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Numerical Optimization of Trapezoidal Microchannel Heat Sinks

Yue-Tzu Yang, Shu-Ching Liao

Abstract**—**This study presents the numerical simulation of three-dimensional incompressible steady and laminar fluid flow and conjugate heat transfer of a trapezoidal microchannel heat sink using water as a cooling fluid in a silicon substrate. Navier-Stokes equations with conjugate energy equation are discretized by finite-volume method. We perform numerical computations for a range of $50 \leqq \mathrm{Re} \leqq 600$, $0.05\mathrm{W} \leqq \mathrm{P} \leqq 0.8\mathrm{W}$, $20\mathrm{W/cm}^2 \leqq q'' \leqq 40\mathrm{W/cm}^2$. The present study demonstrates the numerical optimization of a trapezoidal microchannel heat sink design using the response surface methodology (RSM) and the genetic algorithm method (GA). The results show that the average Nusselt number increases with an increase in the Reynolds number or pumping power, and the thermal resistance decreases as the pumping power increases. The thermal resistance of a trapezoidal microchannel is minimized for a constant heat flux and constant pumping power.

Keywords**—**Microchannel heat sinks, Conjugate heat transfer, Optimization, Genetic algorithm method.

I. INTRODUCTION

MICROCHANNEL heat sink provides efficient cooling for the high power density applications. In practice, the cross-section of microchannels made by modern micromachining technology in silicon substrates is essentially trapezoidal. An experimental investigation has been performed on the laminar convective heat transfer and pressure drop of water in different trapezoidal silicon microchannels [1], [2]. It was found that the values of the Nusselt number and the apparent friction constant dependent greatly on different parameters. Koo and Kleinstreuer [3] found that the viscous dissipation was strongly dependent upon the hydraulic diameter and the aspect ratio of the channel. Herwig and Mahulikar [4] investigated that the temperature dependence of the properties of the fluid. Li et al. [5] found that compared with the inlet property method, both average and variable property methods have significantly lower apparent friction coefficients, but higher Nusselt numbers in the longitudinal direction. Optimization methods with numerical analyses are regarded as general design tools and offer a number of advantages, including automated design capability, varieties of constraints, and multi-objective applications. Liu and Garimella [6] presented analytical models and compared these models with the more robust three-dimensional numerical model and optimized the microchannel geometry. Husain and Kim [7], [8] presented a single objective optimization of microchannel heat sink based on the surrogate methods. These studies revealed that pressure and pumping power constrained optimization limits the applicability of pumping source used at the micro-level. Husain and Kim [9], [10] performed shape optimization of micro heat exchanger and microchannel heat sink respectively and obtained Pareto-optimal solutions. Husain and Kim [11] demonstrated the numerical multi-objective optimization of a microchannel heat sink design. The steady and laminar fluid flow and conjugate heat transfer were studied by a three-dimensional numerical analysis.

The objective of this work is to numerically investigate the fluid flow and the heat transfer characteristics of water in trapezoidal microchannels made of silicon plates and attempt to explain the optimum results.

II. PROBLEM DESCRIPTION AND NUMERICAL SCHEME

A silicon-based microchannel heat sink model as shown in Fig. 1 has been taken to analyze and optimize based on genetic algorithm. The flow is assumed to be steady and laminar, and a uniform heat flux is applied at the bottom of the heat sink. The governing equations for the 3-D incompressible flow can be written as follows,

Continuity equation

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{1}$$

Momentum equation

$$u_j \frac{\partial}{\partial x_j}(\rho u_i) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{2}$$

Energy equation of fluid

$$u_i\left[\frac{\partial\left(\rho C_p T\right)}{\partial x_i}\right] = \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) + \mu\left[\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{3}$$

Energy equation of solid

$$\frac{\partial}{\partial x_i}\left(k_s\frac{\partial T_s}{\partial x_i}\right) = 0 \tag{4}$$

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Under hydraulic boundary conditions, uniform velocities are applied at the channel inlet. Exposed to the atmosphere, the outlet pressure is the static pressure. The no-slip boundary condition is applied at all solid walls. The thermal boundary condition at the bottom wall is a constant wall heat flux, while an adiabatic boundary condition is imposed on the top wall and $T = T_0$ at the microchannel inlet. The governing equations are discretized by using a control-volume-based finite-difference method with a power-law scheme on an orthogonal non-uniform staggered grid. The coupling of velocity and pressure terms of momentum equation are solved by SIMPLE (Semi-Implicit Method for Pressure-Linked Equation) algorithm [12]. The solution is considered convergent when the normalized residual of the algebraic equation is less than a prescribed value of 10^{-4}.

