World Academy of Science, Engineering and Technology
International Journal of Information and Communication Engineering
Vol:8, No:7, 2014

Modeling and Control Design of a Centralized Adaptive Cruise Control System

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Abstract—A vehicle driving with an Adaptive Cruise Control System (ACC) is usually controlled decentrally, based on the information of radar systems and in some publications based on C2X-Communication (CACC) to guarantee stable platoons. In this paper we present a Model Predictive Control (MPC) design of a centralized, server-based ACC-System, whereby the vehicular platoon is modeled and controlled as a whole. It is then proven that the proposed MPC design guarantees asymptotic stability and hence string stability of the platoon. The Networked MPC design is chosen to be able to integrate system constraints optimally as well as to reduce the effects of communication delay and packet loss. The performance of the proposed controller is then simulated and analyzed in an LTE communication scenario using the LTE/EPC Network Simulator LENA, which is based on the ns-3 network simulator.

Keywords—Adaptive Cruise Control, Centralized Server, Networked Model Predictive Control, String Stability.

I. Introduction

ADAPTIVE Cruise Control (ACC) is a longitudinal control to drive a vehicle at a certain speed, while it is additionally able to reduce speed in order to follow a preceding vehicle. The detection of preceding vehicles is usually achieved by a radar sensor, which is sometimes extended by video cameras to obtain a better angular resolution [1]. Each vehicle is usually controlled decentralized, hence extra work has to be done to guarantee string stability of a vehicular platoon. Vehicular platoons occur when multiple ACC-controlled vehicles drive consecutively. String stability is then achieved, if sudden changes are not amplified within the platoon. The design of string stable ACC-Systems has extensively been discussed in the literature. Here string stability is often achieved by introducing a speed dependent spacing error [2], [3] or some communication technology between the vehicles [4], [5]. If using communication technology to gain string stability, delay and packet loss may have some influence. This has been discussed e.g. in [6], [7].

Model Predictive Control (MPC) is a process control based on iterative optimization of a basic model. At each sampling time, using an appropriate mathematical model, the behaviour of the system is predicted and the control strategy is derived by minimizing a cost over a finite horizon. From this control sequence only the first step is then applied to the system. At the next sampling time, a new control problem based on new output measurements is solved over a shifted horizon. Stability analysis of MPC has extensively been discussed in the literature. A detailed overview is given by [8]. Regarding the research of ACC-Systems, MPC has been discussed e.g. in [9], [10]. Here MPC has been chosen because it is able to integrate constraints (e.g. physical limits, comfort parameters) or certain optimization strategies (e.g. fuel efficiency) to the optimization process. However no stability study was made. [11] uses MPC to compute the spacing-control laws for transitional maneuvers of ACC-controlled vehicles. In this paper we present a new string stable ACC-System, in which the vehicles of a platoon are controlled by a centralized server. The used controller is based on a MPC approach, which enables optimal integration of safety or comfort constraints and reduces the effects of communication delay and packet loss.

The paper is organized as follows: In II we present the thoretical background of our ACC-System. Therefore we first establish a model of a vehicular platoon in II-A, putting some emphasis on gaining full controllability and integral behaviour. In II-B we formulate the optimization problem of the MPC, for which we achieve asymptotic stability, using existing stability theory. The System is then extended in II-C, providing asymptotic stability of the system while introducing communication delay and packet loss. Finally, the simulation and testing of the server-based ACC-System in a realistic communication environment is described in III.

II. Centralized ACC

A. Modeling of a Platoon

In this section we develop a linear model for platoons, consisting of n following vehicles and a leading vehicle. This model should cover the longitudinal behaviour of the vehicles and it should provide integral as well as basic stability behaviour, which can be achieved, if full controllability of the system is given. In addition the leading vehicle is not assumed to be controlled by this ACC-System, but manually or by some other driver assistance system instead. This, however, contradicts the full controllability assumption. A straightforward way to satisfy both assumptions is to consider only relative vehicular values between two consecutive vehicles. Therefore, the manually driven leading vehicle can be part of the system, since its relative values remain controllable.

