The Impact of Treatment of Latent Tuberculosis on the Incidence: The Case of Algeria

S. Selmane

Abstract—We present a deterministic model which describes the dynamics of tuberculosis in Algerian population where the vaccination program with BCG is in place since 1969 and where the WHO recommendations regarding the DOTS (directly-observed treatment, short course) strategy are in application. The impact of an intervention program, targeting recently infected people among all close contacts of active cases and their treatment to prevent endogenous reactivation, on the incidence of tuberculosis, is investigated. We showed that a widespread treatment of latently infected individuals for some years is recommended to shift from higher to lower equilibrium state and thereafter relaxation is recommended.

Keywords—Deterministic model, reproduction number, stability, tuberculosis.

I. INTRODUCTION

HUNDRED and thirty-one years after the identification by Robert Koch in 1882 of the Mycobacterium tuberculosis, pathogenic of tuberculosis (TB), the disease is still a problem of public health world. In 2012, an estimated 8.6 million people developed TB and 1.3 million died from the disease. The number of TB deaths is unacceptably large given that most are preventable [7].

As many countries, Algeria is concerned by TB; the annual number of new cases of TB is around 21,000 cases of which more than 48% are cases of contagious pulmonary TB. In spite of a relatively significant medical cover for the country, more than 180 patients died yearly of smear positive pulmonary TB. Since 1969, vaccination by Bacille Calmette Guerin (BCG) is compulsory. Tuberculosis is a notifiable disease in Algeria and where the total exemption from payment of care; individuals with TB disease get their drugs from special centre implanted in each area of the country. A national program based on WHO recommendations was set up and several efforts were made in order to take charge of individuals with TB disease [5]. The various actions undertaken allowed a significant reduction of the incidence of the disease and this from the Seventies. In Fig. 1, the recorded pulmonary TB and extra-pulmonary cases per 100,000 inhabitants in Algeria extracted from [6] are plotted from 2001 to 2009.

An understanding of the dynamics of TB at the population level will lead to a better revitalization of the control program of this disease [1]. Since people with TB infection are considered at highest risk of developing TB disease in the 2 years which follow the infection, during which approximately 5 to 10 percent develop TB disease, an intervention that targets people with recent latent TB infection could be effective as control measure. Nevertheless to provide treatment for a large fraction of the population is costly and not feasible besides which the identification of LTBI individuals is not an easy task. We propose then to quantify how much treatment of recent TB infection individuals, of all close contacts of smear positive pulmonary cases, reduces the incidence of TB; an intervention would consist in keeping a watch on these close contacts.

The paper is organized as follows: In section III a deterministic model which describes the dynamics of tuberculosis is proposed. The dynamic of the model is governed by ordinary differential equations; therefore the analysis of the disease free equilibrium and the endemic equilibrium and conditions for local and global stability of these points is investigated in this section. Intervention that alter reactivation and re-infection as well as treatment of carries and the impact of treatment of TB infection on the incidence of TB over time are examined in this section. Section IV includes some numerical simulations of the proposed model and discusses the obtained results.

II. TUBERCULOSIS

Tuberculosis is an infectious disease caused by bacteria called Mycobacterium Tuberculosis (MTB). The bacteria usually attack the lungs (pulmonary TB), but can also affect other parts of the body through the blood (extra-pulmonary TB). The MTB is transmitted quasi exclusively by air. The infecting droplets are produced in the form of aerosol by the contagious patients at the time of cough, speech or sneezes. These droplets remain in suspension in the ambient air; ninety percent of them are inactivated as soon as their emission and only a fraction of 1% survive for few hours. The inhalation into the lungs of some bacteria suspended in the air constitutes,
in practice, the only mode of contamination. The individual becomes infected by breathing in the bacteria. The immune system is sometimes able to kill TB bacteria. If not, either, the bacteria remain alive but inactive in the body and the person contracts a TB infection, or, they become active and begin to multiply in the body and cause TB disease. Infected individuals who did not progress to TB disease may remain infected, non-infectious, for their lifetime unless endogenous reactivation or exogenous re-infection occurs [4]. Note that only the contamination by smear positive TB individuals has an epidemiologic importance.

