

# CDM Controller Order and Disturbance Rejection Ability

João Paulo Coelho, Wojciech Giernacki, José Boaventura-Cunha

*Abstract*—The coefficient diagram method is primarily an algebraic control design method whose objective is to easily obtain a good controller with minimum user effort. As a matter of fact, if a system model, in the form of linear differential equations, is known, the user only need to define a time-constant and the controller order. The later can be established regarding the expected disturbance type via a lookup table first published by Koksal and Hamamci in 2004. However an inaccuracy in this table was detected and pointed-out in the present work. Moreover the above mentioned table was expanded in order to enclose any  $k$  order type disturbance.

*Keywords*—Coefficient diagram method, control system design, disturbance rejection.

## I. INTRODUCTION

**T**HE coefficient diagram method (CDM), as a control design and analysis method, was presented by Shunji Manabe in the late nineties of the twentieth century [10], [11]. Since then many articles have been published in both CDM theoretical extensions [12], [16] and practical applications [1], [3], [15].

From the user point-of-view, the main feature of CDM is its simplicity. In fact the design process only requires the designer to define a single parameter: the equivalent time-constant. Then the controller transfer function is automatically obtained via an algebraic method similar to pole placement. However, unlike the latter, the characteristic polynomial in CDM is easily defined. An improved version of Kessler's standard form [6], commonly called Manabe's standard form, is selected as the target polynomial. This choice will lead to a zero overshoot closed-loop step response and a settling time within 2.5 to 3 time-constants.

Besides the CDM algebraic nature, this method also includes a diagram that can be used to understand the system behaviour. In fact the precise name of this method derives from this diagram. The plotted curves in this diagram can be used to analyse the system dynamic behaviour, his robustness regarding modelling errors and stability. The latter is added by taking into consideration the Lypatov-Sokolov sufficient stability conditions [9]. In this article only the algebraic steps

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of the method are considered. Hence the reader is referred to [13] for extended treatment on the diagram nature of this method.

In the same document Shunji Manabe emphasize that CDM is a control design procedure tailored for people without a strong (or even any) theoretical background in control theory. This statement is further reinforced after the publication of [7] where a guideline for controller order selection is presented. However there are some issues regarding the published table that the present article tries to point out. In particular the controller order selection when the system is subject to sinusoidal type disturbances.

The reasons that lead us to make this affirmation will be presented in Section III. However, before that, an overview on the algebraic nature of CDM will be presented in Section II. This paper ends with Section IV where this work conclusions are exposed.

## II. THE CDM CONTROLLER DESIGN METHOD

The CDM design procedure can be summarized as follows. First a plant mathematical model, in polynomial format, is required. Then the characteristic equation is established regarding the desired dynamic performance.

The next step concerns the definition of the controller order and his description also in polynomial format. Then the controller coefficients are obtained by solving a design equation similar to the Diophantine equation.

The last step is to analyse the coefficient diagram and make inference about the desired and obtained system characteristics. Computer simulation of the overall system, taking into consideration disturbances and measurement noise or sensor faults, should be made.

This section presents the first four steps of the above design algorithm and will be divided into four subsections. The first will address the controller structure where a closed-loop block diagram is presented and the  $\mu$  operator is defined. The controller mathematical description is presented in subsection II-B and the characteristic polynomial in II-C. The last subsection concerns the design equation and the shape of the Sylvester matrix.

### A. CDM Controller Structure

This section begins with the block diagram presented in Fig. 1 where the overall CDM closed-loop system structure can be perceived.

