Hexagonal Honeycomb Sandwich Plate Optimization Using Gravitational Search Algorithm
A. Boudjemai, A. Zafrane, R. Hocine

Abstract—Honeycomb sandwich panels are increasingly used in the construction of space vehicles because of their outstanding strength, stiffness and light weight properties. However, the use of honeycomb sandwich plates comes with difficulties in the design process as a result of the large number of design variables involved, including composite material design, shape and geometry. Hence, this work deals with the presentation of an optimal design of hexagonal honeycomb sandwich structures subjected to space environment. The optimization process is performed using a set of algorithms including the gravitational search algorithm (GSA). Numerical results are obtained and presented for a set of algorithms. The results obtained by the GSA algorithm are much better compared to other algorithms used in this study.

Keywords—Optimization, Gravitational search algorithm, Genetic algorithm, Honeycomb plate.

I. INTRODUCTION

Determining the appropriate structural components form is a problem of primary importance for the engineer. In all areas of mechanical structures, the impact of good design of a mechanical part is very important to its strength, durability and its use in service. This challenge is daily in high-tech sectors such as space research, aerospace, automotive, shipbuilding competition, precision engineering, precision mechanics or structures in civil engineering. The development of the engineering art requires considerable effort to constantly improve the technical design of structures. Optimization intervenes paramount in increasing performance and reducing the weight of aerospace and automotive engines, resulting in substantial energy savings. Aerospace structures generally require light designs. The purpose of these designs is to maximize strength by weight, or effectiveness of the design. Satellite structural design has evolved considerably over the past four decades. Traditionally, the efficiency was achieved using a combination of various designs and structural materials. The satellites structures must withstand dynamic launch environment and support adequately the constraints of the space environment in orbit. Recently, honeycomb cellular materials have been an important research topic due to their outstanding potential in energy absorption, thermal isolation, dynamic and acoustic damper [1]. [2]. Periodic cellular metals are, in fact, highly porous structures with 20% or less of their interior volume occupied by metals [3]-[5]. Some, such as hexagonal honeycomb, have been widely used in the manufacture of the aerospace structures due to their lightweight, high specific bending stiffness and strength under distributed loads [2]. In this paper, design optimization of a hexagonal aluminum honeycomb plate for space applications defined by respective geometric conditions and loading is considered by applying a gravitational search algorithm. Genetic and gradient-based algorithms were also used in order to compare with the GSA algorithm. We consider minimum weight optimization of sandwich beams for a given stiffness. The core consists of regular hexagonal honeycomb.

II. EQUIVALENT OF HONEYCOMB SANDWICH PLATE

The generated equivalent model can be mostly used in the preliminary design stage of the design process. It can be used to reduce the time spent for the analysis of the honeycomb structure used in the satellite structural design and a great advantage to decrease in the pre-processor time and computation time. The study of the mechanical behaviour of a composite material commonly uses the homogenisation concept. This concept makes it possible to avoid the problems involved in heterogeneities. One idealizes the real constitution material by considering it continuous (see Fig. 1). The equivalent characteristics of a honeycomb sandwich plate are determined by identifying its membrane and bending stiffness to those of an isotropic plate, as shown in the Table I.

![Fig. 1 Equivalent parameters of a Honeycomb Sandwich Plate](image)

The equivalent characteristics of a honeycomb sandwich plate are determined by identifying its membrane and bending stiffness to those of an isotropic plate, as shown in the Table I.
TABLE I

<table>
<thead>
<tr>
<th>Equivalent Parameters of Sandwich Structure</th>
<th>Honeycomb sandwich plate</th>
<th>Equivalent isotropic plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane stiffness</td>
<td>(2E_h h) ((1 - \nu^2)^{-1})</td>
<td>(E_{eq} t_{eq}) ((1 - \nu^2)^{-1})</td>
</tr>
<tr>
<td>Bending stiffness</td>
<td>(\frac{E_h h}{2(1 - \nu^2)})</td>
<td>(\frac{E_{eq} t_{eq}}{12(1 - \nu^2)})</td>
</tr>
</tbody>
</table>

where

\[ t_{eq} = \left(3h^2\right)^{\frac{1}{3}} \]  

\[ E_{eq} = \frac{2}{\sqrt{3}} \frac{h_p}{h} E \]  

\( h_c \) Height of honeycomb core or thickness of the plate.
\( E_{eq} \) Equivalent elastic modulus.
\( E \) Young modulus.
\( h_p \) Thickness of facing skin.
\( t_{eq} \) Equivalent thickness.

In an anisotropic mechanical behaviour, all honeycombs are closed cell structures. By identifying a unit cell and deriving the volume fraction occupied by metal, the equivalent density is given by [13]:

\[ \rho_{eq} = \frac{2\rho_c h_c + 2\rho_f (H - h_c)}{t_{eq}} \]  

where

\( \rho_c \) density of honeycomb core material
\( \rho_f \) density of facing material,
\( \rho_{eq} \) equivalent density
\( H \) height of sandwich panel including facing skins

For the analytical comparison of the first modal frequency of the equivalent model, we use in the analysis the theory applied in the case of a beam with clamped-free boundary conditions.

