Fundamental Groups in Chaotic Flat Space and Its Retractions

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***Abstract*—**The purpose of this paper is to give a combinatorial characterization and construct representations of the chaotic fundamental groups of the chaotic submanifolds of chaotic flat space by using some geometrical transformations. The chaotic homotopy groups of the limit folding for chaotic flat space are presented. The chaotic fundamental groups of some types of chaotic geodesics in chaotic flat space are deduced.

***Keywords*—**Chaotic flat space, Chaotic folding, Chaotic retractions, Chaotic fundamental groups.

I. Introduction and Definitions

FLAT space represents one of the most intriguing and emblematic discoveries in the history of geometry. If it was introduced for a purely geometricpurpose;it came into prominence in many branches of mathematics and physics. This association with applied science and geometry generated synergistic effect: Applied science gave relevance to flat space and flat space allowed formalizing practical problems [6], [8].

Vector spaces, linear maps, topological spaces, and continuous maps, groups and homomorphisms together with the distinguished family of maps are referred to a category.
An operator which assigns to every object in one category a corresponding object in another category and to every map in the first map in the second in such a way that compositions are preserved and the identity map is taken to the identity map is called a functor. Thus, we may summarize our activities thus far by saying that we have constructed a functor (the fundamental group functor) from the category of pointed spaces and maps to the category of groups and homomorphisms. Such functors are the vehicles by which one translates topological problems into algebraic problem [21]-[24], [26], [28].

Most folding problems are attractive from a pure mathematical standpoint, for the beauty of the problems themselves. The folding problems have close connections to important industrial applications. Linkage folding has applications in robotics and hydraulic tube bending. Paper folding has application in sheet-metal bending, packaging, and air–bag folding [10]-[12], [18]. Isometric folding between two Riemannian manifolds may be characterized as maps that send piecewise geodesic segments to a piecewise geodesic segments of the same length [4].For a topological folding the maps do not preserve lengths[5],[6], i.e. A map $\Im: M \to N$, where M and N are C^{∞}–Riemannian manifolds of dimension mand n respectively is said to be an isometric folding of M into N, iff for any piecewise geodesic path $\gamma: J \to M$, the induced path $\Im \circ \gamma: J \to N$ is a piecewise geodesic and of the same length asγ . If $\Im$ does not preserve length, then $\Im$ is a topological folding [1]-[5], [7].

An n-dimensional topological manifold M is a Hausdorff topological space with a countable basis for the topology which is locally homeomorphic to R^n. If h:$U \to U'$ is a homeomorphism of U $\subseteq M$ onto $U' \subseteq R^n$,then h is called a chart of M and U is the associated chart domain. A collection $(h_\alpha, u_\alpha)$is said to be an atlas for M if $\cup_{\alpha \in A} U_\alpha = M$. Given two charts h_α, h_β such that $U_{\alpha\beta}$=$U_\alpha \cap U_\beta \neq \emptyset$,the transformation chart $h_\beta \circ h_\alpha^{-1}$ between open sets ofR^n is defined, and if all of these charts transformation are C^∞-mappings, then the manifolds under consideration is a C^∞-manifolds. A differentiable structure on M is a differentiable atlas and a differentiable manifold is a topological manifold with a differentiable structure [29]-[31]. M may have other structures as colors, density or any physical structures. The number of structures may be infinite. In this case the manifold is said to be a chaotic manifold and may become relevant to vacuum fluctuation and chaotic quantum field theories. The magnetic field of a magnet bar is a kind of chaotic 1-dimensional manifold represented by the magnetic flux lines. The geometric manifold is the magnetic bar itself [1]-[5].

Fuzzy manifolds are special type of the category of chaotic manifolds. Usually we denote by M =$M_{0123\ldots h}$ to a chaotic manifolds [6], [7], where M_{0h} is the geometric (essential) manifold and the associated pure chaotic manifolds, the manifolds with physical characters, are denoted by M_{1h} ,…, .$M_{\infty h}$[11], [13], [18], [19].