In this paper we do not consider comprehensive vehicle models and reduce the vehicles to the parameters position, velocity, acceleration and jerk for simplicity. Here a platoon consists of n vehicles $V_1, \dots, V_n$ and one preceding leading vehicle V_0, while being ordered ascendent from the front backwards. For the i^{th}-vehicle of the platoon V_i, we then get the following equation of motion by

$$d_i(t) = x_i^{(r)} + v_i^{(r)} t + \frac{1}{2} a_i^{(r)} t^2 + \frac{1}{6}(j_{i-1} - j_i) t^3,$$

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where t is the time, $d_i(t)$ is the distance depending on t and at $t = 0$ we have the distance $x_i^{(r)} = x_{i-1} - x_i$, the relative velocity $v_i^{(r)} = v_{i-1} - v_i$ and the relative acceleration $a_i^{(r)} = a_{i-1} - a_i$ between V_i and V_{i-1} with "(r)" characterizing a relative value. The jerk of vehicle V_i is denoted by j_i. Note that $j_0 \equiv 0$ since the leading vehicle V_0 is not controlled by this ACC-System. If we took those relative values for our state space representation, we would get our platoon modeled like a chain, where each member is connected to its predecessor. Hence this chain would break, if one vehicle had communication failure of some kind. To avoid this weakness, we create additional connections by defining the following state values

$$d_i^w(t) := \sum_{l=1}^{i} w_l d_{i-l+1}(t). \tag{1}$$

We therefore introduce the nonnegative weights $w_1, \cdots, w_n$, which can be chosen arbitrarily. Here we choose monotonically decreasing weights. Accordingly we get the relative velocities $d_i^{w\prime}(t)$ and the relative accelerations $d_i^{w\prime\prime}(t)$ for $i = 1, \ldots, n$.

Replacing t with kT_s with $k > 0$ and some constant time step T_s, we get the discrete representation. Hence we are able to construct the discrete state space representation of the platoon, consisting of n vehicles by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= x(k) \end{aligned} \tag{2}$$

with the state vector

$$x(k) := \begin{pmatrix} d_n^w(k) \\ d_n^{w\prime}(k) \\ d_n^{w\prime\prime}(k) \\ j_n(k) \\ \vdots \\ d_1^w(k) \\ d_1^{w\prime}(k) \\ d_1^{w\prime\prime}(k) \\ j_1(k) \end{pmatrix} \in \mathbb{R}^{4n}. \tag{3}$$

Note that we do not use the jerk as input value, but define $u_i(k) := j_i(k+1) - j_i(k)$ for $i = 1, \cdots, n$ instead, to achieve integral behaviour of the controller. Additionally we have the output vector $y(k)$ and the matrices

$$A := \begin{pmatrix} A_n & A_{n-1} - A_n & \ldots & A_1 - A_2 \\ & \ddots & \ddots & \vdots \\ & & A_n & A_{n-1} - A_n \\ & & & A_n \end{pmatrix} \in \mathbb{R}^{4n \times 4n} \tag{4}$$

$$B := \begin{pmatrix} B_1 & & \\ & \ddots & \\ & & B_1 \end{pmatrix} \in \mathbb{R}^{4n \times n} \tag{5}$$

$$A_i := \begin{pmatrix} 1 & T_s & \frac{1}{2}T_s^2 & -\frac{1}{6}w_i T_s^3 \\ 0 & 1 & T_s & -\frac{1}{2}w_i T_s^2 \\ 0 & 0 & 1 & -w_i T_s \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

$$B_1 := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{7}$$

We can easily check, that this system is fully controllable.