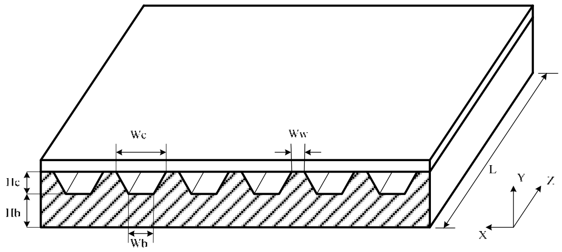

Fig. 1 Schematic of microchannel heat sink

Multi-objective optimization based on evolutionary algorithms requires many evaluations of objective functions to search for the optimal solutions. These evaluations of objective functions become very expensive and time consuming in the absence of a representative response function. Therefore, surrogate-based approximation is used to save time and to avoid the numerical cost. The genetic algorithms (GA) use a population of several individuals to perform the optimization by simulating the benefit and evolution mechanism of biology [13]. It was suggested to apply various surrogate models. In this study, the response surface methodology (RSM) [14] and genetic algorithms (GA) are used to carry out the optimal process. The response surface methodology is a parameter design for efficient experiment. It can find out the mix effect of parameters by fewer experiments, and create an objective function. After creating an objective function, we use the genetic algorithms (GA) to find out the optimal geometries. GA solves optimization problem iteratively based on biological evolution process in nature.

In the solution procedure, a set of parameter values is randomly selected. Set is ranked based on fitness values (i.e. performance factor in this study). The best combination of parameters leading to minimum fitness values is determined. A new combination of parameters is generated from the best combination by simulating biological mechanisms of offspring, crossover and mutation. This process is repeated until the fitness value with a new combination of parameters cannot be further reduced anymore. The final combination of parameters is considered as the optimum solution. It is convenient to adopt GA to resolve the heat sink optimization.

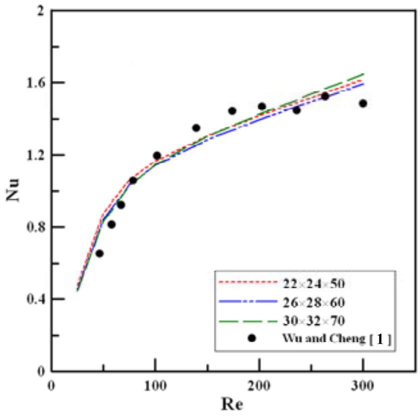

Fig. 2 Grid independent test of case 2

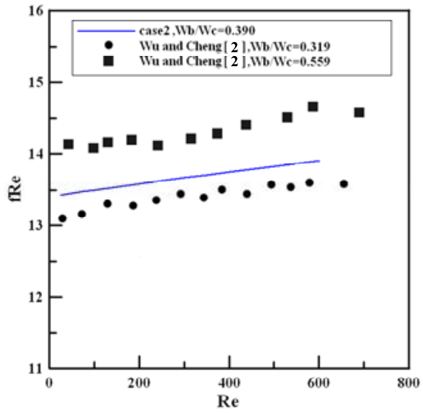

Fig. 3 Friction constant distribution

III. RESULTS AND DISCUSSION

The geometric parameters of the case studied are shown in Table I. For the validation of the theoretical model and the choice of appropriate boundary conditions, the numerical results are compared to the available experimental results in the literature for case 2, which is shown in Fig. 2. It indicates that the Reynolds number dependence of the Nusselt number is obvious. A comparison of theoretical predictions with the experimental data in the literature is used to assess the grid independence of the results. Different size meshes, 22×24×50, 26×28×60 and 30×32×70 are employed in testing the numerical model. It has been validated using experimental data reported in Wu and Cheng [1]. Certain discrepancies between calculations and the available data of Wu and Cheng [1] may be caused by the roundoff and discretization or measurement errors. Considering these factors, the overall comparisons with test data are satisfactory. Fig. 3 shows the distribution of friction constant for laminar flow. It can be seen from the figure that the laminar friction constant fRe of the trapezoidal channels increases with the increase in Re, having a trapezoidal cross-section, deviate greatly from the classical value of fRe =16.

From Fig. 4, it can be seen that the Nusselt number increases with the increase of Re, especially for the optimum case. The change in the Nusselt number due to different geometric parameters are more obvious at large Reynolds numbers than at low Reynolds numbers. For the trapezoidal channels, there are three geometric parameters including α $({W_c}/{W_w + W_c})$, β $({H_c}/{H_b + H_c})$ and γ $({W_b}/{W_c})$ which affect the friction and heat transfer. Fig. 5 shows that the change of the total thermal resistance is not obvious at $\beta > 0.2$. Fig. 6 presents that total thermal resistance decreases while β and γ increase at $\alpha = 0.65$ and P=0.3W. Therefore design variables β and γ can be suitably utilized to economize the optimization procedure in view of multi-variable, multi-objective and multi-disciplinary design optimizations. From Figs. 7 and 8, the results show that a lower thermal resistance can be obtained at the cost of a higher pumping power, whereas low pumping power are associated with high thermal resistances. It provides a designer to pick up the optimal solution in accordance with the available pumping power to drive the coolant.

