Parallel Particle Swarm Optimization Optimized LDI Controller with Lyapunov Stability Criterion for Nonlinear Structural Systems

P.-W. Tsai, W.-L. Hong, C.-W. Chen, C.-Y. Chen

Abstract—In this paper, we present a neural-network (NN) based approach to represent a nonlinear Tagagi-Sugeno (T-S) system. A linear differential inclusion (LDI) state-space representation is utilized to deal with the NN models. Taking advantage of the LDI representation, the stability conditions and controller design are derived for a class of nonlinear structural systems. Moreover, the concept of utilizing the Parallel Particle Swarm Optimization (PPSO) algorithm to solve the common P matrix under the stability criteria is given in this paper.

Keywords—Lyapunov Stability, Parallel Particle Swarm Optimization, Linear Differential Inclusion, Artificial Intelligence.

I. INTRODUCTION

IN 1985, Takagi and Sugeno first propose the new concept for fuzzy inference systems [1]. Their method is called the Takagi-Sugeno (T-S) fuzzy model, which has been widely used in the industry and academia with lots of successful examples. The T-S fuzzy model utilizes linear models in the consequent parts and results in the convenience on analysis via the conventional linear system theory. Different T-S fuzzy model based controllers have been proposed one after another [2]-[7]. The local dynamics in different state space regions are represented by a set of linear sub-models in this type of fuzzy controllers. In other words, this kind of fuzzy controllers are composed of these linear sub-models.

On the other hand, swarm intelligence is also a raising research field after the late 1990s. Unlike the fuzzy models, algorithms in swarm intelligence work base on the tinny intelligences, which are able to be observed from the creatures' behavior or their special characteristics for surviving in Mother Nature, to collaborate a serious of process for solving optimization problems in the fields of management, engineering, and economics. Many newly developed swarm intelligence algorithms are easy to be found in current literatures. For instance, Chu et al. propose Cat Swarm Optimization (CSO) [8], [9] for solving numerical optimization problems by taking the felid as the model; in 2010, Pan et al. propose Fish Migration Optimization (FMO) [10] by simulating the migration behavior and the life cycle of the grayling; and Tsai et al. propose Evolved Bat Algorithm (EBA)

[11] in 2012. Beside the innovative algorithms in swarm intelligence, some algorithms are proposed base on the fusion of different swarm intelligence algorithms, e.g., Baterial-GA Foraging [12] is an example of the fusion of different swarm intelligence algorithms. In addition, algorithms in swarm intelligence can be employed to solve optimization problems in two ways: solving the problem directly by optimizing the mathematic formula of the described problem [2], [13]-[17]. The other way is combining the swarm intelligence algorithm with other systems such as Artificial Neural Network (ANN) [18]-[20]. In this paper, we present a concept of employing swarm intelligence algorithm to find the common symmetric positive definite matrix P for r subsystems. The existing of the P matrix insures that the system is stable.

The rest of this paper is constructed as follows: The brief review on the T-S type fuzzy model, the NN approximation, and the PPSO, which is one of the improved versions of PSO in swarm intelligence, are given in Section II. Base on the LDI representation and Lyapunov's approach, a stability criterion is derived to guarantee the stability of fuzzy systems via the linear matrix inequality (LMI) technique. Finally, the design of our method to find the common symmetric positive definite matrix P is presented at last.

II. LITERATURE REVIEW

A. Motion for Structural Systems

The equation of motion for a shear-type-building modeled as an n-degree-of-freedom system controlled by actuators and subjected to external force $\phi(t)$ can be characterized by (1):

$$M\ddot{\bar{X}}(t) + C\dot{\bar{X}}(t) + K\bar{X}(t) = BU(t) - M\bar{r}\phi(t) \quad (1)$$

where $\bar{X}(t) = [\bar{x}_1(t), \bar{x}_2(t), \dots, \bar{x}_n(t)] \in R^n$ is a n-vector; $\ddot{\bar{X}}(t)$, $\dot{\bar{X}}(t)$, and $\bar{X}(t)$ are the acceleration, velocity, and displacement vectors, respectively; matrices M, C, and K are the $(n \times n)$ mass, damping, and stiffness matrices; $\bar{r}$ is a n-vector denoting the influence of the external force; B is an $(n \times m)$ matrix denoting the locations of m control forces; $\phi(t)$ is the excitation with an upper bound $\phi_{up}(t) \geq \|\phi(t)\|$; $U(t)$ corresponds to the actuator forces (generated by an active tendon system or an active mass damper, for example). It should be noted that this is only a static model.