The aim of this paper is to describe the connection between the chaotic fundamental groups and the chaotic homotopy group geometrically, specifically concerned with the study of the new type of chaotic retractions, chaotic deformation retracts, chaotic foldings and the chaotic fundamental groups of chaotic flat space $F_{0123\ldots h}$ as presented by El-Ahmady [1]-[26] and M. Abu-saleem [27]-[40]. The set of chaotic homotopy classes of chaotic loops based at the point $x_0(\mu)$ with the product operation $[f(\mu)][g(\mu)]=[f(\mu).g(\mu)]$ is called the chaotic fundamental groups and denoted by $\pi_1(X(\mu), x_0(\mu))$ [7], [8], [18]-[44].

A subset A of a topological space X is called a retract of X if there exists a continuous map $r: X \to A$ such that $r(a) = a, \forall a \epsilon A$ where A is closed and X is open [3], [7]. Also, let Xbe

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a space and A a subspace. A map $r: X \to A$ such that $r(a) = a$, for all $a \in A$, is called a retraction of X onto A and A is the called a retract of X.This can be restated as follows. If $i: A \to X$ is the inclusion map, then $r: X \to A$ is a map such that $ri = id_A$. If, in addition, $ri \simeq id_X$, we call r a deformation retract and A a deformation retract of X.Imagine a point light source on a two-dimensional surface. If we switch this light source on, then light will propagate in an ever increasing circle, around the source with a velocity C which is that of light. Imagine now we blot the elapsing time needed for the propagation of light on a vertical axis perpendicular to our surface with its zero point exactly at the light source point. That way we obtain the light cone which plays a fundamental role in relatively theory [25], [26]. Here we will use an E-Infinity fuzzy light cone.The light cone gives us a way of differentiating between three types of geodesics. They correspond to velocities less than C, exactly equal to C and larger than C. We call them time-like, null and space-like geodesics. Special relativity states that all matter must move at speed of light. This means that all bodies must move on time like or null geodesics i.e. unless acted upon by forces other than gravity [25], [26].

Next let us recall the concept of a metric in four-dimensions. Considering only flat space, we have

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$$

Now we see that $ds^2 > 0$, $ds^2 = 0$ and $ds^2 < 0$ correspond to space-like, null and time-like geodesics, we note that massless particles, such as the photon, move on null geodesics. That can be interpreted as saying that in 4-dimensional space, the photon does not move and the time for that photondoes not pass. Particularly intriguing is the mathematical possibility of a negative metric. Now it is extremely interesting that there is a geometry in which two separated points may still have a zero distance analogous to $ds^2 = 0$ the corresponding to a null geodesic [25], [26]. In what follows we would like to introduce a new type of geodesics of chaotic flat space namely "the retraction of chaotic flat space". The 0-dimensional flat space will be discussed.

II. Main Results

Theorem 1. The chaotic fundamental groups of types of the chaotic retractions of chaotic flat space $F_{0123\ldots h}$ are isomorphic to $Z_{0123\ldots h}$ or chaotic identity group $0 = 0_{0123\ldots h}$.

Proof. The chaotic flat space is defined as

$$ds^2 = -\left(c_1(t)x_2 + c_2(t)\right)^2 dx_1^2 - \frac{c_1(t)\left(c_1(t)x_2 + c_2(t)\right)}{x_2 + c_3(t)} dx_2^2 - \frac{x_2 + c_3(t)}{c_1(t)\left(c_1(t)x_2 + c_3(t)\right)} dx_3^2 + \left(c_1(t)x_2 + c_3(t)\right)^2 \cos^2 x_1 dx_4^2 \cdots \quad (1)$$

where $c_1(t), c_2(t)$ and $c_3(t)$ are functions of time. Then, the coordinates of the chaotic flat space $F_{0123\ldots h}$ are