**TB control program in Algeria**

The priorities for TB control program in Algeria are:

- The vaccination at birth in order to reduce the incidence of childhood TB knowing that is relatively ineffective in protecting against adult TB and does not prevent MTB infection.
- The identification in a permanent way of active TB cases and their treatment in order to break the transmission chain of the MTB and thus the sterilization of the sources of infection.

The cases of TB are only detected in the infectious stage; this is due to a lack of efficient system of detection at early stages of infection. People living under the same roof as a contagious tuberculous (the national average being of 10 individuals by household) are examined in order to identify among them the possible cases of TB disease. It is requested to the adults a tuberculinic test. If the examination is negative, they are informed of the possibility of late appearance of the disease 10 years a tuberculinic test. If the examination is negative, they are informed of the possibility of late appearance of the disease and informed of the clinical signs which will have to lead them to consult as soon as possible [5].

**III. MODEL DESCRIPTION AND ANALYSIS**

**A. Model description**

Although the population is vaccinated, this does not avoid infection. Susceptible (S), individuals who have never encountered the natural mycobacterium, can be infected only through contact with individuals having smear positive pulmonary TB disease. Latent TB infection (LTBI) is divided into two stages: (1) an early stage at high risk of developing active TB, referred as recent LTBI (L₁), and (2) later stage at low risk of developing active TB, referred as persistent LTBI (L₂). Likewise, we consider two classes of infectious individuals: smear positive pulmonary TB individuals, referred as (I₁), they can infect others and smear negative pulmonary TB individuals, referred as (Iₙ), who have TB disease and can not transmit it.

Infected individuals initially progress through recent LTBI, either, to active TB at rate ϕ₁ or, to persistent LTBI at rate (1 − ϕ₁)δ. From persistent LTBI class, individual can progress at low risk and slowly to infectious class either, by endogenous reactivation at rate ω, or, by exogenous re-infection at rate σ₁β₁. All detected infectious individuals receive 6 months treatment; 90% have a full recovery and the remaining 10% including 1% for disease-induced death and 9%, gathering the relapses, the failures and those which fail to comply with the treatment, return to infectious class at rate ρ₂ they receive a treatment of second line. Treated (T) individuals acquire some immunity not fully which reduces the risk of re-infection. They can return to the recent LTBI class only by exogenous re-infection at rate σ₂β₂. The factor reducing the risk of infection, as a result of acquired immunity to a previous infection, is taken fixed for persistent latent individuals, σₚ = 0.5 (any value between 0 and 1 would lead to the same conclusions), contrary to that for treated individuals where it was considered variable.

We incorporate into the model treatment of recent LTBI at a variable rate τ₀ and persistent LTBI at a variable rate τ₂. The model is schematically illustrated in Fig. 2, and the interactions of the compartments are specified by the following system (I)

\[
\begin{align*}
S &= μ − βI_pS − μS \\
L₁ &= βI_pS + σ_T βI_pT + σ_L βI_pL₂ − (δ + τ₀ + μ) L₁ \\
L₂ &= (1 − ϕ₁)δL₁ − σ_L βI_pL₂ − (ω + τ₂ + μ) L₂ \\
I₁ &= α₁ϕ₁L₁ + α₂ϕ₂L₂ + α_T ρT − (τ₀ + μ) I₁ \\
Iₙ &= (1 − α₁)ϕ₁L₁ + (1 − α₂)ωL₂ + (1 − α_T)ρT − (τ₀ + μ) Iₙ \\
T &= τ₀L₁ + τ₂L₂ + τ₀I_p + τ₀Iₙ − σ_T βI_pT − (ρ + μ)T \\
S + L₁ + L₂ + I₁ + Iₙ + T &= 1.
\end{align*}
\]

where the rate of infection λ = βI_p depends on the number of cases of smear positive pulmonary TB in the population and where

\[
S + L₁ + L₂ + I₁ + Iₙ + T = 1.
\]

so that the total population size is constant. The natural death term (μ) represents the per capita rate at which individuals die of causes other than TB.