For the controller design, the standard first-order state equation corresponding to (1) is obtained in (2), (3):

P.-W. Tsai, W.-L. Hong, C.-W. Chen, and C.-Y. Chen are with Department of Maritime Information and Technology, National Kaohsiung Marine University, Kaohsiung, Taiwan (e-mail: chengwu@webmail.nkmu.edu.tw).

C.-W. Chen is also with Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia.

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

$$\dot{\bar{X}}(t) = AX(t) + BU(t) + E\phi(t) \qquad (2)$$

where

$$\bar{X}(t) = \begin{bmatrix} \bar{x}(t) \\ \dot{\bar{x}}(t) \end{bmatrix} \qquad (3)$$

in which $\bar{X}(t)$ is a $2n$ state vector; A is a $(2n \times 2n)$ system matrix.

B. Modeling of the Neural Network

Nowadays, NN is widely used in many control systems. Especially, in the nonlinear fields due to their universal approximation capability. A neural-network-based approach is utilized in the discussion of the stability of nonlinear systems to approximate the nonlinear structural system and the stability conditions thus derived. Assume we have a NN model with S layers. Each layer has $R^\sigma (\sigma = 1,2,\dots,S)$ neurons, in which $x_1(t) \sim x_\delta(t)$ and $u_1(t) \sim u_m(t)$ are the input variables. The notation W^σ denotes weight matrix of the σth layer.The transfer function vector of the σth layer is defined in (4):

$$\Psi^\sigma(v) \equiv [T(v_1)\, T(v_2) \dots T(v_{R^\sigma})]^T \qquad (4)$$

where $T(v_\zeta)(\zeta = 1,2,\dots,R^\sigma)$ is the transfer function of the ζth neuron; and v is the net input. The final output of the NN model can then be described by (5):

$$\dot{X}(t) = \Psi^S(W^S \Psi^{S-1}(W^{S-1} \Psi^{S-2}(\dots \dots \Psi^2(W^2 \Psi^1(W^1 \Lambda(t)))))) \quad (5)$$

Although many successful applications of NNs can be found in recent days, there are still some drawbacks to their use in any control scheme. The most significant example is that a class of NNs admit a linear difference inclusion (LDI) state-space representation has been proposed and used in stability analysis with a Lyapunov function method. Base on the LDI model, some systematic model based neural network control design techniques have been developed.

A LDI base system can be described in the state-space representation as follows:

$$\dot{Y}(t) = A(a(t))Y(t) \qquad (6)$$

$$A(a(t)) = \sum_{i=1}^{r} h_i(a(t))\bar{A}_i \qquad (7)$$

The class of the structural system listed in (2) can be considered as one kind of LDI. [21] It means that without loss of generality, we can use $h_i(t)$ instead of $h_i(a(t))$. We will use $h_i(t)$ from now on. This class also contains the T-S fuzzy model, which is commonly used in the field of fuzzy control. Subsequently, the min-max matrix G_ζ^σ can be defined by (8):

$$G_\zeta^\sigma = diag[g(T(v_\zeta))], \quad \sigma = 1,2,\dots,S \qquad (8)$$

By using the interpolation method and (5), we can obtain

$$\dot{X}(t) = [\sum_{\zeta^S=1}^{2} h_{\zeta^S}(t)G_\zeta^S(W^S[\dots[\sum_{\zeta^2=1}^{2} h_{\zeta^2}(t)G_\zeta^2(W^2$$

$$[\sum_{\zeta^1=1}^{2} h_{\zeta^1}(t)G_\zeta^1(W^1\Lambda(t))])] \dots])] = \sum_{\zeta^S=1}^{2} \dots \sum_{\zeta^2=1}^{2} \sum_{\zeta^1=1}^{2} h_{\zeta^S}(t) \dots h_{\zeta^2}(t)h_{\zeta^1}(t)$$

$$G_\zeta^S W^S \dots G_\zeta^2 W^2 G_\zeta^1 W^1 \Lambda(t) = \sum_{\Omega^\sigma} h_{\Omega^\sigma}(t)E_{\Omega^\sigma}\Lambda(t) \qquad (9)$$

where

$$\sum_{\zeta^\sigma} h_{\zeta^\sigma}(t) \equiv \sum_{q_1^\sigma=1}^{2} \sum_{q_2^\sigma=1}^{2} \dots \sum_{q_{R^\sigma}^\sigma=1}^{2} h_{q_1^\sigma}(t)h_{q_2^\sigma}(t) \dots h_{q_{R^\sigma}^\sigma}(t)$$