To guarantee safety and comfort of our system, we introduce constraints for the gap between two consecutive vehicles and for the velocity, the acceleration and the jerk of all vehicles in the platoon. The equivalent constraints for d_i^w, $d_i^{w\prime}$ and $d_i^{w\prime\prime}$ $(i = 1, \cdots, n)$ can be achieved iteratively, starting from $i \equiv 1$. In the following paper we summarize the above constraints as $x_{min} \le x \le x_{max}$ or $x \in \mathbb{X} := \{x | x_{min} \le x \le x_{max}\}$.
Note that

- the constraints are independent of the velocity v_0 and acceleration a_0 of the leading vehicle.
- we can easily achieve boundedness by introducing a sufficiently large upper bound for $d_i(k)$ for all $i = 1, \ldots, n$ and $k > 0$.
- by transforming the state vector x appropriately, we obtain $0 \in \text{int}\mathbb{X}$.

B. Model Predictive Control Design

With increasing computational capacity Model Predictive Control (MPC) is gaining interest also in discrete systems with high frequent sampling rate. Based on a mathematical model of the system, an optimization problem is solved over a finite prediction horizon having the length $N_p \cdot T_s$ with the constant time step T_s of the discrete system representation, introduced in II-A. The input variables can be nonzero over a control horizon with the length $N_c \cdot T_s$ ($N_c < N_p$). Usually only the first value of the predicted input sequence is then applied to the system.

We are introducing the following notation: $x_{t+k|t}$ with $t > 0$, $k \ge 1$ is the k^{th} predicted state value, based on the initial state value $x_{t|t} := x(t)$ and the input sequence $u_t, \ldots, u_{t+k-1}$. $X := \{x_{t+1|t}, \ldots, x_{t+N_p|t}\}$ is the predicted state sequence at initial time t and $U := \{u_t, \ldots, u_{t+N_c-1}\}$ the predicted input sequence respectively.

Evaluating the system matrix A in (4), we recognize that all eigenvalues λ yield $\lambda = 1$. Our forthcoming stability analysis however requires all eigenvalues λ of A being distinct with $|\lambda| < 1$. This can be arranged by applying a suitable linear state controller with controller gain K i.e.

$$u(k) := -Kx(k) + \widetilde{u}(k) \tag{8}$$

with a new input variable $\widetilde{u}(k)$ for all $k \ge 0$. Hence we get the new system equations

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ &= \underbrace{(A - BK)}_{=:\widetilde{A}} x(k) + B\widetilde{u}(k), \\ y(k) &= x(k), \end{aligned} \tag{9}$$

$$\mathcal{P}: \begin{cases} \min_U \left\{ \|Px_{t+N_p|t}\|_1 + \sum_{k=1}^{Np-1} \|Qx_{t+k|t}\|_1 + \sum_{k=0}^{N_c-1} \|Ru_{t+k}\|_1 \right\} \\ \text{s.t.} \\ \quad x_{t|t} = x(t) \\ x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k} & ; \forall k \geq 0, \\ \quad u_{t+k} = 0 & ; N_c \leq k \leq N_p - 1, \\ \quad x_{min} \leq x_{t+k|t} \leq x_{max} & ; k = 1, \ldots, N_p, \end{cases} \tag{10}$$

for all $k \geq 0$.

Since our model is fully controllable, we can choose K, so that all eigenvalues λ of $\widetilde{A}$ are distinct and $|\lambda| < 1$. The calculation of K can be done e.g. with the algorithm introduced in [12]. This algorithm is also used by the MATLAB-function *place* [13]. We choose the controller gain matrix K, such that all eigenvalues λ of $\widetilde{A}$ are distinct and $0.9 \leq \lambda < 1$.

We are now able to formulate the optimization problem $\mathcal{P}$ with $N_c < N_p$ as shown in (10). Henceforth we consider the augmented model (9) with its matrices (4)-(7), but omit the index $\tilde{}$ for simplicity. In addition we use the constraints x_{min}, x_{max} as defined in II-A, the nonsingular weighting matrices $Q \in \mathbb{R}^{4n \times 4n}$, $R \in \mathbb{R}^{n \times n}$ and the full column rank matrix $P \in \mathbb{R}^{4n \times 4n}$.

Note that we use the 1-Norm in our cost function, for which two major advantages are associated.

