A Bathtub Curve from Nonparametric Model

Eduardo C. Guardia, Jose W. M. Lima, Afonso H. M. Santos

***Abstract*—**This paper presents a nonparametric method to obtain the hazard rate “Bathtub curve” for power system components. The model is a mixture of the three known phases of a component life, the decreasing failure rate (DFR), the constant failure rate (CFR) and the increasing failure rate (IFR) represented by three parametric Weibull models. The parameters are obtained from a simultaneous fitting process of the model to the Kernel nonparametric hazard rate curve. From the Weibull parameters and failure rate curves the useful lifetime and the characteristic lifetime were defined. To demonstrate the model the historic time-to-failure of distribution transformers were used as an example. The resulted “Bathtub curve” shows the failure rate for the equipment lifetime which can be applied in economic and replacement decision models.

***Keywords*—**Bathtub curve, failure analysis, lifetime estimation, parameter estimation, Weibull distribution.

I. Introduction

THE aging of electricity infrastructures has become an important issue for utilities regarding the consequences of increasing failure rates after a moment where the system components are more susceptible to fail and the whole operation reliability may be affected.

In the distribution networks in Brazil, the smaller components are responsible for 90% of the Regulatory Asset Base of utilities. These components (cables, poles, meters, switches and distribution transformers) are considered commodity units in the field and will play an important challenge for reliability analysis in the long run. The capacity to manage aging problems at these equipment levels will be used to evaluate the utility’s maturity in asset management [1].

The statistical approach to obtain the failure rate of equipments can be classified in two categories, the parametric and the nonparametric estimation models. However, few methodologies have been found in literatures even for power transformers [2] and its subcomponents [3]. A Monte Carlo simulation method has been used to obtain the time-to-failure (TTF) and time-to-repair (TTR) to account for the uncertainties in failure and repair rates [4].

In addition, the estimation of failures is very close to the estimation of equipment useful lifetime, concerning the “bathtub curve” [5].

This paper demonstrates the application of the method for a distribution transformer.

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A. Reliability Functions

The reliability functions used in statistical distribution lifetime models relies on the time-to-event analysis where installation and failures dates define the time variable.

Six mathematically equivalent functions can be used to describe the distribution of the lifetime and each of them can unequivocally determine the other five. The equations for the Weibull distribution can be seen in [6]. These functions are the failure density, the failure distribution, the reliability (survival) function, the hazard rate, the cumulative hazard rate and the mean residual life function.

B. Bathtub Shape Hazard Rate

Reliability models for a power system component generally take into account two failure modes: the repairable and the non-repairable modes. The first one has a constant failure rate and a repair time. The second one has an increasing failure rate and no repair time, since aging failures take components out forever [5], [7].

Aging has been defined from many points of view for different authors and can be summarized as a time process with increasing internal failure rates [8]-[12].

The “bathtub curve”, shown in Fig. 1, represents three kinds of failures and can be built from the composition of three Weibull curves with scale and shape parameters, α and β, respectively.

The first region is characterized by wear-in (infant mortality) failures and a decreasing failure rate ($\beta < 1$). The second region is the useful life (normal life) where the failure rate is low and constant and failures occur randomly ($\beta = 1$). In the third region, the wear-out (end-of-life), there is an increasing failure rate due to age ($\beta > 1$).

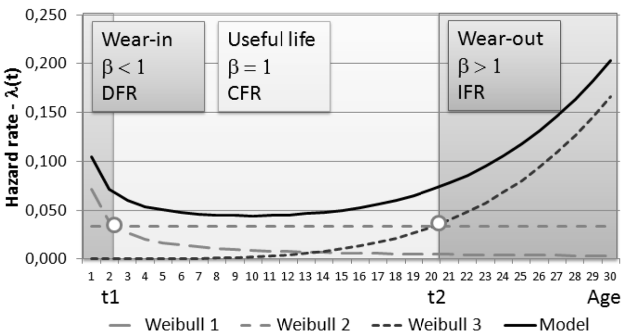

Fig. 1 Bathtub hazard rate curve

C. Parameter Estimation

The reliability density function for the Weibull model is shown in (1) for the three parameter equation. If $\gamma = 0$ the model can be reduced to a 2-parameter function.

