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Analysis of Electrical Networks Using Phasors: A Bond Graph Approach

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***Abstract*—**This paper proposes a phasor representation of electrical networks by using bond graph methodology. A so-called phasor bond graph is built up by means of two-dimensional bonds, which represent the complex plane. Impedances or admittances are used instead of the standard bond graph elements. A procedure to obtain the steady-state values from a phasor bond graph model is presented. Besides the presentation of a phasor bond graph library in SIDOPS code, also an application example is discussed.

***Keywords*—**Bond graphs, phasor theory, steady-state, complex power, electrical networks.

I. Introduction

When an electric system is operating in steady-state, differential equations are not required to describe its behavior since all variables are either constants or, in the AC case, sinusoidal variations in time with constant frequency. In the latter case, a phasor representation [1] is appropriate.

The use of phasor notation not only brings a significant mathematical simplification, but also reduces the capacity requirements for computational processing.

At the other hand, bond graph methodology [2] results in a concise graphical representation of energy storage, dissipation, and exchange in a system. The overall purpose of this methodology is the domain-independent representation of any engineering system which is involved in different domains.

The paper consists of eight sections. In Section II the phasor representation and its usage in electrical networks are presented. In Section III, we give a brief introduction to bond graph methodology. Section IV contains the description of the phasor bond graph elements. In section V, the methodology proposed to obtain the steady-state values is described, and illustrated by means of an example. Section VI shows the phasor bond graph elements implementation in SIDOPS code. Section VII illustrates the simulation results in 20sim® of an electrical network by using phasor bond graphs. Finally, the conclusions are stated in Section VIII.

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II. Phasor Representation

A. Mathematical Description

The sinusoidal analysis by means of phasors is an elegant way to analyze electrical circuits with sinusoidal inputs and responses with a given constant frequency, i.e. when the system is in steady-state, without the need to solve differential equations.

A phasor represents the temporal behavior of an electrical signal relative to a fixed reference. Similar to a vector, a phasor has magnitude and phase. It can be represented as an instant in time of a "rotary vector".

In general, a sinusoidal function with $\hat{A}$ amplitude, θ phase angle, and ω frequency; can be described by

$$f(t) = \hat{A}\cos(\omega t + \theta) \tag{1}$$

where $\hat{A}$ may be expressed in root-mean-square (RMS) value, A. For sinusoidal waves $\hat{A} = \sqrt{2}A$. It is possible rewrite (1) in complex notation by using Euler's formula, and substituting its amplitude value by its RMS value,

$$F(t) = \tfrac{1}{\sqrt{2}} f(t) = \Re\{Ae^{j(\omega t+\theta)}\} = \Re\{Ae^{j\theta}e^{j\omega t}\} \tag{2}$$

where $\Re\{\}$ is the real operator. The part that does not depend on time $Ae^{j\theta}$ in (2) is known as a *phasor* [3]. A phasor $\vec{F}$ may also be written as,

$$\vec{F} = Ae^{j\theta} = A\angle\theta = A(\cos\theta + j\sin\theta) \tag{3}$$

The time integral of $F(t)$ is

$$\int F(t)dt = Ae^{j\theta}\int e^{j\omega t}dt = -j\tfrac{1}{\omega}\vec{F}e^{j\omega t} \tag{4}$$

which implies that the integral of the phasor is lagged by $\pi/2$ radians, and scaled by $1/\omega$. Similarly, the time derivative of $F(t)$ is,

$$\tfrac{d}{dt}F(t) = Ae^{j\theta}\tfrac{d}{dt}e^{j\omega t} = j\omega\vec{F}e^{j\omega t} \tag{5}$$

Hence, the derivative of a phasor is leaded by $\pi/2$ radians, and multiplied by ω. This means that in phasor notation the integration and differentiation operations can be performed by scaling and phase shifting.