**B. Analysis of The model**

1) Determination of the Basic Reproduction Number: The basic reproduction number R₀, which is defined as the average...
number of secondary infections produced by an infected individual in a completely susceptible and homogeneous population [3], is computed with the help of the next generation operator approach [2].

Letting $X = (S, T)$ (the number non-infected individuals), $Y = (L_1, L_2, I_0)$ (the number of infected individuals who do not transmit the disease), $Z = (I_p)$ (the number of infected individuals capable of transmitting the disease), $U_0 = (1, 0, 0, 0, 0, 0) \in \mathbb{R}^{2+3+1}$ the disease free equilibrium and

$$\bar{g}(X^*, Z) = (\bar{g}_1(X^*, Z), \bar{g}_2(X^*, Z), \bar{g}_3(X^*, Z))$$

with

$$\bar{g}_1(X^*, Z) = \frac{(\sigma_1 \beta_1 I_0 + c_3) \beta L_1}{c_1 (\sigma_1 \beta_1 I_0 + c_3) - (1 - \delta) \sigma \sigma_2 \beta_1 \beta}$$

$$\bar{g}_2(X^*, Z) = \frac{c_3 \delta L_2 (c_1 \sigma_2 I_0 + c_3) (1 - \delta) (1 - \alpha)}{c_1 (\sigma_1 \beta_1 I_0 + c_3) - (1 - \delta) \sigma \sigma_2 \beta_1 \beta}$$

$$\bar{g}_3(X^*, Z) = \frac{c_3 \delta L_2 (c_1 \sigma_2 I_0 + c_3) (1 - \alpha)}{c_1 (\sigma_1 \beta_1 I_0 + c_3) - (1 - \delta) \sigma \sigma_2 \beta_1 \beta}$$

gives

$$M = \left( \frac{\alpha_1 \phi c_2 + \alpha_2 \omega (1 - \phi)}{c_1 c_2 \rho} \right) \delta \beta$$

Hence $R_0$, defined as the spectral radius of the matrix $MD^{-1}$ is

$$R_0 = MD^{-1} = \frac{\alpha_1 \phi c_2 + \alpha_2 \omega (1 - \phi)}{c_1 c_2 \rho} \delta \beta.$$  

2) Steady States: In qualitative analysis of the model, the existence of steady states and their stability will be determined and analyzed.

To find an equilibrium $(S^*, L_1^*, L_2^*, I_0^*, T^*)$ of system (I) we have to solve the following system on $I_0^*$

$$\begin{align*}
\mu - \beta_1^* S^* - \mu S^* &= 0 \\
\beta_1^* S^* + \sigma T^* + \sigma I_0^* L_2^* - c_1 L_1^* &= 0 \\
(1 - \phi) \delta L_1^* - \sigma_1 \beta_1^* I_0^* L_2^* - c_2 L_2^* &= 0 \\
\alpha_1 \phi L_1^* + \alpha_2 \omega L_2^* + \alpha \rho T^* - c_3 I_0^* &= 0 \\
(1 - \alpha_2) \phi L_2^* + (1 - \alpha_3) \omega L_2^* + (1 - \alpha_4) T^* - c_3 I_0^* &= 0 \\
-\tau_1 L_1^* + \tau_2 L_2^* + \tau_3 I_0^* - \sigma_1 \beta_1^* T^* &= 0 \\
-\tau_4 T^* &= 0
\end{align*}$$

where $c_1 = \delta + \tau_1 + \mu, c_2 = \omega + \tau_2 + \mu, c_3 = \tau_3 + \mu, c_0 = \tau_0 + \mu$ and $\rho T = \rho \mu$

Equations 1, 3, 4 and 5 give $S^*, L_1^*, L_2^*$ and $T^*$ as function of $I_0^*$ and $L_2^*$.

$$\begin{align*}
S^* &= \frac{\mu}{(\beta_1^* + \mu)} \\
L_1^* &= \frac{L_2^* (1 - \phi)}{\alpha_1 \phi c_2 + \alpha_2 \omega (1 - \phi)} \\
T^* &= \frac{c_3 \rho T^*}{(1 - \alpha_1) \phi c_2 + \alpha_2 \omega (1 - \phi)}
\end{align*}$$

$$\begin{align*}
L_2^* &= \frac{c_3 \rho T^*}{(1 - \alpha_1) \phi c_2 + \alpha_2 \omega (1 - \phi)}
\end{align*}$$