At last, from (6) and (7), the dynamics of the NN model (9) can be rewritten in (10):

$$\dot{X}(t) = \sum_{i=1}^{r} h_i(t)\bar{E}_i\Lambda(t) \qquad (10)$$

where $h_i(t) \geq 0$, $\sum_{i=1}^{r} h_i(t) = 1$, r is a positive interger, $\bar{E}_i$ denotes a constant matrix with an appropriate dimension associated with E_{Ω^σ}. The LDI state-space representation in (10) can be further rearranged into (11):

$$\dot{X}(t) = \sum_{i=1}^{r} h_i(t)\{A_i X(t) + B_i U(t)\} \qquad (11)$$

where A_i and B_i are the partitions of E_i corresponding to the partition $\Lambda(t)$.

Thus, the nonlinear structural system in (2) can be approximated as a LDI representation in (11). Subsequently, based on this LDI representation, sufficient conditions will be presented to guarantee the close-loop system finite-time stabilization and eliminate the effect of the approximation errors and external disturbance on the regulated output. These conditions can be reduced to feasibility problems involving LMIs or the swarm intelligence algorithms. In addition, the LDI representation follows the same rules as the T-S fuzzy model, therefore, combining the flexibility of fuzzy logic theory and the rigorous mathematical analysis tools of a linear system theory into a unified framework. Tanaka and Sugeno [22] proposed a control concept called "parallel distributed compensation" (PDC) for the fuzzy control of T–S fuzzy systems in 1992. The method of fuzzy-model-based control via PDC schemes is thus developed to achieve suitable control performance, taking into consideration stability analysis of a nonlinear structural system. A useful remark is described below: The class of neural network-based LDI representations is similar to Takagi-Sugeno's fuzzy model, which is obtained by interpolating several literalized systems at different operating points through fuzzy certainty functions. [23] Therefore, the results presented above can be applied to their fuzzy models.

C. Parallel Particle Swarm Optimization

PPSO [24] is proposed by Chang et al. in 2005. Based on the original structural of PSO, the authors divide the artificial agents into independent groups, and let each group has its own ecological distribution. Learning from the ecology, it is known that an isolated environment would develop a different ecological distribution comparing to the outside environment. The precedents of splitting the amount of the population into

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

several sub-populations to construct the parallel structure can be found in several algorithms, such as Island-model Genetic Algorithm or Parallel Genetic Algorithm [25]. In addition, Chang et al. also demonstrated three information exchanging strategies for the artificial agents to share the information for finding the near best solution under the designed conditions.

In short, the complete PPSO algorithm with its three communication strategies can be summarized as follows:

Step 1. **Initialization**: Generate N_j particles $X_{i,j}^t$ for the jth group, $i = 0, 1, ..., N_{j-1}; j = 0, 1, ..., S - 1; S = 2^m$, where S is the number of groups, N_j is the number of particles contained in the jth group, m is a positive integer, and t denotes the iteration.

Step 2. **Evaluation**: Evaluate the fitness value of $\left(X_{i,j}^t\right)$ for every particle in each group by the user defined fitness function and update the stored near best solution.

Step 3. **Movement**: Update the velocity and the particles' coordinate by (12)-(14).

$$V_{i,j}^{t+1} = W^t V_{i,j}^t + c_1 r_1 \left(P_{i,j}^t - X_{i,j}^t\right) + c_2 r_2 \left(G_j^t - X_{i,j}^t\right) \quad (12)$$

$$X_{i,j}^{t+1} = X_{i,j}^t + V_{i,j}^{t+1} \quad (13)$$

$$f(G^t) \leq f\left(G_j^t\right) \quad (14)$$

where $V_{i,j}^t$ is the velocity of the ith particle in the jth group at the tth iteration, G_j^t is the best solution among all particles in the jth group at the tth iteration, G^t denotes the near best solution among all groups at the tth iteration, and $f(\cdot)$ stands for the user defined fitness function, which is used to describe the solution space of the targeting problem.

Step 4. **Communication**: Three possible communication strategies proposed by Chang et al. are listed as follows:

Strategy i. **Migrate G^t to each group**: Mutate G^t to replace the poorest particles in each group and replace G_j^t by G^t for each group per R_1 iterations.

Strategy ii. **Migrate G_j^t to its neighboring groups**: Utilize G_j^t to replace the poorest particles in the neighboring groups per R_2 iterations.