B. Application of Phasors to Electrical Networks

In an electrical network, let the instantaneous voltage and the instantaneous current be

$$v(t) = v\cos(\omega t + \theta_v); \qquad i(t) = i\cos(\omega t + \theta_i) \tag{6}$$

The phasor representation of (6) may be obtained by using (2) and (3), thus

$$\vec{V} = V e^{j\theta_v} = V\angle\theta_v; \qquad \vec{I} = I e^{j\theta_i} = I\angle\theta_i \tag{7}$$

The *impedance*, *Z*, is the relationship between the voltage and current. Since this relationship is between two phasors, it will be a phasor too. The impedance may be expressed as

$$\vec{Z} = \vec{V}/\vec{I} = Z\angle\theta = R + j(X_L - X_C) \tag{8}$$

where $\theta = \theta_v - \theta_i$ is called the *impedance angle*. The real part is given by the resistive elements, and the imaginary or reactive part, is given by the inductive and capacitive reactances in the system, respectively X_L and X_C. Table I shows a list of the three basic elements in an electrical network and their impedances.

TABLE I
IMPEDANCES

Time	Phasor	Impedance
$v(t) = R \cdot i(t)$	$\vec{V} = R \cdot \vec{I}$	R
$v(t) = L\frac{d}{dt}i(t)$	$\vec{V} = jX_L\vec{I}$	$jX_L = j\omega L$
$v(t) = \frac{1}{C} \cdot \int i(t)\,dt$	$\vec{V} = -jX_C\vec{I}$	$-jX_C = 1/j\omega C$

Obviously, the impedance of capacitive or inductive elements is a function of the constant frequency ω.

In power engineering, voltages, and currents are often represented in a *phasor diagram*. A phasor diagram is like a “picture” at any instant of these rotary vectors, which we can determine the angular difference between them at that time.

C. Complex Power

The instantaneous power consumed by the network may be written as

$$p(t) = v(t) \cdot i(t) = v \cdot i\cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \tag{9}$$

After applying some trigonometric identities [4], we have

$$p(t) = P(1 + \cos 2(\omega t + \theta_v)) + Q\sin 2(\omega t + \theta_v) \tag{10}$$

with

$$P = VI\cos\theta; \qquad Q = VI\sin\theta \tag{11}$$

where P is called *real* or *active power* defined in *watts* (*W*). It represents the absorbed power by the resistive elements in the load. At the other hand, Q is referred as *reactive power*, defined in *volt-ampere reactive* (*var*), and this power supplies the stored energy in reactive elements.

Since $\cos\theta$ plays an important role in the amount of real power in the system, it is called *power factor* [4], [5].

Real, and reactive power are represented together as a *complex* or *apparent power*, *S*, its unit is volt-ampere (*VA*) [6]. The apparent power may be represented as,

$$\vec{S} = P + jQ = \vec{V}\vec{I}^* = \vec{Z}\,|\vec{I}\,| = R\,|\vec{I}\,|^2 + jX\,|\vec{I}\,|^2 \tag{12}$$

where $\vec{I}^*$ is the conjugate current. These three powers are normally described in a so-called *power triangle*.

III. Bond Graph Modelling

The bond graph methodology is a graphical notation of a port-based description for modeling dynamical systems. This graphical technique is based on representing power transfer by means of *bonds*. Given that energy is a domain-independent quantity, it forms the basis for this domain independent approach. The labelled nodes of a bond graph describe a fundamental behavior with respect to energy like storage, transduction, etc.

In each physical domain, the power is the product of two variables, *effort* and *flow*. These pair of variables is called *power variables*. These two variables are represented as paired signals flowing in opposite direction. Therefore, just one of them may be an input. Hence, the bond also represents a bilateral signal flow.

In a bond graph, the way in which these signal are specified as input and output is by means of the *causal stroke*. It is a perpendicular line put at one end of a bond indicating the direction of the effort signal, or called also *causality*.