The first fact to be highlighted regards the use of variable  $\mu$  within the block diagram architecture. Generally, in the

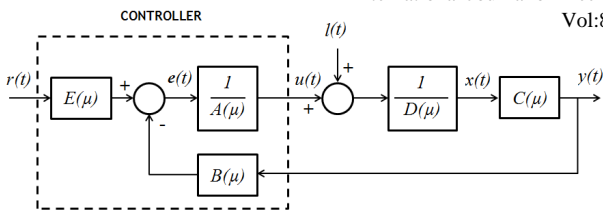


Fig. 1. Coefficient diagram method closed-loop system structure.

literature, a similar block diagram is presented but with the character  $s$  replacing  $\mu$ . Even if this fact can be seen as irrelevant, the use of  $s$  can bias the reader to think that the above block diagram is expressed in the Laplace domain. However CDM handle the system in the time domain. Hence, in order to stress out this issue, the  $\mu$  greek letter is used instead. Note that the relationship between the Laplace operator  $s$  and  $\mu$  is equivalent to the relation between the  $Z$ -transform variable  $z$  and the backward shift operator  $q$ . The use of  $z$  character implies that the system representation is in the frequency domain while the use of  $q$  defines it in the discrete-time domain. For example  $Y(z) = (z^{-1} + 1) X(z)$  and  $y(n) = (q^{-1} + 1) x(n)$  may resemble similar but, in fact, they are in different domains.

Within the CDM framework, the  $\mu$  operator is defined by the equality represented in (1).

$$\frac{d^i}{dt^i} = \mu^i \quad (1)$$

A general linear differential equation with constant coefficients  $a_i$  and  $b_j$ , for  $i = 0, \dots, n$  and  $j = 0, \dots, m$ , with the following generic structure:

$$a_0 y(t) + \sum_{i=1}^n a_i \frac{d^i y(t)}{dt^i} = b_0 u(t) + \sum_{j=1}^m b_j \frac{d^j u(t)}{dt^j} \quad (2)$$

can assume an alternative formulation using the  $\mu$  operator as shown in (3).

$$y(t) \left( a_0 + \sum_{i=1}^n a_i \cdot \mu^i \right) = u(t) \left( b_0 + \sum_{j=1}^m b_j \cdot \mu^j \right) \quad (3)$$

This representation resembles a polynomial in  $\mu$  with  $a_i$  and  $b_j$  as coefficients and  $n$  and  $m$  as their orders. Now if one defines  $A(\mu) = a_n \cdot \mu^n + \dots + a_0$  and  $B(\mu) = b_m \cdot \mu^m + \dots + b_0$  then the system differential equation can be written, in a more compact way, as:

$$A(\mu) \cdot y(t) = B(\mu) \cdot u(t) \quad (4)$$

where the dot operation represent the product between each polynomial term and the related signal.

Remark that, even if (3) resembles a polynomial, in fact it does not represent a *true* polynomial since it does not possess the same properties. For example it is not allowed to express the signal  $y(t)$  in (4) as,

$$y(t) = \frac{B(\mu)}{A(\mu)} \cdot u(t) \quad (5)$$

Another form of system representation requires the introduction of a state variable denoted by  $x(t)$  and defined as:

$$A(\mu) \cdot x(t) = u(t) \quad (6)$$

leading to a relationship between  $y(t)$  and  $x(t)$  expressed in (7).

$$y(t) = B(\mu) \cdot x(t) \quad (7)$$

Manabe in [10], [11] call the formulation expressed in (4) as *left polynomial form* and the one defined by the pair (6) and (7) by *right polynomial form*. This concept is fundamental to properly understand the block diagram system formulation.

In Fig. 1, it is possible to observe that some polynomials are represented in a fraction denominator. However this liberty must be properly understood as representing (6) or (7). For this reason, the quantity  $\frac{1}{A(\mu)}$  must not be taken literally as there is no polynomial inverse in the  $\mu$  domain. That is, this polynomial representation is algebraically defined as a ring.

In the end, handling a polynomial in  $\mu$  domain is different from the one expressed in  $s$  domain. However, in practice, the difference resumes to the fact that the numerator and denominator must be handled separately. They cannot co-exist in the same block since the polynomial inverse in  $\mu$  is not defined.