$$x_1 = \frac{iA_1}{1 - i\left(c_1(t)x_2 + c_2(t)\right)},\ x_2 = \frac{ic_1(t)}{2}(\log B_1 - N \log B_2),$$
$$B_1 = \sqrt{x_2^2 + Lx_2 + M},\ B_2 = \frac{x_2 + \frac{L}{2} + \sqrt{x_2^2 + Lx_2 + M}}{\sqrt{M - L^2/4}} \quad (2)$$
$$L = \left(c_3(t) + \frac{c_2(t)}{c_1(t)}\right), M = \frac{c_2(t)c_3(t)}{c_1(t)}, N = \left(c_3(t) - \frac{c_2(t)}{c_1(t)}\right)$$
$$x_3 = \frac{iA_2}{\left(1 - i\sqrt{\frac{x_2 + c_3(t)}{c_1(t)\left(c_1(t)x_2 + c_3(t)\right)}}\right)},\ x_4 = \frac{A_3}{\left(1 - \left(c_1(t)x_2 + c_2(t)\right)\cos x_i\right)}$$

where A_1, A_2 and A_3 are the constant of integration.

In a position, using Lagrangian equations:

$$\frac{d}{ds}\left(\frac{\partial T}{\partial x_i'}\right) - \frac{\partial T}{\partial x_i} = 0,\quad i = 1,2,3,4,5.$$

To deduce a chaotic geodesic which is a subset of chaotic flat space $F_{0123\ldots h}$. Since $T = \frac{1}{2}\overline{ds}^2$, this yields

$$T = \frac{1}{2}\left\{ -\left(c_1(t)x_2 + c_2(t)\right)^2 - \frac{c_1^2(t)x_2 + c_1(t)c_2(t)}{x_2 + c_3(t)} x_2'^2 - \frac{x_2 + c_3(t)}{\left(c_1^2(t)x_2 + c_1(t)c_2(t)\right)} x_3'^2 + \left(c_1(t)x_2 + c_3(t)\right)^2 \cos^2 x_1 x_4'^2 \right. \quad (3)$$

Then, the Lagrangian equations are

$$\frac{d}{ds}\left\{ -\left(c_1(t)x_2 + c_2(t)\right)^2 x_1' \right\} + \left(c_1(t)x_2 + c_2(t)\right)^2 \cos x_1 \sin x_1 x_4'^2 = 0 \quad (4)$$

$$\frac{d}{ds}\left\{ -\frac{\left(c_1^2(t)x_2 + c_1(t)c_2(t)\right)}{x_2 + c_3(t)} x_2' \right\} + \left(c_1^2(t)x_2 + c_1(t)c_2(t)\right)x_1'^2 + \frac{c_1^2(t)c_3(t) - c_1(t)c_2(t)}{2(x_2 + c_3(t))^2} x_2'^2 + \frac{c_1(t)c_3(t) - c_1^2(t)c_3(t)}{2\left(c_1^2(t)x_2 + c_1(t) + c_3(t)\right)} x_3'^2 - \left(c_1^2(t)x_2 + c_1(t)c_2(t)\right)\cos^2 x_1 x_4'^2 = 0 \quad (5)$$

$$\frac{d}{ds}\left\{ \frac{x_2 + c_3(t)}{c_1^2(t)x_2 + c_1(t)c_3(t)} \right\} x_3' = 0 \quad (6)$$

$$\frac{d}{ds}\left\{ \left(c_1(t)x_2 + c_2(t)\right)^2 \cos^2 x_1 x_4' \right\} = 0 \quad (7)$$

$$-2\left(c_1(t)x_2 + c_2(t)\right)\left(c_1'(t)x_2 + c_2'(t)\right)x_1'^2 +$$

$$\frac{(x_2+c_3(t))(2c_1(t)c_1'(t)x_2+c_1'(t)c_2(t)+c_1(t)c_2'(t)) - (c_1^2(t)x_2+c_1(t)c_2(t))c_3'(t)}{(x_2+c_3(t))^2}x_2'^2 -$$
$$-\frac{(c_1^2(t)x_2+c_1(t)c_2(t))c_3'(t)-(x_2+c_3(t))(2c_1(t)c_1'(t)x_2+ c_1'(t)c_2(t)+c_1(t)c_2'(t))}{(c_1^2(t)x_2+c_1(t)c_3(t))^2}x_3'^2 -$$
$$2(c_1(t)x_2+c_2(t))(c_1'(t)x_2+c_2'(t))\cos^2 x_1 x_4'^2 = 0 \quad (8)$$