Momentum and *displacement* are conserved variables, or *energy variables*. They are obtained by integration of one of the power variables, and represent the energy accumulated in an ideal energy storage element.

There are some basic types of elements necessary to represent energy behavior in a domain independent way. The 1-port elements which dissipate power (resistor *R*), store energy (inertia *I*, capacitor *C*) and supply power (sources S_e, S_f). In addition, it is necessary interconnect two or more elements in a power conservative way in order to create structure. At one hand, we have the *transducers* elements (2-port): transformers (*TF*), and gyrators (*GY*). At the other hand, the *interconnection* elements (multiport): the 0-*junction* is a node where all the efforts are equal, and represents in an electrical circuit a parallel connection; the 1-*junction* describes a series connection in an electrical circuit, it means that all the flows are equal. Note however, that series and parallel are special cases, while the junctions are more general in the sense that they apply to each common flow (1-junction) and common effort (0-junction) situation.

It is possible to obtain the state-space equations of the system using a bond graph model. Consider Fig. 1 scheme of a multiport linear time-invariant system, which includes the key vectors [7].

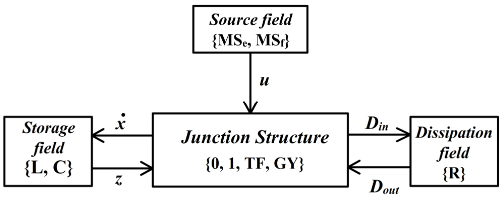

Fig. 1 Bond graph key vectors

In Fig. 1, the state vector $x \in \Re^n$ is composed of energy variables; $u \in \Re^p$ denotes the system input; $z \in \Re^n$, the co-energy vector; finally $D_{in} \in \Re^r$, and $D_{out} \in \Re^r$ represent the energy exchanges between the dissipation field and the junction structure.

The storage and dissipation field relationships are,

$$z = Fx; \qquad D_{out} = LD_{in} \tag{13}$$

The junction structure relationships are defined by

$$\begin{bmatrix} \dot{x} \\ D_{in} \\ y \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} z \\ D_{out} \\ u \end{bmatrix} \tag{14}$$

where the vector $y \in \Re^q$ is the plant output. The entries of the S matrix take values inside the set $\{0, \pm 1, \pm m, \pm n\}$, where m, and n are transformer, and gyrator modules; S_{11} and S_{22} are square skew-symmetric matrices; $S_{21} = -S_{12}^T$ and $S_{41} = -S_{14}^T$.

Conveniently representing the stated equations as

$$\dot{x} = Ax + Bu; \qquad y = Cx + Du \tag{15}$$

where

$$\begin{aligned} A &= (S_{11} + S_{12}MS_{21})F \qquad & C &= (S_{31} + S_{32}MS_{21})F \\ B &= S_{13} + S_{12}MS_{23} \qquad & D &= S_{33} + S_{32}MS_{23} \end{aligned} \tag{16}$$

being $M = (I - LS_{22})^{-1} L$, with I as an identity matrix.

IV. Phasor Bond Graphs Elements

The Laplace transform can also be applied to the bond graph models [8], [9]. With this transformation, the 1-port elements become impedances, or admittances.

If a 1-port has effort-out causality is modelled as impedance, while a 1-port with flow-out causality is modelled as admittance. We must assume that the constitutive relationships of the components are lineal, thus

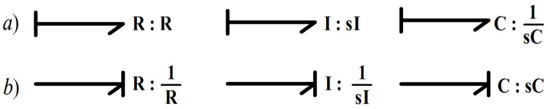

Fig. 2 1-port elements *a*) impedances, and *b*) admittances

Note that the preferred integral causality of an *I*-element is an admittance. We are now able to build up impedance bond graphs by using these elements, and following the same procedure that in the case of dynamic models.

If we substitute $s = j\omega$, Fourier operator, the phasor bond graph elements are obtained.