### B. Controller Description

Consider again the block diagram of Fig. 1 where five different polynomials in the  $\mu$  domain can be distinguished. All the signals involved in this schematic can be written using the following four relations:

$$y(t) = C(\mu) \cdot x(t) \quad (8)$$

$$D(\mu) \cdot x(t) = u(t) \quad (9)$$

$$A(\mu) \cdot u(t) = e(t) \quad (10)$$

$$e(t) = E(\mu) \cdot r(t) - B(\mu) \cdot y(t) \quad (11)$$

Both  $C(\mu)$  and  $D(\mu)$  concern the plant dynamics and the remaining three defines the controller behaviour. Please note that if  $B(\mu)$  was expressed in the Laplace domain it would be noncausal. Moreover, in CDM controller design, the pre-filter  $E(\mu)$  is a zero order polynomial and his only coefficient is computed in order to achieve closed-loop zero steady state error.

Additionally, and for the sake of simplicity, let the order of both polynomials  $A(\mu)$  and  $B(\mu)$  be equal to  $n$  (even if some higher order coefficients of  $B(\mu)$  must be set to zero). Moreover lets make the same assumption about the orders of polynomials  $C(\mu)$  and  $D(\mu)$ . Let  $m$  be now its order value.

Also in (11) there must be a order match between the polynomials  $E(\mu) \cdot r(t)$  and  $B(\mu) \cdot y(t)$ . In practice this mean that maybe higher order coefficients of one of the two polynomials are equal to zero.

After some simple algebraic manipulation, and by defining the  $m + n$  order characteristic polynomial  $P(\mu)$  as:

$$P(\mu) = A(\mu) \cdot D(\mu) + C(\mu) \cdot B(\mu) \quad (12)$$

the closed-loop system behaviour is represented by:

$$P(\mu) \cdot y(t) = C(\mu) \cdot E(\mu) \cdot r(t) \quad (13)$$

C. The Characteristic Polynomial

Obtaining a proper characteristic polynomial is a complex task usually requiring a deep knowledge on control theory in order to find the closed loop poles location. This task can be made easier by imposing a given structure to it. In [14] a comparison between several characteristic polynomial structures, regarding both time and frequency performance indexes, is presented. From the published results one can conclude that Manabe's and binomial polynomials present the best results.

We begin this section by rewriting the  $n + m$  order characteristic polynomial  $P(\mu)$  as:

$$P(\mu) = \sum_{i=0}^{n+m} p_i \cdot \mu^i \quad (14)$$

Let's define two additional figures: the stability index, denoted by  $\gamma_i$  for  $i = 1, \dots, (n+m) - 1$  and the predominant time constant  $\tau$ . Both are described in further detail in [11] and presented hereafter in (15) and (16):

$$\gamma_i = \frac{p_i^2}{p_{i-1} \cdot p_{i+1}} \quad (15)$$

$$\tau = \frac{p_1}{p_0} \quad (16)$$

Each of the characteristic polynomial coefficients  $p_i$  in (14) can be written as a function of both stability indexes and predominant time constant. Hence the (normalized) characteristic polynomial can be expressed alternatively as:

$$\frac{P(\mu)}{p_0} = \sum_{i=2}^{n+m} \left\{ (\tau\mu)^i \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_j^{i-j}} \right) \right\} + \tau\mu + 1 \quad (17)$$

For Manabe's polynomial, the coefficients are chosen in order to have the following stability index values [11]:

$$\gamma_i = \begin{cases} 2.5 & \text{if } i = 1 \\ 2 & \text{if } i = 2, \dots, n + m \end{cases} \quad (18)$$

D. Solving for the Controller

One of the hardest part in a pole placement method is to obtain the characteristic polynomial. The CDM method presents a simple way to obtain it by just defining the desired equivalent time constant. Having the desired characteristic polynomial, the next step is just algebraic and requires solving an system of equations with the formulation described by (12).

Due to the fact that the plant dynamics are fixed, the above mentioned equation has an appearance that resembles the Diophantine equation from the numbers theory field. In this case we don't look for integers but for polynomials in  $\mu$  domain.