From (7), we have $(c_1(t)x_2+c_2(t))^2\cos^2 x_1 x_4' =$ constant (say $= \gamma$),

if $\gamma = 0$, we get $x_4' = 0$, which implies to $x_4 = 0$. Then, we obtain the following coordinates

$$x_1 = \frac{iA_1}{1-i(c_1(t)x_2+c_2(t))},\ x_2 = \frac{ic_1(t)}{2}(\log B_1 - N\log B_2),$$
$$x_3 = \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}},\ x_4 = 0 \quad (9)$$

which is the chaotic hypersurface time-like geodesic F_{i1} in chaotic flat space $F_{0123...h}$, which is the chaotic retraction. Therefore, $\pi_1(F_{i1} \subset F_{0123...h})$ is isomorphic to identity group.

In a special case if $x_2 = \frac{-c_2(t)}{c_1(t)}$, we obtain the chaotic spheres S_{i1}^2 in chaotic flat space F_i, which represented by

$$x_1 = iA_1,\ x_2 = iB_1',\ x_3 = iB_2',\ x_4 = 0 \quad (10)$$

Then, $x_1^2+x_2^2+x_3^2-x_4^2 = (A_1^2+B_1'^2+B_2'^2) = -k^2$, which is the geodesic hypersphere in chaotic time-like flat space which is a retraction, where A_1, B_1' and B_2' are constants. Therefore $\pi_1(S^2{}_{i1} \subset F_{0123...h})$ is isomorphic to identity group.

Also, if $c_2(t) = 0$, then we get the following coordinates.

$$x_1 = iA_1,\ x_2 = 0,\ x_3 = \frac{iA_2}{1-i\sqrt{1/c_1(t)}},\quad x_4 = 0 \quad (11)$$

Hence,

$$x_1^2+x_2^2+x_3^2-x_4^2 = (iA_1)^2 + \left(\frac{iA_2}{1-i\sqrt{1/c_1(t)}}\right)^2,$$

which is the chaotic great circle s_{i1}^1 in chaotic flat space-time geodesic. These geodesic is a retraction in chaotic flat space. Therefore, $\pi_1(S^1{}_{i1} \subset F_{0123...h})$ is isomorphic to $Z_{0123...h}$. But, if we put $A_1^* = iA_1$, $A_2^* = iA_2$ and $c_1(t) = -c_1^*(t)$, we have

$$x_1 = A_1^*,\ x_2 = 0,\ x_3 = \frac{iA_2^*}{1-i\sqrt{1/c_1^*(t)}},\quad x_4 = 0 \quad (12)$$

In this case we obtain a chaotic circle s_{i2}^1, which is a space-time geodesic. Therefore, $\pi_1(S^1{}_{i2} \subset F_{0123...h})$ is isomorphic to $Z_{0123...h}$.

If $\cos^2 x_1 = 0$, then $x_1 = n\frac{\pi}{2}$, n is odd and the chaotic hyper space F_{i2} is represented by the following coordinate

$$x_1 = n\frac{\pi}{2},\ x_2 = \frac{ic_1(t)}{2}(\log B_1 - N\log B_2),$$
$$x_3 = \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}},\ x_4 = A_3 \quad (13)$$

which is a geodesic in time-like chaotic flat space. Also, these geodesics is a retraction on $F_{0123...h}$. Therefore, $\pi_1(F_{i2} \subset F_{0123...h})$ is isomorphic to identity group.