III. FORMULATION OF THE OBJECTIVE FUNCTION

To optimize the honeycomb plate and this in order to be as efficient as possible, it is necessary to minimize its mass. Density and the geometric parameters of the plate are the elements that most influence on the mass. However, optimization of these parameters ensures that the criteria of strength and rigidity are affected [6]-[12].

The key issue in the design of most sandwich panels is the minimization of weight which is given by

\[ W = 2\rho_c g h_c h_p + \rho_f a b h_c \]  

where \( g \) is the acceleration due to gravity. This is known as the “objective function” since this is what we wish to minimize. Taking into account the consideration ratio which is given by

\[ \left( \frac{h_c}{b} \right) = \frac{2}{3} \frac{G_c}{E_f} \left( \frac{h}{b} \right) \left( \frac{\delta}{P} \right) a G \left( \frac{h_c}{b} \right)^{-1} \]  

where

\( G_c \) Out of plane shear modulus of the core
\( E_f \) Elastic modulus of the face.
\( P \) Load.
\( \delta \) Deflection.

IV. OPTIMIZATION PROCEDURE

In this study, a set of algorithms were used to minimize the hexagonal honeycomb composite structures subject to bending load.

A. The Genetic Algorithm

Since 50 years of evolutionary algorithms have evolved: genetic algorithms, mainly developed in the USA by J. H. Holland, evolutionary strategies, developed in Germany by I. Rechenberg, H.-P. Schwefel and evolutionary programming [13]-[16]. Each of these constitutes a different approach; however, they are inspired by the same principles of natural evolution. Fig. 2 shows the evolutionary algorithm scheme.
population, evaluate and assign fitness values to individuals in the population, perform reproduction through the fitness weighted selection of individuals from the population, and perform recombinantion and mutation to produce members of the next generation [13], [16].

In first step, (selection, reproduction) each of the individuals is selected by its fitness value. Reproduction consists in duplicating each individual in relation to the average of the performances for all the chromosomes of population. Then individuals which give the best results have a good probability to be selected for the next generation. After the reproduction step, a crossover allows a generation of new individuals. The crossover step consists to cut two chromosomes, named parents, on a random place, then the end of these two individuals string is reversed and two chromosomes are created and named children. Mutation consists to modify in a random way and with a small probability (0.01-0.1) the bit value of a chromosome. In other words, a "1" becomes a "0" and a "0" becomes a "1".

The value of genetic algorithm parameters is given in Table II.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>size of chromosome</td>
<td>02</td>
</tr>
<tr>
<td>kind of selection</td>
<td>Roulette wheel</td>
</tr>
<tr>
<td>rate of crossover</td>
<td>0.8</td>
</tr>
<tr>
<td>rate of mutation</td>
<td>0.003</td>
</tr>
<tr>
<td>size of population</td>
<td>10</td>
</tr>
</tbody>
</table>

**B. The Gravitational Search Algorithm**

GSA is a novel heuristic [17] optimization method which has been proposed by E. Rashedi and all in 2009 [17]. The basic physical theory which GSA is inspired from is the Newton’s theory that states: Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them [17].

The algorithm considers agents as objects consisting of different masses proportional to their value of fitness function. During generations, all these objects attract each other by the gravity force, and this force causes a global movement of all objects towards the objects with heavier masses. Hence, masses cooperate using a direct form of communication, through gravitational force. The heavy masses - which correspond to good solutions - move more slowly than lighter ones, this guarantees the exploitation step of the algorithm; the GSA was mathematically modeled in [17]-[21].

GSA algorithm can be explained following steps

**Step1. Initialisation**

When it is assumed that there is a system with \( N \) (dimension of the search space) masses, position of the \( i \)-th mass is described as follows. At first, the positions of masses are fixed randomly.

\[
X_i = (x_i^1, x_i^2, ..., x_i^d), i = 1, ..., N
\]  

(6)

where, \( x_i^d \) is the position of the \( i \)-th mass in \( d \)-th dimension.

**Step2. Fitness Evaluation of All Agents**

In this step, for all agents, best and worst fitness are computed at each epoch described as follows.

\[
\begin{align*}
\text{best}(t) &= \min_{j \in \{1,...,N\}} \text{fit}(t) \\
\text{worst}(t) &= \max_{j \in \{1,...,N\}} \text{fit}(t)
\end{align*}
\]  

(7)

where \( \text{fit}(t) \) is the fitness of the \( j \)-th agent of \( t \) time, \( \text{best}(t) \) and \( \text{worst}(t) \) are best (minimum) and worst (maximum) fitness of all agents.

**Step3. Compute the Gravitational Constant (\( G(t) \))**

In this step, the gravitational constant at \( t \) time (\( G(t) \)) is computed as follows.

\[
G(t) = G_0 \exp \left( -\frac{1}{T} \right)
\]  

(8)

where \( G_0 \) is the initial value of the gravitational constant chosen randomly, \( \alpha \) is a constant, \( t \) is the current epoch and \( T \) is the total iteration number.