We propose to express the impedances in matrix form. In this way, we may represent the phasor elements using 2D multibonds [10], [11]. Where, one bond will represent the real part of the phasor, and the second bond will represent the imaginary part of the phasor. From Table I, we can rewrite the impedances in matrix form, see Table II.

TABLE II
1-Port Impedances in Phasor Bond Graph Form

Element	Phasor	2D multibond
Resistive	$\begin{bmatrix} \Re\{\vec{V}\} \\ \Im\{\vec{V}\} \end{bmatrix} = R \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Re\{\vec{I}\} \\ \Im\{\vec{I}\} \end{bmatrix}$	$\vec{V}$, $\vec{I}$ **R**
Inductive	$\begin{bmatrix} \Re\{\vec{V}\} \\ \Im\{\vec{V}\} \end{bmatrix} = X_L \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Re\{\vec{I}\} \\ \Im\{\vec{I}\} \end{bmatrix}$	$\vec{V}$, $\vec{I}$ $\boldsymbol{X_L}$
Capacitive	$\begin{bmatrix} \Re\{\vec{V}\} \\ \Im\{\vec{V}\} \end{bmatrix} = X_C \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \Re\{\vec{I}\} \\ \Im\{\vec{I}\} \end{bmatrix}$	$\vec{V}$, $\vec{I}$ $\boldsymbol{X_C}$

$\Im\{\}$ is the imaginary operator

Note that in the phasor model, we will keep the same preferred causalities as in a dynamic model, effort-out for capacitors, flow-out for inductors, and indifferent for resistors.

The 2-port elements are modelled in the same way as in case of 2D multibonds [11], see Table III.

TABLE III
2-Port Elements in Phasor Bond Graph Form

Element	Phasor	2D multibond
Transformer	$\begin{bmatrix} \vec{V}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{T}^T \\ \mathbf{T} & 0 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{V}_2 \end{bmatrix}$; $\mathbf{T} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$	**TF**
Gyrator	$\begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{G}^T \\ \mathbf{G} & 0 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix}$; $\mathbf{G} = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$	**GY**

m and *n* are the scalar modulus of the transformer and gyrator, respectively

We can change (7) into its matrix form using (3), so we can obtain the 2D multibond sources. See Table IV.

TABLE IV
Sources in Phasor Bond Graph Form

Source	Phasor	2D multibond
Voltage	$\vec{V} = V[\cos\theta_v \quad \sin\theta_v]^T$	$\boldsymbol{V}\angle\theta$
Current	$\vec{I} = I[\cos\theta_i \quad \sin\theta_i]^T$	$\boldsymbol{I}\angle\theta$

V and *I* are the RMS value of voltage and current, respectively

V. Phasor Bond Graph Analysis

In this section we will show how to obtain the system steady-state values by using the phasor bond graphs. In Fig. 3 the field structure for a phasor bond graph is depicted,

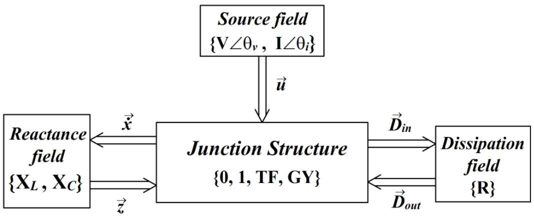

Fig. 3 Phasor bond graph field structure

The reactance, and dissipation filed contain the power demanding elements of the system, and may be defined as

$$\vec{z} = \vec{F}\vec{x}; \qquad \vec{D}_{out} = \vec{L}\vec{D}_{in} \tag{17}$$

where $\vec{F}$ is filled with the impedance, or admittance of the reactive elements. The matrix $\vec{L}$ contains the impedance, or admittance of the resistive elements.