From (6), we obtain

$$\frac{x_2+c_3(t)}{c_1^2(t)x_2+c_1(t)c_3(t)}x_3' = \text{constant (say } \alpha\text{)}.$$

If $\alpha = 0$, we get $x_3' = 0 \Rightarrow x_3 = 0$, we obtain the following coordinates

$$x_1 = \frac{iA_1}{1-i(c_1(t)x_2+c_2(t))},\ x_2 = \frac{ic_1(t)}{2}(\log B_1 - N\log B_2),$$
$$x_3 = 0,\ x_4 = \frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1} \quad (14)$$

which is the chaotic hyperspace F_{i3}, which is the chaotic retraction and chaotic geodesic in time-like. Therefore $\pi_1(F_{i3} \subset F_{0123...h})$ is isomorphic to identity group.

Andif

$$\frac{x_2+c_3(t)}{c_1^2(t)x_2+c_1(t)c_3(t)} = 0, \text{ then } x_2 = -c_3(t)$$

which deduce also a chaotic time-like geodesic hyperspace F_{i4} having the following coordinates

$$x_1 = \frac{iA_1}{1-i(c_1(t)x_2+c_2(t))},\ x_2 = -c_3(t),\ x_3 = \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}},$$
$$x_4 = \frac{A_4}{1-(c_1(t)x_2+c_2(t))\cos x_1} \quad (15)$$

Therefore, $\pi_1(F_{i4} \subset F_{0123...h})$ is isomorphic to identity group.

Corollary 1. The chaotic fundamental groups of chaotic flat space are time-like geodesics.

Theorem 2. The chaotic fundamental group of the limit folding of the chaotic flat space into itself, under (16), is the same as the chaotic fundamental group of the retraction of the chaotic flat space into the geodesic F_{i1}.

Proof. Now, we are going to discuss the folding η_n of the chaotic flat space $F_{0123...h}$. Let $\eta_n : F_{0123...h} \longrightarrow F_{0123...h}$, be given by $\eta_n(x_1,x_2,x_3,x_4)=(x_1,x_2,x_3,\frac{|x_4|}{n})$ where

$$n = 1,2,\cdots \tag{16}$$

An isometric chain folding of $F_{0123...h}$ into itself may be defined by

$$\eta_1:\Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\Big\} \longrightarrow \Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|\Big\},$$

$$\eta_2:\Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|\Big\} \longrightarrow \Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \frac{\left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|}{2}\Big\},\ldots,$$

$$\eta_n:\Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \frac{\left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|}{n-1}\Big\} \to \Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \frac{\left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|}{n}\Big\}$$

Then, we have

$$\lim_{n\to\infty}\eta_n = \Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{ic_1(t)}{2}(\log B_1 - N\log B_2), \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, 0\Big\}.$$

which is a chaotic hypersurface F_{i1} in chaotic flat space $F_{0123...h}$, which is the chaotic retraction. Therefore, $\pi_1(F_{i1} \subset F_{0123...h})$ is isomorphic to identity group.

Theorem 3. The chaotic fundamental group of the limit folding of the chaotic flat space into itself, under condition (17), $\pi_1(F_{i5} \subset F_{0123...h})$, is isomorphic to identity group.

Proof. If we let $\Pi_m : F_{0123...h} \longrightarrow F_{0123...h}$ be given by

$$\Pi_m(x_1,x_2,x_3,x_4)=(x_1,\frac{|x_2|}{m},x_3,\frac{|x_4|}{m}) \tag{17}$$

Then, the isometric chain folding of $F_{0123...h}$ into itself may be defined by

$$\Pi_1:\Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \left|\frac{ic_1(t)}{2}(\log B_1 - N\log B_2)\right|, \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|\Big\} \longrightarrow \Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \left|\frac{ic_1(t)}{2}(\log B_1 - N\log B_2)\right|, \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|\Big\},$$

$$\Pi_2:\Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \left|\frac{ic_1(t)}{2}(\log B_1 - N\log B_2)\right|, \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|\Big\} \longrightarrow \Big\{\frac{iA_1}{1-i(c_1(t)x_2+c_2(t))}, \frac{\left|\frac{ic_1(t)}{2}(\log B_1 - N\log B_2)\right|}{2}, \frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)(c_1(t)x_2+c_3(t))}}}, \frac{\left|\frac{A_3}{1-(c_1(t)x_2+c_2(t))\cos x_1}\right|}{2}\Big\},\ldots,$$

$$\Pi_m:\left\{\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)},\frac{\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|}{m-1},\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}},\frac{\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|}{m-1}\right\}\to$$

$$\left\{\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)},\frac{\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|}{m},\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}},\frac{\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|}{m}\right\}.$$