From (2) we get $L_2^*$ as function of $I_0^*$

$$\begin{align*}
L_2^* &= \frac{\alpha \beta I_0^* + \mu a_0}{b_3 \beta^2 I_0^* + b_2 \beta^2 I_0^* + b_1 \beta I_0^* + \mu b_0}\beta I_0^*
\end{align*}$$

where

$$\begin{align*}
bb &= \rho c_1 \sigma_1 \cdot \beta + \rho (1 - \phi) \delta \alpha T \sigma_1 + \\
\alpha_1 &= (1 - \phi) \delta c_2 \sigma T, \quad a_1 = a_2 \mu, \quad a_0 = (1 - \phi) \delta \alpha T \rho, \\
b_3 &= \phi \alpha \sigma_1 \cdot \sigma T, \quad b_2 = bb + \mu b_3, \quad b_1 = b_0 + \mu b_0, \\
b_0 &= \rho c_1 \cdot c_2
\end{align*}$$

Finally, substituting $L_1^*, L_2^*, I_0^*$ and $T^*$ in (6) we get either $I_0^* = 0$, from which it may be concluded that system (I) always has the disease free equilibrium (DFE), or $I_0^*$ is a root of the third degree polynomial

$$P(I_0) = p_3 I_0^3 + p_2 I_0^2 + p_1 I_0^1 + p_0$$

therefore the possibility of existence of endemic equilibria. The coefficients of $P$ are

$$\begin{align*}
p_3 &= (\mu a_2 d_2 + a_2 d_1 + b_2 e_2 + b_2 e_1) \beta^3 \\
p_2 &= (\mu a_2 d_2 + a_2 d_1 + a_2 d_0 + b_2 e_2 + b_2 e_1) \beta^2 \\
p_1 &= (\mu a_2 d_2 + a_2 d_1 + a_2 d_0 + b_2 e_2 + b_2 e_1) \beta \\
p_0 &= \mu a_2 d_0 \beta + b_2 e_1 - \mu a_0 d_0 \beta (1 - R_0)
\end{align*}$$

where $c_3 = (\tau_0 + \alpha T) - c_0 \omega \sigma_1 + \rho (1 - \alpha T) \sigma_1 c_2 + \rho c_0 \omega (1 - \alpha T) \sigma_1 c_2$

and $R_0 = \frac{\beta}{\alpha_1}$ defining a new reproduction number. Note that for $\rho = 0$, letting $\beta_0 = (\phi c_1 \cdot c_2 + \alpha_2 \omega (1 - \phi))$, we find again the basic reproduction number $R_0 = \frac{\beta_0}{\alpha_1}$.

3) Stability of the disease-free equilibrium (DFE): At the disease free equilibrium $DFE$, we have $I_0^* = 0$ and previous computation yields $S^* = 1$ and $L_1^* = L_2^* = I_0^* = T^* = 0$. Hence $DFE = (1, 0, 0, 0, 0, 0, 0)$.

The stability of the disease free equilibrium is achieved through the determination of the sign of the eigenvalues of the jacobian matrix $J_0$ of system (I) evaluated at $DFE$.

$$J_0 = \begin{pmatrix}
-\mu & 0 & 0 & 0 & -\beta & 0 & 0 \\
0 & -c_1 & 0 & 0 & 0 & -\beta & 0 \\
0 & (1 - \phi) \delta & -c_2 & 0 & 0 & 0 & 0 \\
0 & \alpha_1 \delta & \alpha \omega & -c_3 & 0 & \alpha T \rho \\
0 & (1 - \alpha_1) \delta & (1 - \alpha_2) \omega & 0 & -c_0 & (1 - \alpha T) \rho \\
\tau_1 & \tau_2 & \tau_3 & \tau_4 & -\beta & 0 & 0 \\
\tau_1 & \tau_2 & \tau_3 & \tau_4 & -\beta & 0 & 0
\end{pmatrix}$$

