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Evaluation Performance of PID, LQR, Pole Placement Controllers for Heat Exchanger

Mohamed Essahafi, Mustapha Ait Lafkih

***Abstract*—**In industrial environments, the heat exchanger is a necessary component to any strategy of energy conversion. Much of thermal energy used in industrial processes passes at least one times by a heat exchanger, and methods systems recovering thermal energy.

This survey paper tries to presents in a systemic way an sample control of a heat exchanger by comparison between three controllers LQR (linear quadratic regulator), PID (proportional, integrator and derivate) and Pole Placement. All of these controllers are used mainly in industrial sectors (chemicals, petrochemicals, steel, food processing, energy production, etc…) of transportation (automotive, aeronautics), but also in the residential sector and tertiary (heating, air conditioning, etc...) The choice of a heat exchanger, for a given application depends on many parameters: field temperature and pressure of fluids, and physical properties of aggressive fluids, maintenance and space. It is clear that the fact of having an exchanger appropriate, well-sized, well made and well used allows gain efficiency and energy processes.

***Keywords*—**LQR linear-quadratic regulator, PID control, Pole Placement, Heat exchanger.

I. Introduction

THE manufacturers staff of an industrial unit is called to improve the energy performance and variability of heat exchange processes. And identify the type of control that will be implanted for an optimal control and of course optimize setting parameters.

Process control expertise is the way that allows producers to retain their operations running within specified restrictions and to set more precise limits to maximize effectiveness, ensure quality and security. Precise control of temperature, pressure and flow is important in many process applications. However a Controller automatically compares the rate of the Process Variable (Temperature) to the Set Point (desired Temperature) to control if an error exists. If there is an error, the controller regulates its output according to the parameters that have been set in the controller. The tuning parameters essentially determine: How much correction should be made? And how long the correction should be applied?

The paper is organized as follows: Section II gives an overview of system modeling; Section III describes briefly the underlying mathematics, for the Model of three controller algorithm PID, LQR and Pole placement using state space models. Section IV focuses on simulation of the Model, and some conclusions are given in Section V.

M. Essahafi is preparing a PhD thesis in adaptive control at the faculty of Sciences and Technology, University of Sultan Moulay Slimane, Morocco (e-mail: mohamed.essahafi@yahoo.fr).

Mustapha Ait Lafkih is Professor at the Electrical Engineering Department, Faculty of Sciences and Technology, University of Moulay Slimane.

II. System Modeling

The SISO (single-input, single-output) system studied in this paper is the heat exchanger. The system exchange thermal energy from the vapor flowing through the walls of the exchanger, and the water flowing through a pipe as shown in Fig. 1.

TABLE I
LIST OF SYMBOLS

Symbol	Quantity
A	State matrix of state-space model
B	Input-to-state matrix of state-space model
C	State-to-output matrix of state-space model
D	Direct feed-through matrix of state-space model
p	Laplace operator
q^{-1}	Backward shift operator such that $q^{-1}f(t) = f(t-1)$
Kp	proportional gain
Ti	integral time parameter
Td	derivative time
r(t)	reference set point
y(t)	output signal
e(t)	error difference between r (t) and y (t)
U(P)	Laplace transform of the reference set point(t)
E(P)	Laplace transform of the error difference(t)
J(x(k))	Discrete time for quadratic cost function for LQR controller
Ko	feedback gain matrix of LQR controller
K	feedback gain matrix of pole placement
R	the average thermal resistance of the heat exchanger.

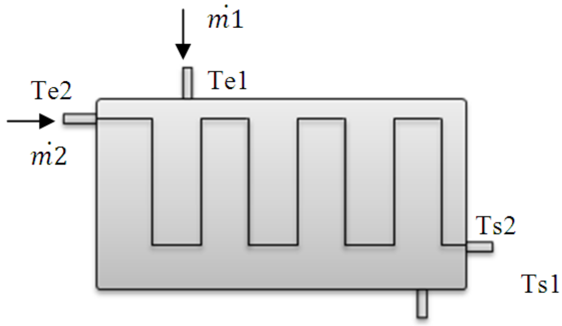

Fig. 1 Diagram of a heat exchanger

The vapor enters to the heat exchanger (by Temperature *Te1*) and out (by Temperature Ts1). Water enters at ambient (Temperature Te2) out from spring pipe (by temperature*Ts2*). The vapor flow rate is controlled by a valve located in the inlet vapor pipe by constant mass flow $\dot{m}1$.

The temperature of the water as of the vapor in the enclosure varies continuously and it will be understood that a specific model of the heat exchange should take into account the temperature variation all along the coil.