$$f(t) = \frac{\beta}{\alpha}\left(\frac{t-\gamma}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}} \quad (1)$$

where:
α = scale parameter or characteristic life
β = shape parameter
γ = location parameter

Estimates of α and β may be obtained directly by fitting the observed data to the cumulative failure function or the cumulative survivor function using least-squares. However this method demands reasonable starting values [13]. The maximum likelihood estimation (MLE) is a method that tests various combinations of α and β fitting the density function to the original sample data [14].

The Kaplan-Meier estimator can be used to obtain the nonparametric model for the survivor function, which is the MLE for the survivor function.

The nonparametric methods present some advantages like their insensitivity to outlier observations and the requirement of few assumptions about the underlying population to be normally distributed [15].

II. Proposed Method

The first step on the lifetime methodology is the estimation of a theoretical nonparametric hazard rate that will fit the parametric hazard rate model. This model corresponds to a mixture of three Weibull hazard rate curves simultaneously adjusted by the parameters estimation to build the “bathtub curve”. From the point where the constant hazard rate crosses the increasing hazard rate the useful lifetime is defined.

The condition to plot the “bathtub curve” is shown in Fig. 2, where the three cumulative failure curves of the “bathtub curve” crosses at the same point (α, 0.632*F(t)) [16].

The three curves are considered a family of curves where the value of the parameter α is common and the values of β are different. In Fig. 3 we present the density functions and its properties for β = 1, MTTF = α, f(0) = 1/α and f(α) = 0.368/α.

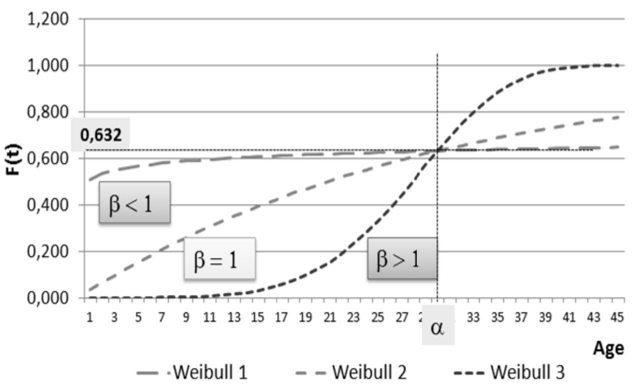

Fig. 2 Weibull reliability functions: Cumulative failure

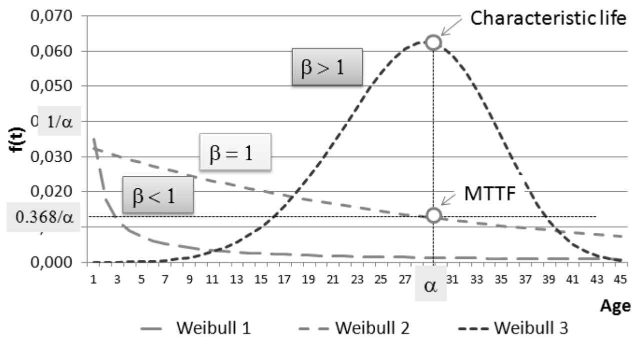

Fig. 3 Weibull reliability functions: Failure density

Regarding this condition, the fitting process has to adjust three parameters (α_1=α_2=α_3=α, β_1 and β_3) for three curves. If one wants to fit the model freely, five parameters have to be adjusted (α_1, α_2, α_3, β_1 and β_3). Table I summarizes these relationships.

An example of the fitting process will be shown in Section III for distribution transformers of one phase, 13.2 kV, from rural areas that were removed of service. The removal period occurred between 2003 and 2012 and approximately 75 kinds of events were registered.