Giving  $p_0$ ,  $\tau$  and  $\gamma_i$  beforehand, the problem resumes to the pole-placement problem [7]. However in the CDM method the structure of the Sylvester matrix differs from the pole placement one. This is due to the knowledge of some  $A(\mu)$  coefficients due to a priori assumptions about the type of system disturbances.

Assuming the Sylvester matrix  $\Sigma$  has the structure represented in (20) and that the unknown polynomial coefficients, considering zero the lower  $k$  coefficients of  $A(\mu)$ , are arranged in a vector  $\mathbf{x}$  as expressed in (21) then the CDM controller solution is obtained by solving equation,

$$\mathbf{x} = \Sigma^{-1} \cdot \mathbf{p} \quad (19)$$

were

$$\Sigma = \begin{bmatrix} d_m & c_m & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ d_{m-n+k} & c_{m-n+k} & & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ d_0 & c_0 & \dots & 0 & 0 \\ 0 & 0 & & d_m & c_m \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & d_0 & c_0 \end{bmatrix} \quad (20)$$

and

$$\mathbf{x} = [ a_n \quad b_n \quad \dots \quad a_k \quad b_k \quad b_{k-1} \quad \dots \quad b_0 ]^T \quad (21)$$

$$\mathbf{p} = [ p_{n+m} \quad \dots \quad p_1 \quad p_0 ]^T \quad (22)$$

III. CONTROLLER ORDER AND DISTURBANCE REJECTION

Consider again the block diagram shown in Fig. 1 and let's analyse the output effect of a disturbance in the controlled variable  $u(t)$ . In order to do this the reference signals is set to zero. This leads to a relationship between  $l(t)$  and  $y(t)$  given by the differential equation presented in (23).

$$C(\mu) \cdot A(\mu) \cdot l(t) = \left( A(\mu) \cdot D(\mu) + C(\mu) \cdot B(\mu) \right) \cdot y(t) \quad (23)$$

Assuming  $C(\mu)$  and  $A(\mu)$  are polynomials with order  $m$  and  $n$  respectively and with the following form:

$$A(\mu) = a_n \cdot \mu^n + \dots + a_0 \quad (24)$$

$$C(\mu) = c_m \cdot \mu^m + \dots + c_0 \quad (25)$$

Moreover let's impose  $c_0 \neq 0$ . The product  $A(\mu) \cdot C(\mu)$ , present in (23), is a  $m + n$  order polynomial. This polynomial will be denoted by  $G(\mu)$  and expanded as:

$$G(\mu) = \sum_{i=0}^{m+n} g_i \cdot \mu^i \quad (26)$$

At this point, the effect of particular disturbance types on the output  $y(t)$ , will be discussed. For the system to be able to absorb the disturbance effect,  $y(t)$  must tend to the reference signal as quickly as possible. Assuming  $r(t) = 0$  this steady state system behaviour will be easily handled in the Laplace domain. By applying the final value theorem the following equality should hold:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \cdot Y(s) \quad (27)$$

where  $Y(s)$  is the Laplace transform of  $y(t)$ . That is  $Y(s) = \mathcal{L}\{y(t)\}$ . However, for this equality to hold, all the  $Y(s)$  poles must have negative real parts and no more than one pole can be at the origin [2].

Assuming causality and zero initial conditions, the application of Laplace transform to the differential equation (23) leads to,

$$Y(s) = \frac{G(s)}{A(s) \cdot D(s) + C(s) \cdot B(s)} L(s) \quad (28)$$

where  $Y(s) = \mathcal{L}\{y(t)\}$  and  $L(s) = \mathcal{L}\{l(t)\}$ .

Applying the final value theorem to the above expression and remembering that the equality (27) must hold, than,

$$\lim_{s \rightarrow 0} s \cdot G(s) \cdot L(s) = 0 \quad (29)$$

where  $G(s) \cdot L(s) = \mathcal{L}\{G(\mu) \cdot l(t)\}$ .