Such a model would depend on the geometry of the water circuit and local properties of the enclosure. This would result in a complex modeling.

As a first approximation, we suppose that the enclosure is perfectly insulated, the temperature is perfectly uniform and the heat *qv* supply vapor is proportional to the temperature difference between the vapor inlet and outlet of the exchanger.

$$qv = \dot{m}1 C_1 (Te1 - Ts1) \quad (1)$$

The system is assumed without loss on the exchange, a heat balance can be used to deduce model of the exchanger. The heat exchanged is proportional to the temperature difference between water and vapor

$$qe = \frac{(Ts1 - Ts2)}{R} \quad (2)$$

where R is the average thermal resistance of the heat exchanger. The net heat flow is given by the difference between the heat supplied by the hot vapor and the heat exchanged with water. The evolution of the temperature of the vapor output is linked to the flow of heat by:

$$C_1 \frac{dTs1}{dt} = qv - qe \quad (3)$$

$$qv = \dot{m}1 C_1 (Te1 - Ts1) \quad (4)$$

$$C_1 \frac{dTs1}{dt} = \dot{m}1 C_1 (Te1 - Ts1) - \frac{(Ts1 - Ts2)}{R} \quad (5)$$

δ Constant dependent on the coefficient of heat transfer,

$$\frac{1}{R} = \delta \quad (6)$$

$$\frac{dTs1}{dt} = \dot{m}1 \frac{c_1}{C_{th1}} (Te1 - Ts1) - \frac{\delta}{C_{th1}} (Ts1 - Ts2) \quad (7)$$

$$\frac{dTs2}{dt} = \dot{m}2 \frac{c_2}{C_{th2}} (Te2 - Ts2) - \frac{\delta}{C_{th1}} (Ts1 - Ts2) \quad (8)$$

$$\begin{pmatrix} \dot{Ts}1 \\ \dot{Ts}2 \end{pmatrix} = A \begin{pmatrix} Ts1 \\ Ts2 \end{pmatrix} + BTe1 \quad (9)$$

Once having developed the equations describing the system, SISO linear model is needed to properly design the process control.

$$A = \begin{pmatrix} -(\dot{m}1 \frac{c_1}{C_{th1}} + \frac{\delta}{C_{th1}}) & \frac{\delta}{C_{th1}} \\ \frac{\delta}{C_{th1}} & -(\dot{m}2 \frac{c_2}{C_{th2}} + \frac{\delta}{C_{th2}}) \end{pmatrix}$$

$$B = \begin{pmatrix} \dot{m}1 \\ 0 \end{pmatrix} C = (0 \quad 1) \quad D = 0$$

where,

$x = \begin{pmatrix} Ts1 \\ Ts2 \end{pmatrix}$ State variable vector

$u = Te1$ Control input vector manipulated

$y = \begin{pmatrix} 0 \\ Ts2 \end{pmatrix}$ Measurement vector

The numerical values for all parameters are given in appendix.

III. Controllers Algorithms

A. Control Algorithm of PID

A proportional controller, integrator, differentiator generates the control *u (t)* from the deviation between the reference and the output measure in accordance with (10), refer to the reference number, as [2], [4], [10]

$$u(t) = Kp(e(t) + \frac{1}{Ti} \int e(t)dt + Td \frac{de(t)}{dt}) \quad (10)$$

$$e(t) = r(t) - y(t) \quad (11)$$

Its transmittance of Laplace is

$$\frac{U(P)}{E(P)} = Kp(1 + \frac{1}{Ti.p} + Td.p) \quad (12)$$

$$\frac{U(P)}{E(P)} = \left(\frac{Kp/Ti + Kp.p + Kp.p^2.Td)}{p} \right) \quad (13)$$

However for illustrative purposes we give the calculation of the discretization of this PID.

With the delay operator q^{-1} is expressed in numerical discretization by

$$p \rightarrow \frac{1 - q^{-1}}{Te} \quad (14)$$

Then we get

$$\frac{U(P)}{E(P)} = \frac{a0 + a1.q^{-1} + a2.q^{-2}}{1 - q^{-1}} \quad (15)$$

$$a0 = Kp \left(1 + \frac{Te}{Ti} + \frac{Td}{Te} \right) \quad (16)$$

with

$$a1 = -Kp \left(1 + \frac{2.Td}{Te} \right) \quad (17)$$

$$a2 = \frac{Kp.Td}{Te} \quad (18)$$

Setting parameters *a0, a1* and *a2* are defined by auto tuning, until the system follows in his behavior a system of 2nd order. Which brought the settings of the parametersKp, Ti and Td

$$Kp = -(a1 + 2.a2) \quad (19)$$

$$Ti = -Te \left(\frac{a1 + 2.a2}{a0 + a1 + a2} \right) \quad (20)$$

$$Td = -Te \left(\frac{a2}{a1 + 2.a2} \right) \quad (21)$$

The control algorithm will be implemented in the following way

- At sampling time,
- Acquisition of the measured outputy (k)
- Acquisition of the reference signal $r(k)$
- Application of PID control u (k) to the plant as shown in Fig. 2.