one negative eigenvalue $-\mu$ of $J_0$ is straightforwardly determined, the other five eigenvalues are those of the matrix

$$J_0 = \begin{pmatrix}
-\alpha_1 \delta & \alpha \omega & -c_3 & 0 & \alpha T \rho \\
0 & -c_1 & 0 & 0 & 0 \\
(1 - \phi) \delta & -c_2 & 0 & 0 & 0 \\
\alpha_1 \delta & \alpha \omega & -c_3 & 0 & \alpha T \rho \\
(1 - \alpha_1) \delta & (1 - \alpha_2) \omega & 0 & -c_0 & (1 - \alpha T) \rho \\
\tau_1 & \tau_2 & \tau_3 & \tau_4 & -\beta & 0 & 0 \\
\tau_1 & \tau_2 & \tau_3 & \tau_4 & -\beta & 0 & 0
\end{pmatrix}$$
The stability conditions of \( J_1 \) are determined by use of the following result of M-matrices theory.

**Proposition**

Let \( A = [a_{ij}] \) be a \( n \times n \) matrix. The real part of each of the eigenvalues of \( A \) is greater than zero if and only if all diagonal entries of \( A \) are positive, and there exists a positive diagonal matrix \( D \), such that \( AD \) is strictly diagonal dominant, that is,

\[
a_{ii}d_i > \sum_{j=1, j \neq i}^n |a_{ij}|d_j \quad i = 1, \ldots, n.
\]

Since the matrix \( J_1 \) has negative diagonal entries, we consider the matrix \( -J_1 \). According to the previous proposition, \( J_1 \) has negative real part if and only if there exists a positive diagonal matrix \( D = (d_i^*)_{1 \leq i \leq 5} \) such that \( -J_1D \) is strictly diagonal dominant, namely,

\[
\begin{align*}
d_1^* & > 1 \\
d_1^* & = \frac{\beta + \varepsilon}{c_1} \\
d_2^* & = \frac{(1 - \phi)\delta \beta + \varepsilon(1 - \phi)\delta + c_1}{c_1c_2} \\
d_4^* & = \frac{(1 - \alpha_1)\delta \beta_1 + (1 - \alpha_2)\omega d_2 + (1 - \alpha \tau)\rho d_3 + \varepsilon}{c_2} \\
d_5^* & = \frac{\tau_1 d_1^* + \tau_2 d_2^* + \tau_3 d_3^* + \tau_4 d_4^* + \varepsilon}{c_T} 
\end{align*}
\]

where \( \varepsilon > 0 \), \( t = c_1 \left[ (1 - \alpha_2)\omega c_T + (1 - \alpha \tau)\rho p_T \right] + c_T c_1c_2 \\
y = (1 - \alpha \tau)p_T c_2, \quad z = [c_6 c_T - (1 - \alpha \tau)p_T c_1]c_1c_2, \\
x = (1 - \alpha_1)\delta \beta_1 + (1 - \alpha_2)\omega d_2 + (1 - \alpha \tau)\rho d_3 + \varepsilon \\
\) and

\[
\begin{align*}
c_1 & = (1 - \alpha_2)\omega c_T + (1 - \alpha \tau)\rho p_T \\
c_2 & = (1 - \alpha \tau)p_T c_2, \quad z = [c_6 c_T - (1 - \alpha \tau)p_T c_1]c_1c_2, \\
x & = (1 - \alpha_1)\delta \beta_1 + (1 - \alpha_2)\omega d_2 + (1 - \alpha \tau)\rho d_3 + \varepsilon \\
\) \\

Thus the expression of \( R_0 \) is obtained that the real part of each of the eigenvalues of \( -J_1 \) is greater than zero and therefore the DFE is locally asymptotically stable. Otherwise, it is unstable and an epidemic is triggered. The special case \( R_0 = 1 \) implies that both of the reproduction numbers are less than 1, which implies that both of the transmission rates are less than 1, and therefore the DFE is locally asymptotically stable.

