} + \frac{\mu\alpha_1 u_{0c}^2 H^2}{4}$$

$$A_6 = -\frac{P_{01} H^2 d\omega^2}{2}\{\delta - \mu\alpha_1\} - 2dP_{01}\{\delta\sigma - \delta u_{0h}^2 - \alpha_1\sigma u_{0c}^2 + \alpha_1 u_{0h}^2 u_{0c}^2\} - 2\omega d^2\{\delta u_{0h} - \alpha_1 u_{0h} u_{0c}^2 + \alpha_1\sigma u_{0c} - \alpha_1 u_{0c} u_{0h}^2\} - \frac{H^2\omega}{2}\{\delta u_{0h} P_{01}^2 + \mu\alpha_1 u_{0c}\}$$

$$A_7 = 4P_{01}\omega d\{\alpha_1 u_{0h} u_{0c}^2 - \delta u_{0h} - \alpha_1 u_{0c}\sigma + \alpha_1 u_{0c} u_{0h}^2\} + d^2\omega^2\{\delta - \alpha_1 u_{0c}^2 - 4\alpha_1 u_{0c} u_{0h} + \alpha_1\sigma - \alpha_1 u_{0h}^2\} + \frac{P_{01}^2 H^2 \omega^2}{4}\{\delta + \mu\alpha_1\} - P_{01}^2\{\delta\sigma + \delta u_{0h}^2 + \alpha_1\sigma u_{0c}^2 + \alpha_1 u_{0h}^2 u_{0c}^2\} - d^2\{\delta - u_{0c}^2 + \sigma - u_{0h}^2\}$$

$$A_8 = 2\omega^2 dP_{01}\{\delta - \alpha_1 u_{0c}^2 - 4\alpha_1 u_{0c} u_{0h} + \alpha_1\sigma - \alpha_1 u_{0h}^2\} + 2\alpha_1 d^2\omega^3\{u_{0c} + u_{0h}\} + 2\omega P_{01}^2\{\delta u_{0h} - \alpha_1 u_{0h} u_{0c}^2 + \alpha_1\sigma u_{0c} - \alpha_1 u_{0c} u_{0h}^2\} - 2d^2\omega u_{0c}\cdot\{1 + \alpha_1\}$$

$$A_9 = 4\alpha_1\omega^3 dP_{01}\{u_{0c} + u_{0h}\} - \alpha_1 d^2\omega^4 - P_{01}^2\omega^2\cdot\{\delta - \alpha_1 u_{0c}^2 - 4\alpha_1 u_{0c} u_{0h} + \alpha_1\sigma - \alpha_1 u_{0h}^2\} + d^2\omega^2\cdot\{1 + \alpha_1\}$$

$$A_{10} = -2dP_{01}\alpha_1\omega^4 - 2\alpha_1\omega^3 P_{01}^2\{u_{0c} + u_{0h}\}$$

$$A_{11} = \alpha_1\omega^4 P_{01}^2 \quad (27)$$

in which

$$\alpha_1 = \mu\delta \quad (28)$$

Equation (26) is a tenth order algebraic equation in k and corresponds to different spatial modes of wave propagation.

We have also numerically solved the dispersion law (26). It is shown that for a particular set of values of μ, δ, σ, H, $u_e^{(0)}$ and $u_h^{(0)}$, two roots are real positive and the rest roots are complex. Two real positive roots correspond to two stable modes of propagation. Instability of the wave, when it propagates through the plasma medium is determined by the imaginary part of the complex roots. Positive values of the imaginary part of k signify a decaying mode whereas negative values of the imaginary part of k signify growing instability of the wave. The growth rate of instability is determined by $|k_i|$ where k_i is the imaginary part of k. It is shown numerically with typical plasma parameters that there are four complex roots of k having negative imaginary part. These modes are linearly unstable. The dependence of the growth rate of this instability on different plasma parameters has been studied numerically.

In Fig. 2, we show the variation of $|k_i|$ (for one typical mode) with quantum parameter H for different values of the ion streaming velocity u_{i0}. We find that $|k_i|$ i.e. instability growth rate decreases with increase in quantum diffraction parameter (*H*) but it increases with increase in electron-hole streaming velocity (u0).

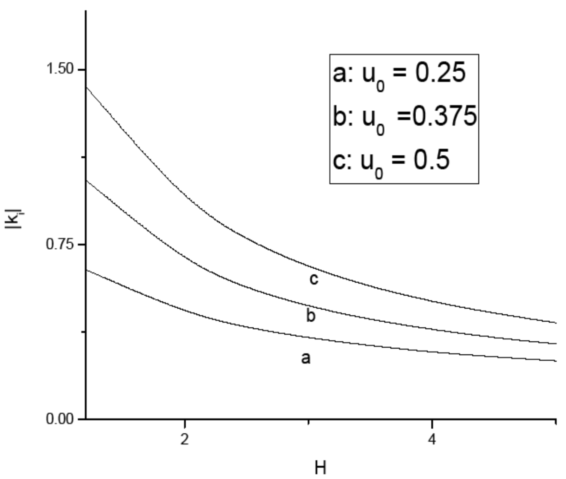

Fig. 2 Variation of $|k_i|$ (for one typical mode) with quantum parameter H for different values of the streaming velocity u_0 (0.25, 0.375 and 0.5), where, d=5, p_{01}=2.4048, σ=0.01, δ=0.1, ω=0.1 and μ=0.5

Fig. 3 shows the dependence of instability growth rate on charge imbalance parameter δ for different values of electron-hole effective mass ratio μ. It shows that the instability growth rate is lower for higher values of δ and μ.

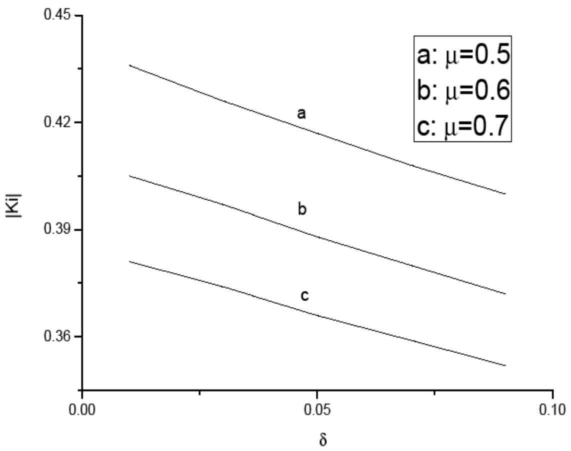

Fig 3 Variation of instability growth rate on charge imbalance parameter δ for different values of electron-hole effective mass ratio μ (0.5, 0.6 and 0.7), where, H=2.5, σ=0.01, ω=0.1, $u_{0c}=u_{oh}=0.25$, d=5

V. Some Concluding Remarks

In the present paper linear instability of electron plasma wave has been theoretically studied in an electron-hole quantum dusty plasma in a cylindrical geometry considering the presence of streaming motion of electrons and holes. Inclusion of streaming motion of electrons and holes opens up the possibility of exciting a number of wave modes some of which are linearly unstable. The growth rate of this instability is shown to depend significantly on different plasma parameters such as stream velocity of plasma particles, quantum diffraction parameter, electron-hole mass ratio and the charge imbalance parameter. The results presented in this paper might be helpful in understanding of the electron-hole motion in semiconductors. Finally we like to point out that it would be interesting to extend the present linear analysis to the nonlinear regime.

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