- A vehicle in the platoon only reacts on preceding vehicles and does not react on subsequent vehicles of the platoon. This is a performance advantage we appriciate, however this controller will react not as smooth as using e.g. a quadratic cost function, which was shown in [14].
- A linear optimization problem needs less computation time.

In [15] the authors present asymptotical stability of problem $\mathcal{P}$ by choosing the terminal weight P appropriately. For that we must guarantee the feasibility of our system by choosing N_c sufficiently large. Referring to [16], we are able to calculate the admissible set, which is finitely determined, since

i) A is asymptotically stable (all eigenvalues λ of A are distinct and $|\lambda_i(A)| < 1$ for all $i = 1, \ldots, 4n$),
ii) $\mathbb{X}$ is bounded,
iii) $0 \in \text{int}\mathbb{X}$.

Hence we are left to solve $8n \cdot N_c$ linear maximization problems for each amount of vehicles in the platoon separately. This is combined with a huge computational effort, which however can be done offline.

C. Introduction of Delay and Packet Loss

Introducing delay and packet loss, we extend the MPC design of section II-B by typical network parameters. In literature this type of controller is called NetworkedMPC and generally discussed in multiple publications e.g. [6], [17]. In Fig. 1 we illustrate the communication channels between the server and the vehicles of the platoon. Here individual delay and packet loss occurs between the server and the vehicles and vice versa.

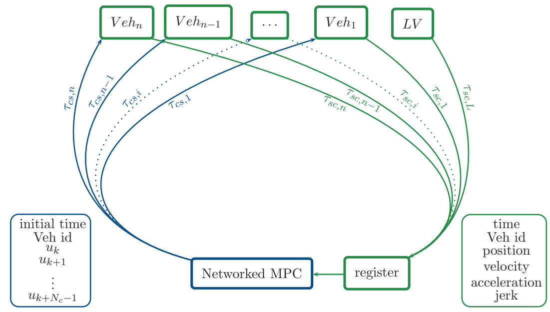


Fig. 1. Networked MPC with communication parameters

On the right side of Fig. 1, the information, which every vehicle of the platoon and the leading vehicle sends to the server, is displayed. To collect the data, it is stored in a register first. Before the MPC computes the new control sequence, the registered position and velocity of each vehicle is predicted to the time of the newest data, by using acceleration only. This prediction is not very accurate, but still better than using unpredicted data.

After the computation of the MPC is finished, the whole control sequence is sent to each vehicle of the platoon, which is illustrated on the left side of Fig. 1. Based on the initial time (the time of the initial data), each vehicle chooses the appropriate input value. Hence e.g. if the total delay (round trip time) is longer than an iteration step or if packet loss occures, the applied input value may not be u_k.

Asymptotic stability of the MPC still can be achieved by the stability theorem of [15], including a slightly different feasibility assumption: *Let $t_1, t_2 > 0$ be chosen such that $t_1 < t_2$ and $t_2 - t_1 < N_c$. Let $U^*(t_1) = \{u^*_{t_1}, \cdots, u^*_{t_1+N_c-1}\}$ be the solution of problem $\mathcal{P}$ at time t_1 and let N_c be chosen such that $\{u^*_{t_2}, \cdots, u^*_{t_1+N_c-1}, 0, \cdots, 0\}$ is feasible at time t_2.* The proof still follows by standard Lyapunov argumentation.

III. Simulation and Results

The functionality of the above developed centralized ACC-System depends heavily on the performance of the used communication technology. To be able to perform tests in a realistic mobile communication environment, we integrate the LTE/EPC Network Simulator LENA [18] based on ns-3 [19] in our testbed. Two adjustments to LENA were necessary.

i) The CQI feedbacks in LENA are evaluated according to the SINR identified in the control channel. This results in an interference behaviour, when all evolved NodeB