TABLE I
CONDITIONS TO PLOT THE BATHTUB CURVE

Curve	Scale	Shape	Position
Wear-in	$\alpha_1 > 0$	$0 < \beta_1 < 0.99$	$\gamma_1 = 0$
Useful life	$\alpha_2 > 0$	$\beta_2 = 1.0$	$\gamma_2 = 0$
Wear-out	$\alpha_3 > 0$	$\beta_3 > 1.0$	$\gamma_3 = 0$

III. Model Application Case

In this section the proposed method is applied to a sample of 14.067 distribution transformers. The in-service year is represented by the manufacturing date with a profile shown in Fig. 4. It gives an idea about the time when the sample was installed.

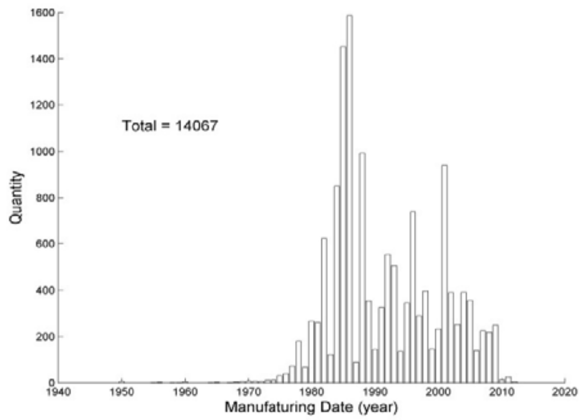

Fig. 4 Manufacturing date distribution

A. First Estimation of Probability Density Functions

The best probability density function parameter estimation for the raw data using a parametric Weibull model establishes

α_0 = 18.4 and β_0 = 2.0. Fig. 5 shows that the parametric function does not fit the data perfectly.

Two nonparametric methods were then used to compare the results. One method is the Kaplan-Meier discrete function, which represents the density function in a normalized histogram. The other one is the Kernel continuous density function estimation. Table II shows basic statistics for them.

The end-of-life Weibull curve is characterized for $\beta > 4$ implying old age and rapid wear out. A $1.0 < \beta < 4.0$ implies early wear out [17]-[19]. A $\beta_0 = 2.0$ indicates that the curve is not appropriate for end-of-life estimation.

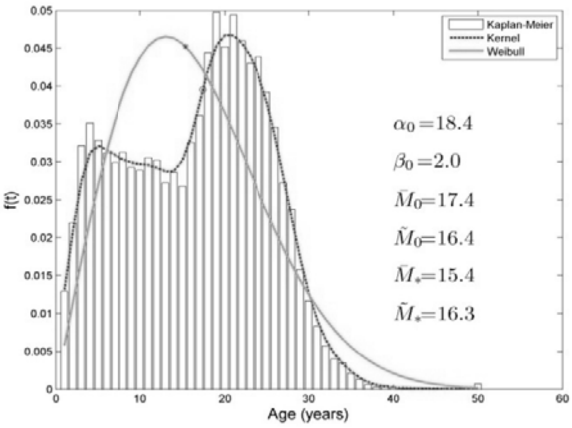

Fig. 5 Data fitting for Weibull and Kernel

B. Failure Distribution

The probability of failing up to time t (F(t)) is given by the cumulative distribution function (CDF) in (2). Using the first estimation parameters (α_0 and β_0) the function F(t) doesn't fit the nonparametric curves (Kaplan-Meier and Kernel).

$$F(\mathrm{t}) = 1 - e^{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}} \tag{2}$$

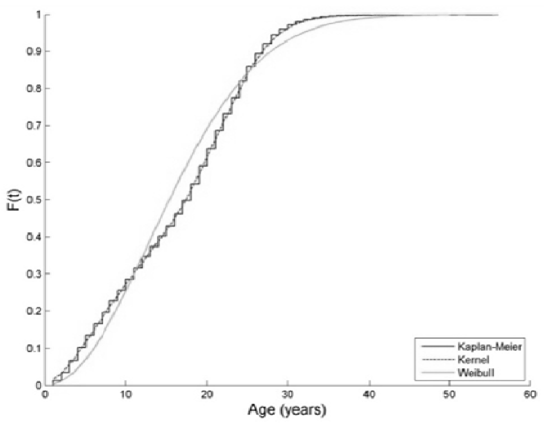

Fig. 6 Failure curve

We can see again in Fig. 6 that the Weibull function with the original parameters (α_0 and β_0) does not fit perfectly on the nonparametric curves. It shows that the first parametric estimated function is not appropriate to represent the data.