Looking on the expression of \( R_0 \), we note that it is independent of the parameters \( \sigma_T \) and \( \sigma_L \); although the exogenous re-infection does not affect the stability, it affects the speed of the disease.

4) **Endemic equilibrium:** The existence of endemic equilibria for system (I) is linked to the existence of positive roots of the polynomial \( P \); \( (I_p^* > 0) \) must be biologically feasible.

A numerical computation of the polynomial discriminant \( \text{Disc} \) of \( P \)

\[
\text{Disc} = \left( \frac{3p_1p_3 - p_2^2}{9p_3^2} \right)^3 + \left( \frac{9p_1p_2p_3 - 27p_0p_3^2 - 2p_2^2}{54p_3^2} \right)^2
\]

yields \( \text{Disc} < 0 \), consequently all roots of \( P \) are real and unequal. Using the fact that the sign of the product of all roots of \( P \) is that of \( -\text{sign}(p_0)\text{sign}(p_3) \) and Since \( p_3 < 0 \) and \( p_0 = \mu a_0 \beta_0 (R_0 - 1) \) we deduced that the polynomial \( P \) has at least one real positive root if \( p_0 > 0 \), that is, \( R_0 > 1 \) and therefore the existence of one endemic equilibrium. Using Descartes’ Rule of signs we proved that \( P \) has three real negative roots for \( 0 < R_0 < 1 \) and thus non equilibria.

5) **Expected population after eradication of the disease:**

Eradication of the disease occurs when there are no more exposed and infectious individuals in the population, namely \( L_1 = L_2 = I_p = I_n = 0 \). Therefore \( S + T = 1 \) and the system (I) is reduced to

\[
\begin{align*}
S & = \mu - \mu S \\
T & = -(\rho + \mu) T
\end{align*}
\]

solving these equations we get

\[
\begin{align*}
S(t) & = 1 + (S(0) - 1)e^{-\mu t} \\
T(t) & = T(0)e^{-(\rho + \mu) t}
\end{align*}
\]

where \( S(0) \) and \( T(0) \) are the initial number of susceptible, treated individuals respectively. As \( t \to +\infty \), \( S(t) \to 1 \), and \( T(t) \to 0 \). Hence, in such situation, the whole population will be comprised of susceptible individuals.

6) **Model without exogenous re-infection:**

Case \( \sigma_T = \sigma_L = 0 \)

We investigate the situation where the re-infection is not possible or does not occur in the population. Equation (1) becomes

\[
\begin{align*}
b_1 e_1 I_p + \mu a_0 d_0 \beta_0 + \mu b_0 e_1 = 0
\end{align*}
\]

We have \( p_1 = b_1 e_1 < 0 \).

Indeed \( b_1 = \alpha_T p_T c_2 > 0 \) and \( e_1 < 0 \) where

\[
e_1 = -\mu (1 - \phi) \delta [p_T (1 - \alpha \tau) + \rho c_T \tau_n + c_p \tau_n + \mu + \mu c_p]
\]

Moreover we have proved that \( p_0 = \mu a_0 d_0 \beta_0 + \mu b_0 e_1 = \mu a_0 d_0 \beta_0 (R_0 - 1) \), therefore for \( R_0 > 1 \), we have one real positive root then one endemic equilibrium and for \( R_0 < 1 \) there exist no equilibrium.

Case \( \sigma_T = 0 \)

Now let us assume that individuals who followed a treatment acquired a full immunity against re-infection; so \( \sigma_T = 0 \). Equation (1) becomes

\[
\begin{align*}
p_2 I_p^2 + p_1 I_p + p_0 = 0
\end{align*}
\]

where \( p_2 = b_2 e_1 \beta_0^2, \quad p_1 = (\beta \mu a_0 d_0 + b_1 e_1) \beta, \quad p_0 = \mu a_0 d_0 \beta_0 + \mu b_0 e_1 \).

Using Descartes’ Rule of signs we proved that \( P \) has
one real positive root and therefore one endemic equilibrium for $R_p > 1$ and two real negative roots and thus non equilibria for $0 < R_p < 1$.