TABLE I
MAIN LTE SIMULATION PRAMETERS

Parameter		Value	
Bandwidth		10	MHz
Frequency	DL	2.14	GHz
	UL	1.95	GHz
Cell radius		500	m
Sectors per eNB		2	
TxPower	eNB	40	dBm
	UE	25	dBm
Pathloss Model		Okumura Hata Urban	
Scheduler		Proportional Fair	
Fading		Rayleigh	
Distance between eNB and Road		20	m
Latency between P-GW and Remote Host		20	ms

(eNB) are transmitting simultaneously, even if no User Entity (UE) is attached. In order to get a more realistic interference model, we changed the CQI calculation appropriately. Hence only transmitted packages can cause interference in a certain frequency band.

ii) Frequency reuse has been added, such that adjacent sectors use different frequency bands.

The most important simulation parameters concerning LTE are illustrated in Table I. Note that the eNBs are positioned along a straight road with parallel oriented antennas.

We then simulate a platoon of 5 vehicles following a leading vehicle with a constant setpoint value of distance of 30 m. Referring to II-A, we choose the following constraints for the gap between two consecutive vehicles (d) and for the velocity (v), the acceleration (a) and the jerk (j) of all vehicles in the platoon, being oriented mostly by safety and comfort considerations.

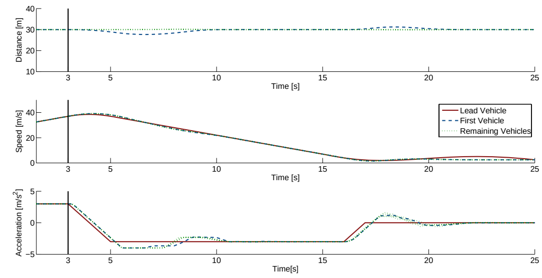
$$\begin{aligned} 5\text{m} &\leq d \\ 0\frac{\text{m}}{\text{s}} &\leq v \leq 50\frac{\text{m}}{\text{s}} \\ -4\frac{\text{m}}{\text{s}^2} &\leq a \leq 4\frac{\text{m}}{\text{s}^2} \\ -3\frac{\text{m}}{\text{s}^3} &\leq j \leq 3\frac{\text{m}}{\text{s}^3} \end{aligned} \quad (11)$$

In the simulation the vehicles send with 10 Hz constantly. The server sends either as soon as new data of all vehicles is available or at least every 200 ms.

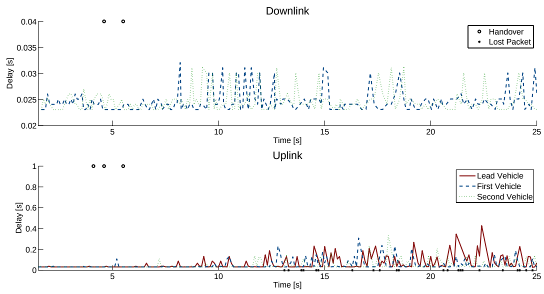
Using the method introduced in [16], we compute the minimal value for the control horizon N_c to gain feasibility. Hereby we obtain $N_c = 27$, given $n \equiv 5$ and $T_s = 0.1s$.

We are now testing the introduced server-based ACC-System in a step response, which corresponds to a simple braking scenario. At the beginning the leading vehicle is constantly accelerating with $3\frac{m}{s^2}$ until it starts having negative jerk at $3s$. The reaction time of the other vehciles now determines success or failure of the braking process and hence the most crucial moment is at $3s$ simulation time. In Fig. 2 and Fig. 3 the leading vehicle is shown in red and solid, the first vehicle of the platoon is shown in blue and dashed and the remaining vehicles of the platoon are shown in green and dotted.

In Fig. 2 the platoon starts at the cell center and drives towards the cell edge and vice versa in Fig. 3. Thereby bad communication conditions occur at the cell edges and are caused by fading and pathloss. We observe in Fig. 3a that high delay values at $3s$ increase the reaction time of the platoon as expected, which causes a significant reduction of the intervehicle distance between the leading vehicle and the first vehicle of the platoon.