C. The Nonparametric Hazard Rate Function

In parametric models, the hazard rate h(t) or λ(t), is calculated from the ratio between the density function and the survival function (3). We propose that applying the same concept to the nonparametric function, a "nonparametric hazard rate" is obtained.

$$\lambda(\mathrm{t}) = \frac{\mathrm{f(t)}}{\mathrm{R(t)}} = \left(\frac{\beta}{\alpha}\right)\left(\frac{\mathrm{t}-\gamma}{\alpha}\right)^{\beta-1} \tag{3}$$

However, the hazard rate is a monotonic function, meaning that it decreases, increases or is constant. Also, once it increases in certain rate with age, the rate does not reduce. Regarding this concept, one might define a valid interval for the nonparametric rate.

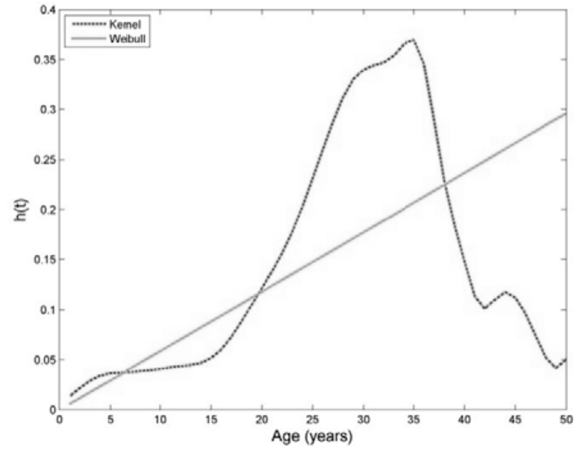

Fig. 7 Hazard rate curves

Fig. 7 shows the hazard rate for the first estimative parametric function and the nonparametric function.

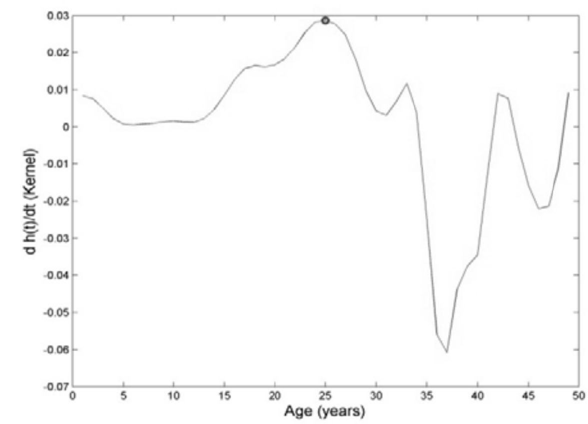

Fig. 8 Hazard rate differential curve and maximum value

A routine to define this valid interval comes with the calculation of the differential of the nonparametric hazard rate, taking the maximum value. It is illustrated in Fig. 8.

The interpretation of h'(t) is presented in [6], where there is a positive aging when h'(t) >0, no aging when h'(t) = 0 and a negative aging when h'(t) < 0. However, for the distribution assets it can be assumed that there is always a continuous positive aging in the wear-out region.

For this sample the maximum value of the differential function is located in time equals to 26 years, defining the model valid interval from 0 to 26 years.

D.Parametric to Nonparametric Fitting ("Bathtub Curve")

At this point it is known that the first parametric function has not fitted the nonparametric hazard rate well. A new fitting test was run to adjust the parameter of the three regions containing the decreasing rate, the constant rate and the increasing rate. Using the sum of parametric hazard rates to achieve the nonparametric model the fitting process was improved, as shown in Fig. 9.