The junction structure relationships may be changed, thus

$$\begin{bmatrix} \vec{x} \\ \vec{D}_{in} \\ \vec{y} \end{bmatrix} = \begin{bmatrix} \vec{S}_{11} & \vec{S}_{12} & \vec{S}_{13} \\ \vec{S}_{21} & \vec{S}_{22} & \vec{S}_{23} \\ \vec{S}_{31} & \vec{S}_{32} & \vec{S}_{33} \end{bmatrix} \begin{bmatrix} \vec{z} \\ \vec{D}_{out} \\ \vec{u} \end{bmatrix} \tag{18}$$

Equation (18) keeps the same characteristics as (14). After some algebraic manipulations, the system output is given by

$$\vec{y} = (\vec{C}(\vec{F}^{-1} - \vec{A})^{-1}\vec{B} + \vec{D})\vec{u} \tag{19}$$

where

$$\begin{aligned} \vec{A} &= \vec{S}_{11} + \vec{S}_{12}\vec{M}\vec{S}_{21} \qquad & \vec{C} &= \vec{S}_{31} + \vec{S}_{32}\vec{M}\vec{S}_{21} \\ \vec{B} &= \vec{S}_{13} + \vec{S}_{12}\vec{M}\vec{S}_{23} \qquad & \vec{D} &= \vec{S}_{33} + \vec{S}_{32}\vec{M}\vec{S}_{23} \end{aligned} \tag{20}$$

being $\vec{M} = (I - \vec{L}\vec{S}_{22})^{-1}\vec{L}$.

In order to clarify this procedure, we will show an example. Consider the RLC circuit shown in Fig. 4.

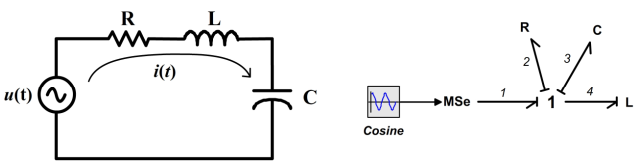

Fig. 4 RLC circuit

We can change the dynamic model to the phasor model by using the 2D multibonds, Fig. 5.

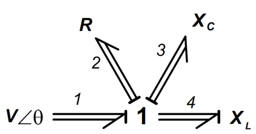

Fig. 5 Phasor model of the RLC circuit

The key vectors are,

$$\begin{aligned} \vec{x} &= \begin{bmatrix} \vec{f}_3 & \vec{e}_4 \end{bmatrix}^T = \begin{bmatrix} \Re\{\vec{f}_3\} & \Im\{\vec{f}_3\} & \Re\{\vec{e}_4\} & \Im\{\vec{e}_4\} \end{bmatrix}^T \\ \vec{z} &= \begin{bmatrix} \vec{e}_3 & \vec{f}_4 \end{bmatrix}^T = \begin{bmatrix} \Re\{\vec{e}_3\} & \Im\{\vec{e}_3\} & \Re\{\vec{f}_4\} & \Im\{\vec{f}_4\} \end{bmatrix}^T \end{aligned} \tag{21}$$

$$\begin{aligned} &\vec{D}_{in} = \vec{f}_2 = \begin{bmatrix} \Re\{\vec{f}_2\} & \Re\{\vec{f}_2\} \end{bmatrix}^T; \; \vec{D}_{out} = \vec{e}_2 = \begin{bmatrix} \Re\{\vec{e}_2\} & \Re\{\vec{e}_2\} \end{bmatrix}^T \\ &\vec{u} = \vec{e}_1 = \begin{bmatrix} \Re\{\vec{e}_1\} & \Re\{\vec{e}_1\} \end{bmatrix}^T; \quad \vec{y} = \vec{f}_1 = \begin{bmatrix} \Re\{\vec{f}_1\} & \Re\{\vec{f}_1\} \end{bmatrix}^T \end{aligned} \tag{22}$$