$$e\ (k)\ =\ r\ (k) - y\ (k) \tag{22}$$

$$u(k) =\ a0.e\ (k) a1.e\ +\ (k-1) +\ a2.e\ (k-2) +\ u\ (k-1) \tag{23}$$

$$u\ (k-1)\ =\ u\ (k) \tag{24}$$

$$e\ (k-2)\ =\ e\ (k-1) \tag{25}$$

$$e\ (k-1) =\ e\ (k) \tag{26}$$

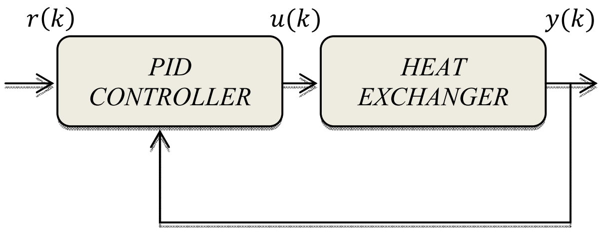

Fig. 2 PID controller, with heat exchanger

B. *Linear Quadratic Regulator Problem*

Let us now consider the discrete LQR problem for a linear time-invariant system, refer simply to the reference number, as [1], [3], [5], [6], [9].

$$\begin{cases} x(k+1) = A.x(k) + B.u(k) \\ y(k+1) = C.x(k) + D.u(k) \end{cases} \tag{27}$$

Find an optimal control that minimizes the quadratic cost function

$$J\big(x(k)\big) = \tfrac{1}{2}\textstyle\sum_{i=k}^{\infty}(x_i^T Q x_i + u_i^T R u_i) \tag{28}$$

where $q \geq 0$ is a symmetric and positive semi-definite matrix and $R > 0$ is a symmetric and positive definite matrix.

These are usually chosen as diagonal matrix with *qi* Maximum expected or accepted value of *1/xi²,ri* Maximum expected or accepted value of 1/ui² and Initial conditions *x(0)=x0* specified.

Note that this cost function also depends on all the future control inputs $u(k), u(k+1)$.. which we do not show in its argument.We assume here that all the states are measurable and seek to find a state-variable feedback control

$$u(k) = -K_o x(k) \tag{29}$$

K is derived from P using

$$K_o = R^{-1}B^T P \tag{30}$$

where p is the unique positive, definite solution to the following algebraic Riccati equation

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \tag{31}$$

That gives desirable closed-loop properties. Therefore Fig. 3 represents the closed-loop system using this form

$$x(k+1) =\ (A - BK_o)x(k) \tag{32}$$

with $(\mathrm{A} - \mathrm{B}K_o)$ the closed-loop plant matrix

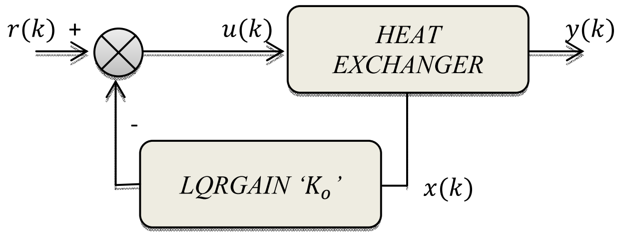

Fig. 3 LQR controller, with heat exchanger

C. *Pole Placement*

The present design technique begins with a determination of the desired closed-loop poles based on the transient-response or frequency-response requirements, such as speed, damping ratio, or bandwidth, as well as steady-state requirements. Refer to the reference number, as [7], [8].