Strategy iii. **Hybrid Migrations:** Separate the groups into two clusters. Apply strategy i to the first cluster per R_1 iterations; apply strategy ii to both clusters per R_2 iterations.

Step 5. **Termination**: Repeat Step 2 to Step 5 until the termination condition is satisfied. Record the best value of the function $f(G^t)$ and the best particle position among all groups.

D. LDI-Based Stability Conditions for Nonlinear systems

In this article, the PDC technique is employed to design the global controller for the model listed in (11). The ith rule of the fuzzy logic controller (FLC) is obtained as follows:

Control Rule i:

IF $x_i(t)$ is M_{i1} and …. And $x_g(t)$ is M_{ig}

THEN $U(t) = -K_i X(t), i = 1, 2, ..., r$ (15)

where K_i denotes the local feedback gain vector in the ith subspace. The final model-based fuzzy controller is analytically represented in (16):

$$U(t) = -\frac{\sum_{i=1}^{r} w_i(t) K_i X(t)}{\sum_{i=1}^{r} w_i(t)} = -\sum_{i=1}^{r} h_i(t) K_i X(t) \quad (16)$$

Thus, the complete closed-loop fuzzy system can be obtained as follows:

$$\dot{X}(t) = \sum_{i=1}^{r} \sum_{l=1}^{r} h_i(t) h_l(t)[(A_i - B_i K_i) X(t)] + E_i \phi(t) \quad (17)$$

Therefore, the control system for structural system (1) described by T-S fuzzy representation with the technique of PDC control is obtained.

In 2006, Fang et al. propose a quadratic stabilization condition for T-S fuzzy control systems in the presentation form of LMI Formulation [26]. The LMI is any constraint of the form listed in (18):

$$F(v) = F_0 + \sum_{i=1}^{m} v_i F_i > 0 \quad (18)$$

where $v = [v_1, v_2, ..., v_m] \in R^m$ is a variable vector, and the symmetric matrices $F_i = F_i^T \in R^{n \times n}$, $i = 0, ..., m$ are given. The solution set $\{v | F(v) > 0\}$ may be empty, however, it is always convex. A typical stability condition for the fuzzy system listed in (11) is analyzed as follows:

Theorem 1: The equilibrium point of the fuzzy control system listed in (11) is stable in the large if there exist a common positive definite matrix P and the feedback gains K such that the following two inequalities are satisfied:

$$(A_i - B_i K_i)^T P + P(A_i - B_i K_i) + \frac{1}{\eta^2} P E_i E_i^T P < 0 \quad (19)$$

$$\left[\frac{(A_i - B_i K_i) + (A_i - B_i K_i)}{2}\right]^T P + P\left[\frac{(A_i - B_i K_i) + (A_i - B_i K_i)}{2}\right] + \frac{1}{\eta^2} P E_i E_i^T P < 0 \quad (20)$$

where $P = P^T > 0$ for $i < l \leq r$ and $i = 1, 2, ..., r$.

III. OUR PROPOSED METHOD

Theorem 1 states the stability of a T-S fuzzy controller system and the stabilization can be achieved by finding a common symmetric positive definite matrix P for r subsystems. Hence, the stability analysis is converted to the problem of solving eigen values using the interior-point method associated with LMI techniques. The stability condition can be reduced to that of linear system when $r = 1$. In this paper, we propose the concept of utilizing swarm intelligence method to find the common symmetric positive definite matrix P, which satisfies the LMI stability conditions, instead of using the conventional methods.

To employ PPSO solving problems of optimization, a fitness function should be defined at the first beginning. The fitness function is the mathematic representation of the evaluation condition for the target problem. In our design, we're going to use PPSO to find a common P matrix, which satisfies the

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

condition listed in (21):

$$(A_i - B_i K_i)^T P + P(A_i - B_i K_i) < 0 \qquad (21)$$

where $P = P^T > 0$ and $i, l = 1,2, \ldots, r$. According to Hsiao et al.'s report [2], The equilibrium point of a closed-loop fuzzy system is asymptotically stable in the large, if there exists a common positive definite matrix P, which satisfies (21). The objective of our proposed method is to choose the proper common matrix P for the T-S fuzzy controller system which satisfies the stability condition listed in (21). The Duffing equation can describe a mechanical system with a hardening spring and can display rich nonlinear phenomena such as chaos and bifurcation. As a result, in recent years, the Duffing equation has become a test-bed for various advanced nonlinear and/or adaptive control techniques [27]. In this section, our proposed concept is illustrated for a simulated chaotic system in which the Duffing equation is considered for a large displacement when the high order term cannot be ignored. An example of the nonlinear chaotic system is given in (22):

$$\begin{cases} \dot{x}_1(t) = 2.5x_2(t) \\ \dot{x}_2(t) = -[0.4x_1(t)]^3 - 0.4x_1(t) - 0.01x_2(t) \\ \qquad +0.4\cos(1.29t) + f \cdot u(t) \end{cases} \qquad (22)$$

where $u(t)$ denotes the control force and f is a constant.