We can construct the constitutive relations of the 2-port bond graph nodes by applying (18), (19), and (20), thus

$$\vec{F} = diag\left\{ X_C \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \frac{1}{X_L} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}; \quad \vec{L} = R \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

and the junction structure is given by

$$\begin{aligned} &\vec{S}_{11} = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \\ -I_{2\times 2} & 0_{2\times 2} \end{bmatrix}; \quad \vec{S}_{12} = -\vec{S}_{21}{}^T = \begin{bmatrix} 0_{2\times 2} \\ -I_{2\times 2} \end{bmatrix} \\ &\vec{S}_{13} = \vec{S}_{31}{}^T = \begin{bmatrix} 0_{2\times 2} \\ I_{2\times 2} \end{bmatrix}; \quad \vec{S}_{22} = \vec{S}_{23} = \vec{S}_{32} = \vec{S}_{33} = 0_{2\times 2} \end{aligned} \tag{24}$$

Finally, we substitute (21), (23) and (24) into (19) and to obtain the steady-state value of the output:

$$\vec{y} = \frac{1}{R^2 + (X_C - X_L)^2} \begin{bmatrix} R & X_C - X_L \\ X_C - X_L & R \end{bmatrix} \vec{u} \tag{25}$$

VI. Implementation on 20-sim®

Normally, the electrical networks contain a lot of elements, and sources. Due to this fact, it would be convenient to automate phasor analysis with bond graphs by using a software tool. For this purpose, we will use 20-sim® [12] software. In Table V we show the SIDOPS code [9] for the effort, and flow.

TABLE V
Sources Phasor Bond Graph SIDOPS Code

Element	SIDOPS code

Effort source	`parameters`
	`real V = 1.0 {V};`
	`real ang = 0.0 {deg};`
	
	`variables`
	`real flow[2,1];`
	
	`equations`
	`p.u = V * [cos(ang); sin(ang)];`
	`flow= p.i;`

Flow source	`parameters`
	`real I = 1.0 {A};`
	`real ang = 0.0 {deg};`
	
	`variables`
	`real effort[2,1];`
	
	`equations`
	`p.i = I * [cos(ang); sin(ang)];`
	`effort= p.u;`

The SIDOPS code for the passive phasor elements is shown in Table VI.

TABLE VI
1-PORT ELEMENTS PHASOR BOND GRAPH SIDOPS CODE

<table>
<tr><th>Element</th><th>SIDOPS code</th></tr>
<tr><td>Resistor</td><td><pre>parameters
  real r = 1.0 {ohm};

variables
  real R[2,2];

equations
  R = r * [1, 0; 0, 1];
  p.u = R * p.i;</pre></td></tr>
<tr><td>Inductive reactance</td><td><pre>parameters
  real xl = 1.0 {ohm};

variables
  real XL[2,2];

equations
  XL = xl * [0, -1; 1, 0];
  p.u = XL * p.i;</pre></td></tr>
<tr><td>Capacitive reactance</td><td><pre>parameters
  real xc = 1.0 {ohm};

variables
  real XC[2,2];

equations
  XC = xc * [0, 1; -1, 0];
  p.u = XC * p.i;</pre></td></tr>
</table>

Due to the importance of complex power in the analysis of electrical networks, it is necessary define a bond graph sensor able to determine the complex power during simulation. We can rewrite (12) into its matrix form, thus

$$\begin{aligned}\vec{S} = \vec{V}\vec{I}^* &= \left[\Re\{\vec{V}\} + j\Im\{\vec{V}\}\right]\left[\Re\{\vec{I}\} - j\Im\{\vec{I}\}\right] \\ &= \underbrace{\left[\Re\{\vec{V}\}\Re\{\vec{I}\} + \Im\{\vec{V}\}\Im\{\vec{I}\}\right]}_{P} + j\underbrace{\left[\Im\{\vec{V}\}\Re\{\vec{I}\} - \Re\{\vec{V}\}\Im\{\vec{I}\}\right]}_{Q}\end{aligned} \tag{26}$$

The power sensor is depicted in Fig. 6, and its SIDOPS representation is shown in Table VII.