**Step4. Update the Gravitational and Inertial Masses**

In this step, the gravitational and inertial masses are updated as follows.

\[
M_g(t) = \frac{\text{fit}(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)}
\]  

(9)

where \( \text{fit}(t) \) is the fitness of the \( i \)-th agent of \( t \) time.

\[
M_g(t) = \sum_{j \in \{1,...,N\}} \text{mg}(t) \frac{\text{mg}(t)}{\text{mg}(t)}
\]  

(10)

where \( M_g(t) \) is the mass of the \( i \)-th agent of \( t \) time.

**Step5. Calculate the Total Force**

In this step, the total force acting on the \( i \)-th agent (\( F_i^d(t) \)) is calculated as follows.

\[
F_i^d(t) = \sum_{j \in \{1,...,N\}, j \neq i} \text{rand}(d) (\text{fit}(t) - \text{fit}(t))
\]  

(11)

where \( \text{rand}(d) \) is a random number between interval \( [0, 1] \) and \( \text{best} \) is the set of first \( K \) agents with the best fitness value and biggest mass.

The force acting on the \( i \)-th mass (\( M_i(t) \)) from the \( j \)-th mass (\( M_j(t) \)) at the specific \( t \) time is described according to the gravitational theory as follows.

\[
F_i^d(t) = G(t) \frac{M_i(t) M_j(t)}{R_{ij}(t) + \varepsilon} (x^d_j(t) - x^d_i(t))
\]  

(12)

where \( R_{ij}(t) \) is the Euclidian distance between \( i \)-th and \( j \)-th agents and \( \varepsilon \) is the small constant.
Step 6. Calculate the Acceleration and Velocity

In this step, the acceleration \( a_{i}^{d}(t) \) and velocity \( v_{i}^{d}(t) \) of the \( i \)th agent at \( t \) time in \( d \)th dimension are calculated through law of gravity and law of motion as follows.

\[
a_{i}^{d}(t) = \frac{F_{i}^{d}(t)}{Mg_{i}^{d}(t)}
\tag{13}
\]

\[
v_{i}^{d}(t+1) = \text{rand} \cdot v_{i}^{d}(t) + a_{i}^{d}(t)
\tag{14}
\]

where \( \text{rand} \) is the random number between interval \([0,1]\).

Step 7. Update the Position of the Agents

In this step, the next position of the \( i \)th agents in \( d \)th dimension \( (x_{i}^{d}(t+1)) \) are updated as follows.

\[
x_{i}^{d}(t+1) = x_{i}^{d}(t) + v_{i}^{d}(t+1)
\tag{15}
\]

The value of gravitational search algorithm parameters is given in Table III.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>PARAMETERS SETTING FOR GSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSA parameters</td>
<td>Value</td>
</tr>
<tr>
<td>Dimension of problem</td>
<td>02</td>
</tr>
<tr>
<td>Number of agents</td>
<td>100</td>
</tr>
<tr>
<td>Max-iteration</td>
<td>100</td>
</tr>
<tr>
<td>Velocity clock</td>
<td></td>
</tr>
<tr>
<td>Acceleration flag</td>
<td>gateway node flag</td>
</tr>
<tr>
<td>Mass. ( M_{a}=M_{p}=M_{i}=M ) time master node flag</td>
<td></td>
</tr>
<tr>
<td>Position of agents for internal clock</td>
<td>synchronization</td>
</tr>
<tr>
<td>Distance between agents in search space for external clock</td>
<td>synchronization</td>
</tr>
</tbody>
</table>

V. NUMERICAL RESULTS AND DISCUSSION

In this section, the simulation was performed using the GSA, genetic and gradient based algorithms. The designs parameters considered in the simulation are the length \((a)\), width \((b)\), thickness of the skin \((t)\) and the core thickness \((h)\). The material used is given in Table IV.

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>MATERIAL PROPERTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core, Skins (Aluminium)</td>
<td></td>
</tr>
<tr>
<td>( E ) (MPa)</td>
<td>72000</td>
</tr>
<tr>
<td>( \rho ) (g/mm(^3))</td>
<td>0.0028</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table V shows a comparison between the results of the three algorithms used for the honeycomb plate optimization.

Figs. 3-5 represent the evolution of geometrical parameters of the honeycomb plate using the genetic, the GSA and the gradient based algorithms. Figs. 6 and 7 show the mass evolution regarding the three algorithms. The results obtained by the GSA are clearly better than the two other algorithms.

We note also that the GSA algorithm converge quickly compared to the other two algorithms.
VI. CONCLUSION

In this study, an optimization methodology for weight minimization of a honeycomb plate under load was presented. Methods have been proposed for the purpose of comparison in order to choose the best algorithm that gives the best solution.

The gravitational search algorithm (GSA) was adopted as search algorithm. The results illustrate the efficiency of this algorithm.

The results, convergence rate and reliability of the algorithm are quiet promising and show that the GSA performs very well and in all the distinguished advantage of the technique is significant gain in the speed of convergence.

A significant reduction in the core density is a prerequisite. Moreover, the ability to make high-aspect ratio cores is also a necessary step.

REFERENCES