IV. NUMERICAL SIMULATIONS

The simulations of the stability of the endemic equilibria as well as simulations following the proportion of active TB individuals over time were carried out using the following parameters values. The natural death rate ($\mu$) is obtained from the expression $\frac{1}{\mu} = $ life expectancy. We take the life expectancy of 75 years corresponding to the one of Algerian population. The transmission rate $\beta$ is variable. The treatment period by the program being 6 months which gives a recovery rate of 2 per individual per year, so $\tau_p = \tau_n = 2$ (it is taken as the inverse of the time between the tuberculosis detection by the program and recovery by treatment). The proportion of individuals going towards the class $I_p$ were taken from algerian data. The rate of endogenous reactivation for treated individuals $\sigma = 0.0002$ per individual per year, so $\tau_p = \tau_n = 2$ (it is taken as the inverse of the time between the tuberculosis detection by the program and recovery by treatment). The proportion of individuals going towards the class $I_p$ were taken from algerian data. The rate of endogenous reactivation for treated individuals $\sigma = 0.0002$ per individual per year, so $\tau_p = \tau_n = 2$ (it is taken as the inverse of the time between the tuberculosis detection by the program and recovery by treatment). The rate of progression to active TB is estimated from $\phi \phi \approx 0.6$ [4]; if we assume that $\phi = 5\%$ of the latent population eventually develops active TB then $\delta = 12$year$^{-1}$. The proportion of individuals going from $L_1$ (resp. $L_2, T$) towards $I_p$ is $\alpha_1 = 0.4$ (resp. $\alpha_2 = 0.4, \alpha_T = 0.9$). The proportion of individuals going to compartment $I_p$ and $I_n$ is $\phi = 0.05$. The incidence in Algeria for the year 2006 being 26.1 per 100000 inhabitants [6], the corresponding values of infectious $I_p$ is 8834 cases and of the transmission coefficient $\beta$ is 150.42 and reproduction number $R_p = 1.9$; we used the system (I) to compute these values.

Fig. 3. describes the incidence of TB at equilibrium as function of the reproduction number $R_p$ in the absence of treatment of LTBI individuals; the circle $\circ$ corresponds to the incidence in Algeria for the year 2006.

Fig. 4., Fig. 5, and Fig. 6 describe the proportion of individuals with smear positive pulmonary TB at equilibrium as a function of the transmission coefficient $\beta$ in the absence of treatment of individuals with LTBI ($\tau_1 = \tau_2 = 0$) with heavy line and with treatment of recent LTBI at rate $\tau_1 = 1$ and persistent LTBI at various values of $\tau_2$ with dashed line. Through these figures, we see how the proportion of active TB individuals changes if the susceptibility to re-infection of treated individuals differs from that of persistent LTBI individuals.

Finally, Fig. 7. shows how the proportion of active TB individuals changes over time when the treatment of LTBI individuals is introduced and this is done for series of rates ($\tau_1 = \tau_2 = 0; 0.1; 0.2; 0.3; 0.4; 0.5; 1; 10$ and the limit $\tau_1 \to \infty$).

A widespread treatment of LTBI individuals for some years is recommended to shift from higher to lower equilibrium state and thereafter relaxation is recommended whenever $\sigma_T \leq \sigma_L$. 

Fig. 5. Incidence at equilibrium as function of the reproduction number $R_p$ in the absence of treatment of LTBI individuals; the circle $\circ$ corresponds to the incidence in Algeria for the year 2006.

Fig. 4. The proportion of individuals with smear positive pulmonary TB at equilibrium as a function of the transmission coefficient $\beta$ for $\sigma_T = \sigma_L$ and $\tau_1 = 1$. 
Fig. 5. The proportion of individuals with smear positive pulmonary TB at equilibrium as a function of the transmission coefficient $\beta$ for $\sigma_T = 2\sigma_L$ and $\tau_1 = 1$.

Fig. 6. The proportion of individuals with smear positive pulmonary TB at equilibrium as a function of the transmission coefficient $\beta$ for $\sigma_T = \sigma_L/2$ and $\tau_1 = 1$.

Fig. 7. The proportion of active TB individuals as function of time for different values of the treatment rate of LTBI individuals.

REFERENCES