Then, we get

$$\lim_{m\to\infty}\Pi_m=\left\{\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)},0,\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}},0\right\}$$

which is a chaotic hypersurface F_{i5} in chaotic flat space $F_{0123\ldots h}$. Therefore, $\pi_1(F_{i5}\subset F_{0123\ldots h})$ is isomorphic to identity group.

Theorem 4. The chaotic fundamental group of the limit folding of chaotic flat space into itself, under (18), is equivalent to the zero-dimensional sphere S_i° in chaotic flat space which isomorphic to identity group.

Proof. If the folding is defined by $\gamma_n: F_{0123\ldots h}\longrightarrow F_{0123\ldots h}$ such that

$$\gamma_n(x_1,x_2,x_3,x_4)=\left(\frac{|x_1|}{n},\frac{|x_2|}{n},\frac{|x_3|}{n},\frac{|x_4|}{n}\right) \qquad (18)$$

Then, the isometric chain folding of $F_{0123\ldots h}$ into itself may be defined by

$$\gamma_1:\left\{\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)},\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|,\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}},\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right\}$$

$$\to\left\{\left|\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)}\right|,\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|,\left|\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}}\right|,\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|\right\},$$

$$\gamma_2:\left\{\left|\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)}\right|,\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|,\left|\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}}\right|,\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|\right\}$$

$$\to\left\{\frac{\left|\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)}\right|}{2},\frac{\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|}{2},\frac{\left|\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}}\right|}{2},\frac{\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|}{2}\right\},\ldots,$$

$$\gamma_n:\left\{\frac{\left|\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)}\right|}{n-1},\frac{\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|}{n-1},\frac{\left|\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}}\right|}{n-1},\frac{\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|}{n-1}\right\}$$

$$\to\left\{\frac{\left|\frac{iA_1}{1-i\left(c_1(t)x_2+c_2(t)\right)}\right|}{n},\frac{\left|\frac{ic_1(t)}{2}(\log B_1-N\log B_2)\right|}{n},\frac{\left|\frac{iA_2}{1-i\sqrt{\frac{x_2+c_3(t)}{c_1(t)\left(c_1(t)x_2+c_3(t)\right)}}}\right|}{n},\frac{\left|\frac{A_3}{1-\left(c_1(t)x_2+c_2(t)\right)\cos x_1}\right|}{n}\right\}.$$

Then, we get $\lim_{n\to\infty}\gamma_n=\{0,0,0,0\}$, which a zero-dimensional hypersphere S_i° in chaotic flat space. Therefore $\pi_1(S_i^0\subset F_{0123\ldots h})$ is isomorphic to identity group.

Theorem 5. The end of the limits of the foldings of chaotic flat space of dimension n is a 0-dimensional chaotic flat space and $\pi_1(F_i^0\subset F_{0123\ldots h})$ is isomorphic to identity group.

Proof: If we let $\eta_1: F_i^n\longrightarrow F_i^n$, $\eta_2:\eta_1(F_i^n)\longrightarrow\eta_1(F_i^n)$,

A. $\eta_3:\eta_2\left(\eta_1(F_i^n)\right)\longrightarrow\eta_1\left((F_i^n)\right),\ldots,$

$\eta_n : \eta_{n-1}(\eta_{n-2}(\eta_1(F_i^n))\cdots) \longrightarrow \eta_{n-1}(\eta_{n-2}\cdots(\eta_1(F_i^n)),\cdots,)$, then

$\lim_{n\to\infty} \eta_n : \eta_{n-1}(\eta_{n-2}(\cdots(\eta_1(F_i^n))\cdots) = F_i^{n-1}$, which is the chaotic flat space of dimension $n-1$.