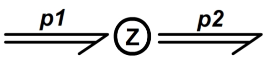

Fig. 6 Complex power sensor in bond graph

TABLE VII
POWER SENSOR PHASOR BOND GRAPH SIDOPS CODE

<table>
<tr><th>Element</th><th>SIDOPS code</th></tr>
<tr><td>Power sensor</td><td><pre>variables
  real P {W}, Q {VAR}, S {VA}, PF;
  real V_mag {V}, I_mag {A};
  real V_ang {deg}, I_ang {deg}, th {deg};

equations
  p2.u = p1.u;
  p1.i = p2.i;

  P = p1.u[1] * p1.i[1] + p1.u[2] * p1.i[2];
  Q = p1.u[2] * p1.i[1] - p1.u[1] * p1.i[2];
  S = sqrt(P^2 + Q^2);
  th = atan2(Q,P);
  PF = cos(atan2(Q,P));</pre></td></tr>
</table>

The transformer and gyrator are both already included in the 20-sim® library.

VII. SIMULATION OF AN ELECTRICAL NETWORK

In this section, we will compare the response of a phasor bond graph model with a dynamical bond graph model. Consider the electrical network shown in Fig. 7.

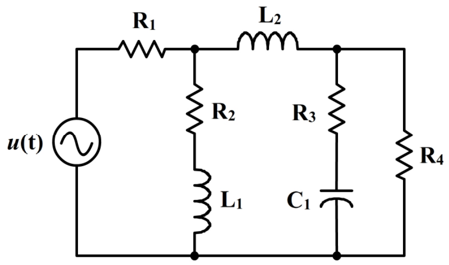

Fig. 7 Electrical network

where $f = 50Hz$, $\omega = 2\pi f$, $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 40\Omega$, $R_4 = 50\Omega$, $L_1 = 63.662mH$, $L_2 = 31.831mH$, $C_1 = 318.3098\mu F$, $v(t) = \sqrt{2}\cdot 230\cos\omega t$.

The circuit is converted into a bond graph by using the standard methodology for electrical systems [9] in Fig. 8.

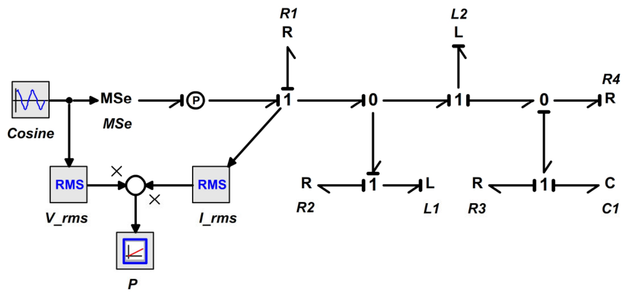

Fig. 8 Bond graph of the electrical network

We added the RMS blocks from the 20-sim® library in order to obtain the real power. Fig. 9 shows the instantaneous voltage source, total current and total power; together with their RMS values.

We can observe from Fig. 9 that the system contains a transient, and the RMS blocks are calculating the RMS value directly from the instantaneous signals.

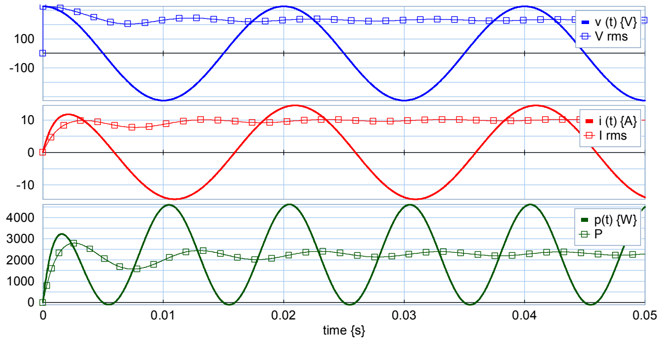

Fig. 9 Voltage, current, and real power in the electric network

In order to construct the phasor bond graph we substitute all the elements by their equivalent 2D representation. Thus, Fig. 10 shows the phasor bond graph including a complex power sensor.