In the conventional approach to the design of a SISO system control system, we design a controller such that the dominant closed-loop poles have a desired damping ratio ζn and a desired undampednatural frequency wn.Consider a control system of (18)

$$\begin{cases} \dot{x} = A.x + B.u \\ y = C.x + D.u \end{cases} \tag{33}$$

We shall choose the control signal to be

$$u(k) = -K.x \tag{34}$$

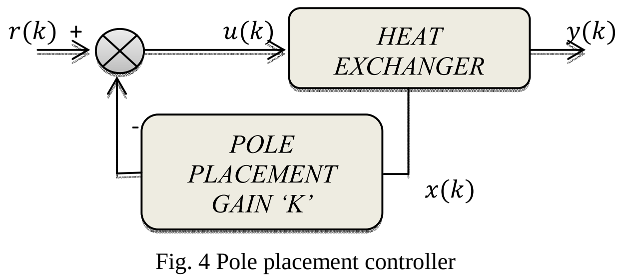

Fig. 4 Pole placement controller

The (32) in continuous time is

$$\dot{x}(t) =\ (A -\ BK)x(t) \tag{35}$$

The solution of (33) is given by,

$$x(t) =\ e^{(A-BK)t}x(0) \tag{36}$$

where $x(0)$ is the initial state caused by external disturbances. The stability and transient-response characteristics are determined by the eigenvalues of matrix $\mathrm{A}-\mathrm{BK}$. Let us Determinate the Matrix K Using Ackermann's Formula.

$$\tilde{A} = (A - BK) \tag{37}$$

The desired characteristic equation is

$$|s\boldsymbol{I} - A + BK| = |s\boldsymbol{I} - \tilde{A}| = (s-\mu 1)..(s-\mu n) \tag{38}$$

$$|s\boldsymbol{I} - \tilde{A}| = s^{\,n} + \alpha_1 s^{\,n-1} + \cdots + \alpha_n n \tag{39}$$

Since the Cayley–Hamilton theorem states that satisfies its own characteristic equation, we have

$$\varphi(\tilde{A}) = \tilde{A}^n + \alpha_1\tilde{A}^{n-1} + \cdots + \alpha_{n-1}\tilde{A} + \alpha_n\boldsymbol{I} = 0 \tag{40}$$

To simplify the derivation, we consider the case where n=3.and Multiplying the $\tilde{\mathrm{A}}^{\mathrm{n}}$ $(n = 1\ldots3)$ matrix in order by $\alpha 3, \alpha 2, \alpha 1$, and $\alpha 0$ (where$\alpha 0 = 1$), respectively, and adding the results, we obtain

$$\alpha_3 I + \alpha_2\tilde{A} + \alpha_1\tilde{A}^2 + \tilde{A}^3 \tag{41}$$

$$\alpha_3 I + \alpha_2 A + \alpha_1 A^2 + A^3 - \alpha_2 BK + \alpha_1 ABK - ABK\tilde{A} - BK\tilde{A}^2 \tag{42}$$

Referring to (31*),* we have

$$\alpha_3 I + \alpha_2\tilde{A} + \alpha_1\tilde{A}^2 + \tilde{A}^3 = \varphi(\tilde{A}) = 0 \tag{43}$$

Also, we have

$$\alpha_3 I + \alpha_2 A + \alpha_1 A^2 + A^3 = \varphi(A) = 0 \tag{44}$$

Substituting the last (42) and (43) into (41), we have

$$\varphi(A) = \langle B|AB|A^2B\rangle \begin{pmatrix} \alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K \end{pmatrix} \tag{45}$$

Since the system is completely state controllable, the inverse of the controllability matrix exists

$$\langle B|AB|A^2B\rangle \tag{46}$$

Premultiplying both sides of (43) by the inverse of the controllability matrix, we obtain

$$\langle B|AB|A^2B\rangle^{-1}\varphi(A) = \begin{pmatrix} \alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K \end{pmatrix} \tag{47}$$

Premultiplying both sides of (47) by [0 0 1], we obtain

$$[0\ 0\ 1]\langle \mathrm{B}|\mathrm{AB}|\mathrm{A}^2\mathrm{B}\rangle^{-1}\varphi(\mathrm{A}) = [0\ 0\ 1]\begin{pmatrix} \alpha_2 \mathrm{K} + \alpha_1 \mathrm{K}\tilde{\mathrm{A}} + \mathrm{K}\tilde{\mathrm{A}}^2 \\ \alpha_1 \mathrm{K} + \mathrm{K}\tilde{\mathrm{A}} \\ \mathrm{K} \end{pmatrix} \tag{48}$$

Equation (48) can be rewritten as

$$\mathrm{K} = [0\ 0\ 1]\langle \mathrm{B}|\mathrm{AB}|\mathrm{A}^2\mathrm{B}\rangle^{-1}\varphi(\mathrm{A}) \tag{49}$$

Equation (49) gives the required state feedback gain matrix K = 3.