To construct a PDC fuzzy controller for dynamic systems with disturbances, we first employ the NN-based approach to represent the dynamics of the chaotic system. In order to stabilize the chaotic system, two model-based fuzzy controllers are designed, based on the concept of the PDC scheme. The membership function for PDC control is plotted in Fig. 1.
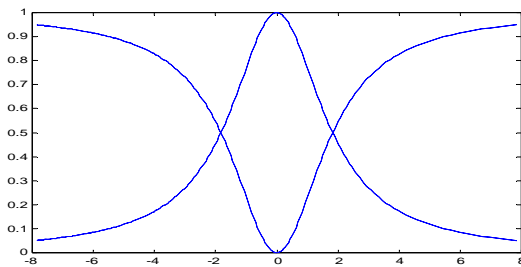

Fig. 1 Membership function of the chaotic system

For the purpose of fulfilling the stability conditions of the theorem, PPSO is employed to find the feasible P matrix. Each particle contains a symmetric positive definite matrix. The fitness function we design for this application is listed in (23):

$$F = \alpha \times \beta \qquad (23)$$

where F denotes the fitness value, and $\times$ stands for the AND operation in Boolean logic; α and β come from (24) and (25).

$$\alpha = \begin{cases} 1, \text{ if}(A_i - B_i K_l)^T P + P(A_i - B_i K_l) < 0 \\ 0, \text{ otherwise} \end{cases} \qquad (24)$$

$$\beta = \begin{cases} 1, P = P^T > 0 \\ 0, \text{ otherwise} \end{cases} \qquad (25)$$

IV. DISCUSSION AND CONCLUSION

This paper discusses the stability problem for a nonlinear system described by an NN type fuzzy model. An LDI representation and the global fuzzy logic controller is constructed by blending all local state feedback controllers. The concept of using PPSO to find the common P matrix, which satisfies the stability criteria of the nonlinear system, is presented in this paper. Based on this criterion, the fuzzy controller design, with the LMI technique, can be used to stabilize the proposed fuzzy systems.