Now let's analyse this last expression for different type of disturbance signals. If  $l(t)$  is the impulse signal  $\delta(t)$  than its Laplace transform is equal to the unity. In addition, taking into consideration that  $\lim_{s \rightarrow 0} G(s) = g_0$ , expression (29) resumes to:

$$\lim_{s \rightarrow 0} s \cdot G(s) = g_0 \cdot \lim_{s \rightarrow 0} s = 0 \quad (30)$$

This expression allows us to conclude that the closed loop system will always be able to absorb disturbance impulses regardless the controller type.

Let's proceed by increasing the disturbance order now for a step type input  $h(t)$ . Since  $\mathcal{L}\{h(t)\} = \frac{1}{s}$ , expression (29) take the following format:

$$\lim_{s \rightarrow 0} s \cdot G(s) \cdot \frac{1}{s} = g_0 \quad (31)$$

Since  $g_0$  is equal to the product of  $a_0$  and  $c_0$  and since  $c_0 \neq 0$  is assumed than, in order for (31) result to be zero, the controller coefficient  $a_0$  must be equal to zero. For this reason, to completely suppress step disturbances, the controller type must be one. In other words it must have a pole at the origin.

Now for a ramp type input disturbance, and performing the same steps as above, expression (32) is obtained.

$$\lim_{s \rightarrow 0} s \cdot G(s) \cdot \frac{1}{s^2} = \lim_{s \rightarrow 0} \frac{G(s)}{s} \quad (32)$$

The former limit can be expanded as:

$$\lim_{s \rightarrow 0} \frac{g_{n+m} \cdot s^{n+m} + \dots + g_2 \cdot s^2 + g_1 \cdot s + g_0}{s} \quad (33)$$

Hence, for a complete ramp disturbance rejection, it is straightforward to see that, at least, both  $g_1$  and  $g_0$  must be zero. If all the  $C(\mu)$  coefficients are assumed non-zero than this disturbance rejection only can be achieved if, at least,  $a_0$  and  $a_1$  are equal to zero. In this case the controller type will increase to 2.

This last case, together with the previous two, allows us to foresee a pattern for the controller type as a function of disturbance order. In fact, if the disturbance can be mathematically expressed as a  $n$  order impulse integral, than at least a  $n$  type controller is needed to fully suppress the disturbance effect.

This conclusion cannot be extrapolated for other signal types. For example, let's assume a  $\omega$  frequency sinusoidal signal  $l(t) = \sin(\omega t)$ , represented in Laplace domain by  $L(s) = \frac{\omega}{s^2 + \omega^2}$ . If one attempts to apply the final value theorem then:

$$\omega \cdot \lim_{s \rightarrow 0} \frac{s \cdot G(s)}{s^2 + \omega^2} = \omega \cdot \lim_{s \rightarrow 0} \frac{g_{m+n} \cdot s^{n+m+1} + \dots + g_1 \cdot s^2 + g_0 \cdot s}{s^2 + \omega^2} \quad (34)$$

It's easy to see that the above expression is always equal to zero. So one may think that even a zero type controller can be able to suppress the disturbance effect from the output. However this is not the case. It's not possible to bypass sinusoidal type disturbances with the same controller structure as for the impulse type despite the result obtained from (34). Sinusoidal disturbance cancelation is a hard problem and the reader is addressed to [8], [5] and [4] for more details. The reason we cannot treat impulse and sinusoidal disturbances in the same way is that one has applied the final value theorem to an expression that has imaginary conjugate poles. When this situation happens, the final value theorem cannot be applied. For this reason the fourth column of Table I in [7] is not accurate since it gives the same polynomial controller condition for both impulse and sinusoidal disturbances.