D. Choosing the Locations of Desired Closed-Loop Poles

The first step in the pole-placement design approach is to choose the locations of the desired closed-loop poles. The most frequently used approach is to choose such poles based on experience in the root-locus design, placing a dominant pair of closed-loop poles and choosing other poles so that they are far to the left of the dominant closed-loop poles.

IV. Simulation and Discussion

A. States Space Representation and Transfer Function of the Heat Exchanger

A state space representation *sys* of a system is (see more details in appendix)

$$A = \begin{pmatrix} -1.395 & 1.178 \\ 0.5209 & -0.7406 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.2 \\ 0 \end{pmatrix} \quad C = (1 \quad 0) \quad D = 0$$

From (33) we obtain

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{50}$$

So we have

$$G = \frac{0.2s + 0.1481}{s^2 + 2.135s + 0.4191} \tag{51}$$

It is apparent that the heat exchanger is a system of 2nd order and the system matrix A and input matrix B is used to design the LQR. Fig. 5 represents the extraction of state variables of the system

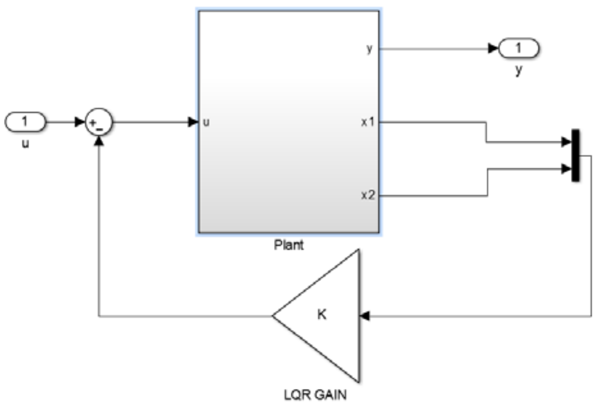

Fig. 5 Extract of state variable x1 and x2

B. Results and Discussion

The simulation has been done with respect to the flowing consideration

$$Q = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$$
$$R = 1$$

The auto tuning PID parameters method we have:

$$Kp = 3.3418 \quad , \quad Ti = 1.7624 \quad , Td = 0$$

and poles selected are:

$$p1 = 0.7142 \qquad p2 = 0.$$

The reference is chosen as step signal with size 1.

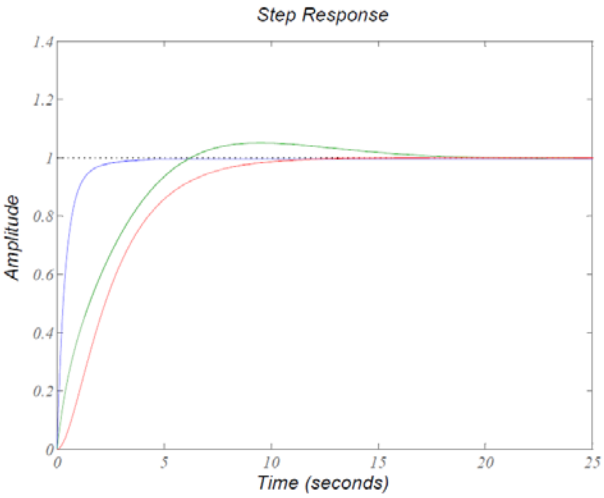

Fig. 6 Step response of heat exchanger controlled by LQR, PID and Pole Placement controllers

In the Fig. 6, it can be observed that comparative results between LQR control, PID control and pole placement control.

The time response of the closed loop system with the simulated PID controller, Pole Placement controller and LQR controller are shown in Fig.6. It can be realized that PID, Pole Placement and LQR controllers are adequate to utilize in control. However, the results proven that the LQR method acts better than PID and Pole Placement controllers in terms of its faster response and minimizing overshoot.

V. Conclusion

In this paper, based on the model of heat exchanger the PID, LQR and pole placement controllers demonstrate that all of these controllers are operative and appropriate for refining the time domain characteristics of system response, such as settling time and overshoots. According to the results, LQR method give the better performance compared to PID controller and pole placement.

Appendix

Numerical values for parameters

$C_{th1} = 1.85$ J·kg^{-1}·K^{-1}

$C_{th1} = 4.185$J·kg^{-1}·K^{-1}

$c1 = 2$J·kg^{-1}·K^{-1}

$c2 = 4.18$ J·kg^{-1}·K^{-1}

$$\delta = 2.18 \ m^3/s$$
$$\dot{m}1 = 0.2 \ m^3/s$$
$$\dot{m}2 = 0.22 \, m^3/s$$

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