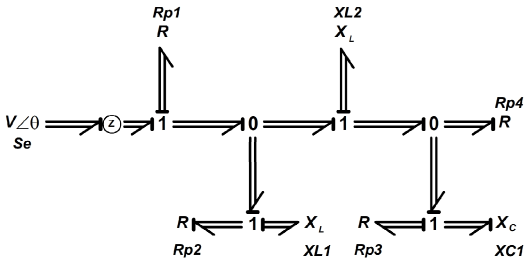

Fig. 10 Phasor bond graph model

It is important remark that in phasor bond graph model all our signals are RMS values.

Comparing the responses from the phasor bond graph model, with the RMS value given by the blocks in the regular bond graph in Fig. 9, we obtain Fig. 11.

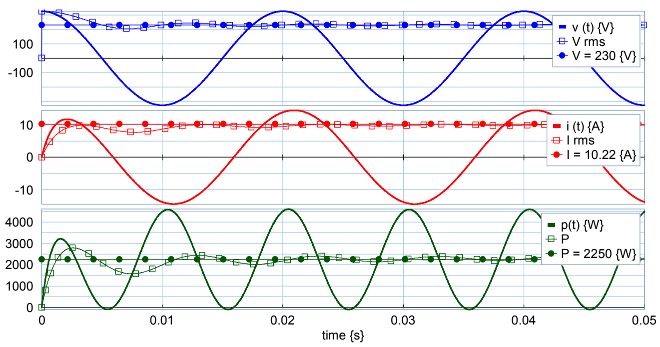

Fig. 11 Comparison between the instantaneous responses and phasor values

Obviously, the signals from the phasor model are constants, while the RMS values of the dynamic model converge to these values once the steady-state has been reached by the system.

One advantage of a phasor model is that the computational time has been reduced drastically. Besides, we are now able to show the reactive power, the impedance angle, and thus the power factor in a more direct manner. Fig. 11 shows these last three quantities taken from the complex power sensor.

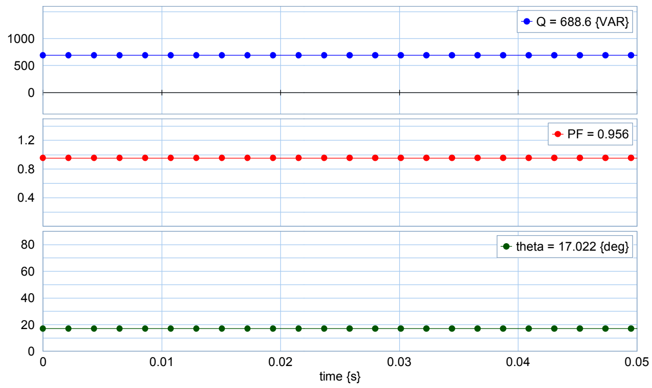

Fig. 12 Reactive power, power factor, and impedance angle

VIII. Conclusion

We have given a brief description of the phasor theory, widely used in the study of electrical systems. The bond graph methodology was introduced, together with the standard methodology for obtaining the steady-state equations from a phasor bond graph model. We saw that the bond graph elements could be replaced by their equivalent impedances.

In order to automate the phasor analysis by using bond graphs, the Fourier transform was substituted into the impedance bond graphs. In this way, we were able to create a 2D multibond bond graph by splitting the real, and imaginary part of the impedances, this model was called phasor bond graph.

The phasor bond graph model allows knowing the angle, and magnitude of important variables as voltage, currents, and complex power. The phasor modelling of the electrical networks give us the steady-state behavior of the system. This information may be useful for fault studies, stability, and widely used in the description of power conversion of electrical machines.

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