REFERENCES

[1] T. Takagi, and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116-132, Feb. 1985.
[2] F.H.Hsiao, C.W.Chen, Y.W. Liang, S.D. Xu, and W.L. Chiang, "T-S fuzzy controllers for nonlinear interconnected systems with multiple time delays," *IEEE Trans. Circuits & Systems- I:Regular Papers* vol. 52, no. 9, pp. 1883- 1893, Sep. 2005.
[3] F.H.Hsiao, C.W.Chen, Y.H. Wu, W.L. Chiang, "Fuzzy controllers for nonlinear interconnected TMD systems with external force,"*Journal of The Chinese Institute of Engineers*, vol. 28, pp. 175-181, 2005.
[4] F.H.Hsiao, W.L. Chiang, C.W.Chen, "Fuzzy Control for Nonlinear Systems via Neural-Network-Based Approach," *International Journal for Computational Methods in Engineering Science and Mechanics*, vol. 6, pp. 145- 152, 2005.
[5] F.H.Hsiao, W.L. Chiang, C.W.Chen, S.D. Xu, and S.L. Wu, "Application and robustness design of fuzzy controller for resonant and chaotic systems with external disturbance,"*Int. J. Uncertainty, Fuzziness and Knowledge-Based System*, vol. 13, pp. 281-295, 2005.
[6] F.H.Hsiao, J.D. Hwang, C.W.Chen, Z.R. Tsai, "Robust Stabilization of Nonlinear Multiple Time-Delay Large-scale Systems via Decentralized Fuzzy Control," *IEEE Trans. Fuzzy Systems*, vol. 13, pp. 152- 163, 2005.
[7] F.H.Hsiao, J. D.Hwang, C. W.Chen, and Z. R.Tsai, "Robust stabilization of nonlinear multiple time-delay large-scale systems via decentralized fuzzy control," *IEEE Transactions on Fuzzy Systems*, vol. 13, pp. 152-163, 2005.
[8] S.-C. Chu, and P.-W. Tsai, "Computational Intelligence Based on the Behavior of Cats," *International Journal of Innovative Computing, Information and Control*, vol. 3, no. 1, 2007, pp. 163-173.
[9] S.-C.Chu, P.-W.Tsai, and J.-S.Pan, "Cat Swarm Optimization," in *Proc.* 9th Pacific Rim International Conference on Artificial Intelligence, LNAI 4099, 2006, pp. 854-858.
[10] J.-S. Pan, P.-W. Tsai, and B.-Y. Liao, "Fish Migration Optimization Based on the Fishy Biology,"in*Proc.* of 2010 4th International Conference on Genetic and Evolutionary Computing, 2010, pp. 783-786.
[11] P.-W. Tsai,J.-S. Pan, B.-Y. Liao, M.-J. Tsai, and V. Istanda, "Bat Algorithm Inspired Algorithm for Solving Numerical Optimization Problems," *Applied Mechanics and Materials*, vol. 148-149, 2012, pp. 134-137.
[12] T.-C. Chen, P.-W. Tsai, S.-C. Chu, and J.-S. Pan, "A Novel Optimization Approach: Bacterial-GA Foraging," in *Proc.* 2nd International Conf. on Innovative Computing, Information and Control (ICICIC-2007), Japan.
[13] C.W. Chen, W.L. Chiang, and F.H. Hsiao, "Stability analysis of T-S fuzzy models for nonlinear multiple time-delay interconnected systems," *Mathematics and Computers in Simulation*, vol. 66, 2004, pp. 523-537.
[14] C.W. Chen, W.L. Chiang, and C.H. Tsai et al., "Fuzzy Lyapunov method for stability conditions of nonlinear systems," *International Journal on Artificial Intelligence Tools*, vol. 15, 2006, pp. 163-171.
[15] D. Dubois and H. Prade, "Fuzzy Sets and Systems: Theory and Applications," Academic Press, New York 1980.
[16] W. I. Lee, C. W. Chen, C. Y. Chen, and Y. C. Sui, "Application of half-circle fuzzy numbers and development of triangular fuzzy numbers to fuzzy control," in *Proc.* 10th WSEAS International Conf. on Applied Computer Sciences (ACS'10), Japan, 2010, pp. 335-340.

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:8, No:5, 2014

[17] J.M. Mendel, "Fuzzy logic systems for engineering: A tutorial,"in *Proc. of the IEEE*, 83 March 1995.

[18] J. J. Buckley, "Fuzzy Hierarchical Analysis,"*Fuzzy Sets and System*, vol. 17, 1985, pp. 233-247.

[19] C.W. Chen, C.L. Lin, C.H. Tsai, C.Y. Chen, and K. Yeh, "A novel delay-dependent criteria for time-delay T-S fuzzy systems using fuzzy Lyapunov method," *International Journal on Artificial Intelligence Tools*, vol. 16, 2007, pp. 545-552.

[20] H. J. Zimmermann., Fuzzy Set Theory and Its Applications, (1991) Kluwer, Dordrecht.

[21] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, "Linear Matrix Inequalities in System and Control Theory," *Studies in Applied Mathematics (SIAM)*, Philadelphia, PA, 1994.

[22] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," Fuzzy Sets and Systems, vol. 45, no. 2, 1992, pp. 135-156.

[23] X.L. Luan, F. Liu, P. Shi,"Robust Finite-Time H (a) Control for Nonlinear Jump Systems via Neural Networks," *Circuits Systems and Signal Processing*, vol. 29, pp. 481-498, 2010.

[24] J.-F. Chang, S.-C. Chu, J. F. Roddick, and J.-S. Pan, "A Parallel Particle Swarm Optimization Algorithm with Communication Strategies," *Journal of Information Sciences and Engineering*, vol. 21, pp. 809-818, 2005.

[25] D. Abramson, and J. Abela, "A Parallel Genetic Algorithm for Solving the School Timetabling Problem," *Technical Report*, Division of Information Technology, CSIRO, 1991.

[26] C.-H. Fang, Y.-S. Liu, S.-W. Kau, L. Hong, and C.-H. Lee, "A New LMI-Based Approach to Relaxed Quadratic Stabilization of T-S Fuzzy Control Systems," *IEEE Tran. on Fuzzy Systems*, vol. 14, no. 3, pp. 386-397, June 2006.

[27] W.-J. Sun, "A Global Asymptotic Synchronization Problem via Internal Model Approach," *International Journal of Control Automation and Systems*, vol. 8, pp. 1153-1158, 2010.