TABLE I
GEOMETRIC PARAMETERS

	W_c	W_b	H_c	H_b	W_b/W_c
Case1	423.2μm	327.4μm	56.13μm	193.87μm	0.774
Case2	157.99μm	61.62μm	56.28μm	193.72μm	0.390
Case3	437.21μm	270.19μm	110.7μm	139.9μm	0.618

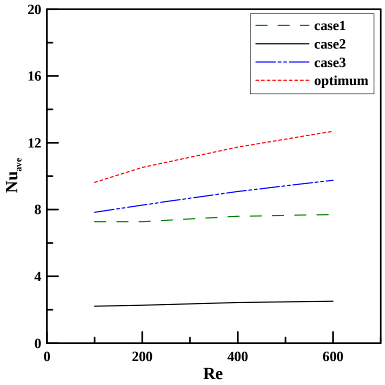

Fig. 4 Averaged Nusselt number distribution

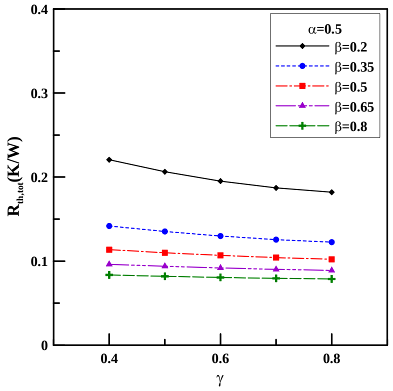

Fig. 5 Effects of the depth of the microchannel tothewhole depth (β) on the total thermal resistance

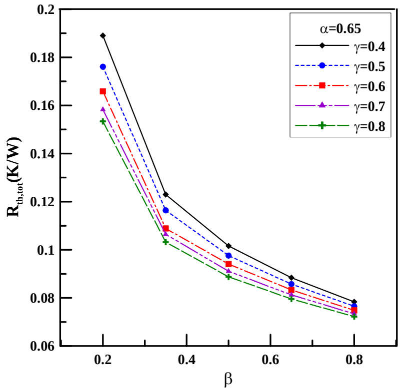

Fig. 6 Effects of the ratio of upper width and lower width (γ) on the total thermal resistance

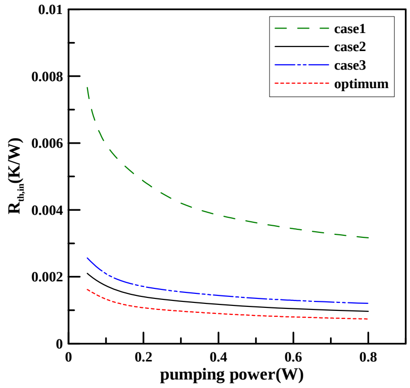

Fig. 7 Distribution of inlet thermal resistance at different pumping power

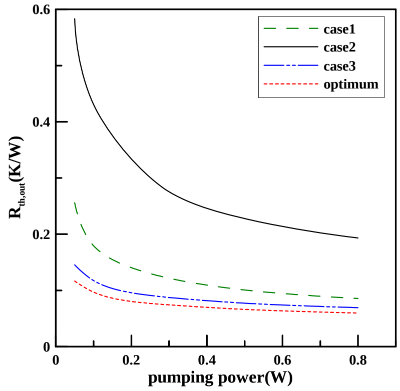

Fig. 8 Distribution of outlet thermal resistance at different pumping power

IV. CONCLUSION

Numerical simulations of Nusselt number Nu, friction constant fRe, and thermal resistance R_{th} for the laminar flow of water through the trapezoidal microchannels with different geometric parameters have been obtained. The laminar Nusselt number and friction constant of the trapezoidal microchannels increase with Re. This increase is more obvious at large Reynolds numbers than that at low Reynolds numbers. The enhancement in heat transfer is more significant at large Reynolds numbers. A comparison of results shows that geometric parameters have more significant effects on the performance. The present study demonstrates the numerical optimization of a trapezoidal microchannel heat sink design using the response surface methodology (RSM) and the genetic algorithm method (GA). Three design variables are selected from the geometric variables, the ratio of the upper width of the microchannel to the whole width α , the depth of the microchannel to the whole depth β and the ratio of upper width and lower width of the microchannel γ . The thermal resistance of a trapezoidal microchannel is minimized for constant heat flux and constant pumping power. Based on the optimal results, the optimum condition is $\alpha = 0.8$, $\beta = 0.586$ and $\gamma = 0.79$.

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