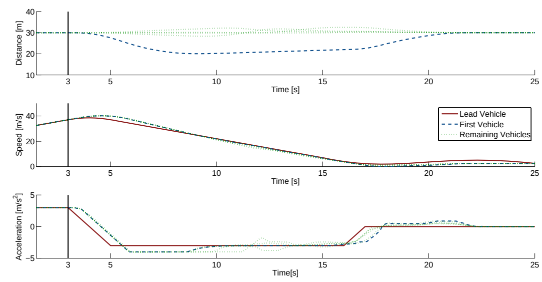


(a) Intervehicle Distance, Speed and Acceleration

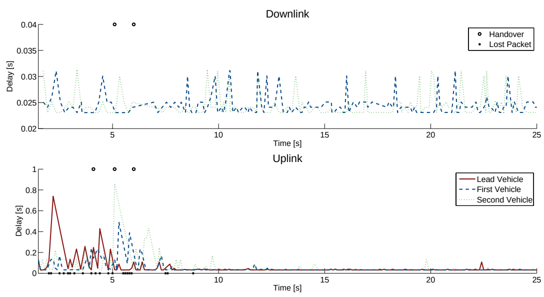


(b) Communication delay in Downlink and Uplink

Fig. 2. Braking scenario with low delay values at the beginning



(a) Intervehicle Distance, Speed and Acceleration



(b) Communication delay in Downlink and Uplink

Fig. 3. Braking scenario with high delay values at the beginning

Note that the platoon reacts as a whole, which is possible due to good communication conditions in downlink. These are e.g. caused by the high Tx power of the eNB. Also note that the system diverges to the setpoint values at the end even with high delay values in uplink as illustrated in Fig. 2a.

In addition we artificially add a temporal disconnection of the communication in uplink to the communication conditions of Fig. 2b. This disconnection starts at $3s$ and effects all vehicle of the platoon and the leading vehicle, which is worst case. In a realistic LTE environment this could be caused by e.g. heavy network load. To compare different disconnection lengths, we find the minimal setpoint distance, which is necessary to guarantee the compliance with the comfort and safety constraints of (11). The result is illustrated in Fig. 4, in which the simulated nonfeasible results are shown in red and dotted and the simulated feasible results are shown in green and dashed. Hence the minimal setpoint distance lies in the grey area in between. Additionally an analytically calculated minimum, if the platoon drives optimally, is shown in blue and solid.

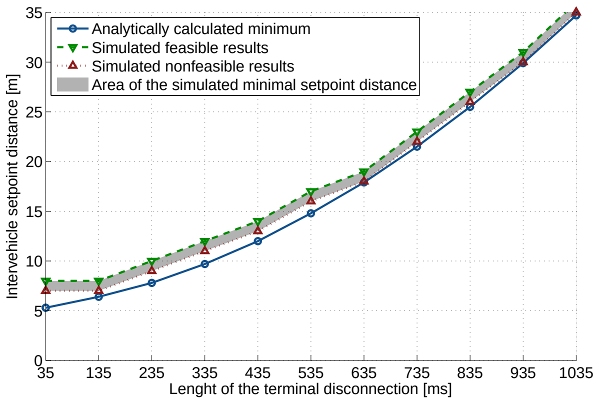


Fig. 4. Analytically calculated and simulated minimal setpoint distances of the server-based ACC-System based on different temporal disconnections in uplink

We observe that the controller acts almost optimal, if single temporal disconnections occur.

IV. Conclusion

In this paper we presented a Networked MPC approach for a centralized, server-based ACC-System. We then put some emphasis on gaining asymptotic stability of the closed loop system. In the last section, we tested the designed controller by simulating in a realistic communication scenario with LTE using the network simulator ns-3 with LENA. Analyzing the test results, we observed safe behaviour in a challenging braking scenario, also when having high delay values in uplink. In addition we observed almost optimal behaviour in scenarios with single temporal disconnections in uplink.

For future work we are left to analyze the presented controller in many different traffic and communication scenarios as well as using real communication data. Additionally the presented model can be extended by considering vehicular dynamics and uncertainties.

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