Also, if we consider

$\gamma_1 : F_i^{n-1} \longrightarrow F_i^{n-1}$, $\gamma_2 : \gamma_1(F_i^{n-1}) \longrightarrow \gamma_1(F_i^{n-1})$, $\gamma_3 : \gamma_2(\gamma_1(F_i^{n-1})) \longrightarrow \gamma_1((F_i^{n-1})),\cdots$,

$\gamma_m : \gamma_{m-1}(\gamma_{m-2}(\gamma_1(F_i^{n-1}))\cdots) \longrightarrow \gamma_{m-1}(\gamma_{m-2}\cdots(\gamma_1(F_i^{n-1})),\cdots,)$, then

$\lim_{m\to\infty} \gamma_m : \gamma_{m-1}(\gamma_{m-2}(\cdots(\eta_1(F_i^{n-1}))\cdots) = F_i^{n-2}$, which is the chaotic flat space of dimension n-2.

Consequently, $\lim_{m\to\infty}\lim_{n\to\infty} \cdots \gamma_m(\eta_n(\cdots(\eta_1(F_i^n)) = F_i^{\circ}$, which is a zero-dimensional chaotic space. Therefore $\pi_1(F_i^0 \subset F_{0123\ldots h})$ is isomorphic to identity group.

Theorem 6. Under (18), the chaotic fundamental group of the limit of foldings of chaotic flat space into itself coincide with minimal retraction.

Proof. Let $\eta_i : F^n_{0123\ldots h} \longrightarrow F^n_{0123\ldots h}$ be a type of foldings and r_i are the retractions. Then, we have the following chains

$$F_i^n \xrightarrow{\eta_1'} F_{i1}^n \xrightarrow{\eta_2'} F_{i2}^n \longrightarrow \cdots F_{i(n-1)}^n \xrightarrow[i\to\infty]{Lim\ \eta_i'} F_i^{n-1}$$

$$F_i^n \xrightarrow{r_1'} F_{i1}^n \xrightarrow{r_2'} F_{i2}^n \longrightarrow \cdots F_{i(n-1)}^n \xrightarrow[i\to\infty]{Lim\ r_i'} F_i^{n-1},$$

$$F_i^{n-1} \xrightarrow{\eta_1^2} F_{i1}^n \xrightarrow{\eta_2^2} F_{i2}^{n-1} \longrightarrow \cdots F_{i(n-1)}^{n-1} \xrightarrow[i\to\infty]{Lim\,\eta_i^2} F_i^{n-2},$$

$$F_i^{n-1} \xrightarrow{r_1^2} F_{i1}^n \xrightarrow{r_2^2} F_{i2}^{n-1} \longrightarrow \cdots F_{i(n-1)}^{n-1} \xrightarrow[i\to\infty]{Lim\ r_i^2} F_i^{n-2}$$

$$\vdots$$

$$F_i^1 \xrightarrow{\eta_1^n} F_{i1}^1 \xrightarrow{\eta_2^n} F_{i2}^1 \longrightarrow \cdots F_{i(n-1)}^1 \xrightarrow[i\to\infty]{Lim\ \eta_i^n} F_i^0$$

$$F_i^1 \xrightarrow{r_1^n} F_{i1}^1 \xrightarrow{r_2^n} F_{i2}^1 \longrightarrow \cdots F_{i(n-1)}^1 \xrightarrow[i\to\infty]{Lim\ r_i^n} F_i^0.$$

Thus from the above chain the end of the limits of folding coincides with the zero-dimensional space which is the limit of retractions and therefore $\pi_1(F_i^0 \subset F_{0123\ldots h})$ is isomorphic to identity group.

III. Conclusion

In this paper we achieved the approval of the important of the chaotic fundamental group of the chaotic geodesic flat space. The relation between the folding of chaotic flat space and chaotic fundamental group are discussed. Theorems which govern these relations are presented.

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