The above referred table is replicated in this document, with some modifications, and labelled Table I. In this case without the sinusoidal disturbance type and including an extra table line for a generic  $k$  order disturbance. This table, just like the one published in [7], only gives a suggestion regarding the controller order to be used.

TABLE I  
CONTROLLER ORDER SELECTION AS A FUNCTION OF SIGNAL  
DISTURBANCE TYPE

Disturbance type	$A(\mu)$ degree	$B(\mu)$ degree	$P(\mu)$ degree	Condition
None	$n - 1$	$n - 1$	$2n - 1$	-
Impulse	$n - 1$	$n - 1$	$2n - 1$	-
Step	$n$	$n$	$2n$	$a_0 = 0$
Ramp	$n + 1$	$n + 1$	$2n + 1$	$a_0 = 0,$ $a_1 = 0$
$k$ order	$n + k - 1$	$n + k - 1$	$2n + k - 1$	$a_0 = 0$ $\dots$ $a_{k-1} = 0$

This value must also take into consideration that, in order to have an invertible Sylvester matrix, for a  $n$  order system the controller must have order equal to  $n - 1$  leading to a characteristic polynomial of order  $2n - 1$ .

However, some of the  $A(\mu)$  controller polynomial coefficients may be already known after assuming complete elimination of some disturbance type. For example, assuming a step input disturbance,  $a_0 = 0$ . In this context a lower number of equations are required to make square the Sylvester matrix. Taking into consideration this step type disturbance input, and for a  $m$  order controller, one have  $m - 1$  unknowns about polynomial  $A(\mu)$ . Assuming that the order of polynomial  $B(\mu)$  is also equal to  $m$  and that the system order is  $n$  than, by

observing (21), and since  $a_0$  is known, the vector of unknowns  $\mathbf{x}$  has now  $2n + 1$  entries. Hence the Sylvester matrix has  $(n + m + 1)$  lines and  $2m + 1$  columns. In order for it to be square  $m$  must be equal to  $n$ . For this reason, the characteristic polynomial has order  $2n$ . The third line in the  $P(\mu)$  degree column of Tab. I condensates this conclusion.

The same idea can be applied to the case one wants to completely suppress a ramp type disturbance effects. In this case the lower order controller that can accomplish this task has  $a_0 = a_1 = 0$ . The vector  $\mathbf{x}$  includes  $2m$  elements. This lead to the constraint  $m + n + 1 = 2m$  for  $\Sigma$  to be invertible. That is  $m = n + 1$  and  $P(\mu)$  is now a  $2n + 1$  order polynomial. Once again this result can be observed in the fourth line of Table I.

Finally, for a  $m$  order controller to be able to suppress a  $k$  order disturbance, the lower  $k$  coefficients of  $A(\mu)$  must be equal to zero. This fact leads to a  $k$  type controller. That is, one in which  $a_0 = \dots = a_{k-1} = 0$ . For this reason the vector of unknowns  $\mathbf{x}$  has now  $2m - k + 2$  elements. For  $\Sigma$  invertibility,  $2m - k + 2$  must be equal to  $n + m + 1$ . That is  $m = n + k - 1$  leading to a  $2n + k - 1$  order characteristic polynomial.

Before ending this section we want to emphasize that Table I is only a guideline. In fact there is no absolute need for a square Sylvester matrix if only an approximated solution is enough. Usually this least squares solution can be sufficient to ensure closed-loop system specifications while leading to lower order controllers. Additionally the controller numerator order do not need to be equal to the denominator order. However giving too much freedom will violate the basilar principle of CDM: to be an easy application technique.

#### IV. CONCLUSION

The CDM method cornerstone relies on the design simplicity from the user point-of-view. Indeed, if the system model is known, the control designer only needs to define two things: the value of  $\tau$  and the controller order. The later can be established taking into account the expected system disturbance shape. The relationship between the CDM controller order and the disturbance type was first published by [7]. However an error was detected and pointed-out in the present work. Moreover, the above mentioned table, was expanded in order to enclose any  $k$  order type disturbance.

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