Considering the point where the constant hazard rate is exceeded by the increasing hazard rate, the useful lifetime was calculated as 17.3 years. The hazard rate at this point is 0.039 failures per year. For the wear-out region a value of $\beta = 5.3$ is more appropriated for old age end-of-life behavior because it pushes the life to an old age with a rapidly IFR close to α.

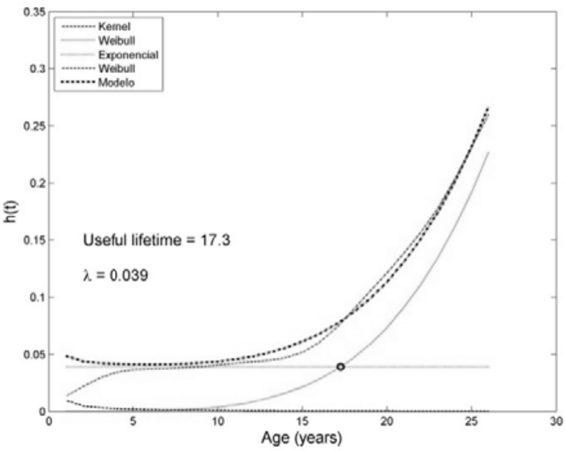

Fig. 9 Bathtub hazard rate curve

E. Second Estimation of Probability Density Functions

Once the fitting problem was solved, the parameters are known for each curve and a new plot of probability density function can show in Fig. 10 that there is information from three different failure modes in the data. The aging and the useful density functions have area equal to one, so the shapes do not look to fit the histogram data.

TABLE II
PROBABILITY DENSITY FUNCTIONS

Curve	Parameters	Median	Mean
Weibull	$\alpha_0 = 18.4$ $\beta_0 = 2.0$	16.3	15.4
Kernel	-	16.4	17.4

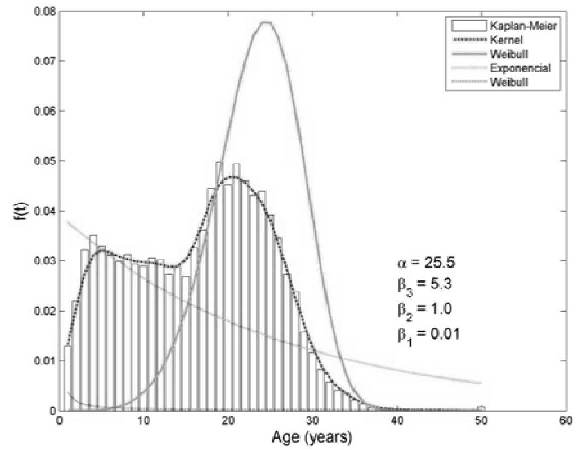

Fig. 10 Modeled Density functions curve

The parameters obtained are presented on Table III.

TABLE III
SECOND PARAMETER ESTIMATION FOR THE BATHTUB CURVE

Curve	Scale	Shape	Position
Wear-in	$\alpha_1 = 25.5$	$\beta_1 = 0.01$	$\gamma_1 = 0$
Useful life	$\alpha_2 = 25.5$	$\beta_2 = 1.0$	$\gamma_2 = 0$
Wear-out	$\alpha_3 = 25.5$	$\beta_3 > 5.3$	$\gamma_3 = 0$

IV. CONCLUSION

This paper has presented a new method to estimate the useful lifetime for distribution assets from the nonparametric approach, which allows the user to obtain important information about the hazard rate summarized on the bathtub curve. The method consists of a graphical fitting process to find the bathtub Weibull parameters. The assumption of same parameter α for the three distributions makes the MTTF and the characteristic life equal. However, modeling the parameters freely (5 parameters) would lead to a more precise hazard rate for random failures for some groups of distribution transformers.

Future researches on this topic will investigate the influence of different kinds of random failures identified on the utility database and apply the methodology to other assets.

ACKNOWLEDGMENT

The authors gratefully acknowledge the data contributions and comments from COPEL – Paraná State distribution utility and ABRADEE – Brazilian Distribution Utilities Association. On the PhD program the support from CAPES, UNIFEI and The University of Texas.

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