

A Constitutive Model for Time-Dependent Behavior of Clay

T. N. Mac, B. Shahbodaghkhan, N. Khalili

Abstract—A new elastic-viscoplastic (EVP) constitutive model is proposed for the analysis of time-dependent behavior of clay. The proposed model is based on the bounding surface plasticity and the concept of viscoplastic consistency framework to establish continuous transition from plasticity to rate dependent viscoplasticity. Unlike the overstress based models, this model will meet the consistency condition in formulating the constitutive equation for EVP model. The procedure of deriving the constitutive relationship is also presented. Simulation results and comparisons with experimental data are then presented to demonstrate the performance of the model.

Keywords—Bounding surface, consistency theory, constitutive model, viscosity.

I. INTRODUCTION

TIME-DEPENDENT behavior of geomaterials such as clay has widely been recognized experimentally [1], [2]. Many different classes of constitutive models have been introduced to capture the time-dependent viscous phenomena observed in soils [3]. Most of these models are based on empirical, rheological and general stress-strain-time concepts [4].

Elastic-viscoplastic (EVP) models have been developed because the elastoplastic models are not capable of predicting the time-dependent behavior of soils such as creep, stress relaxation and strain rate dependency. Most of the existing EVP models are based on the concept of Perzyna's overstress theory [5], [6] and the critical state framework, e.g. models developed by Adachi & Oka [7], Oka et al. [8], Kaliakin & Dafalias [9], [10], Kutter & Sathialingam [11] and Yin & Graham [12]-[15]. However, the main feature of the Perzyna-type models is that the rate-independent yield function used for describing the viscoplastic strain can become larger than zero, which is known as 'overstress'. If the external loading remains constant, the stresses return to the yield surface as a function of time. Another concern is that most of these models are complex and a number of parameters are required to be determined.

In this study, an alternative viscoplastic model is developed within the consistency viscoplastic framework using a rate-dependent yield function. Unlike the overstress based models, this model will meet the consistency condition in formulating the constitutive equation for EVP model. In addition, it will provide a convenient and easy way to apply model for numerical implementation. It will also allow smooth transition from elastoplasticity to viscoplasticity.

T. N. Mac, B. Shahbodaghkhan, and N. Khalili are with the School of Civil and Environmental Engineering, The University of New South Wales, NSW 2052, Australia (e-mail: t.mac@unsw.edu.au, b.shahbodagh@unsw.edu.au, n.khalili@unsw.edu.au).

The proposed model is based on the bounding surface plasticity [16] and the concept of viscoplastic consistency framework [17], [18] to establish continuous transition from plasticity to rate dependent viscoplasticity. The approach adopted in this study concentrates on constitutive equations with a new hardening parameter and hardening rule. Based on the formulation, a numerical program has been developed to simulate the rate-dependent behavior of soils. Simulation results and comparisons with experimental data of remolded Fukakusa clay are presented to demonstrate the application of the model.

II. A BOUNDING SURFACE VISCOPLASTIC MODEL

The model is developed using bounding surface plasticity [16] with a seamless transition from rate-independent plasticity to rate dependent viscoplasticity. The strain-rate contribution (viscosity) is implemented through a rate-dependent yield surface [17]. The yield function and the consistency condition for a rate-dependent material are written as functions of strain and strain rate parameters [18]. Similar to a traditional bounding surface plasticity model, the following essential ingredients are described: elastic properties, yield surface or bounding surface, flow rule for viscoplastic potential and a hardening rule.

In the proposed elastic-viscoplastic model, the total strain increment is decomposed into elastic part (e) and viscoplastic part (vp)

$$\delta \boldsymbol{\varepsilon} = \delta \boldsymbol{\varepsilon}^e + \delta \boldsymbol{\varepsilon}^{vp} \quad (1)$$

A. Elastic Properties

The elastic part $\delta \boldsymbol{\varepsilon}^e$ is expressed in relationship with the stress increment as

$$\delta \boldsymbol{\sigma} = \mathbf{D}^e \delta \boldsymbol{\varepsilon}^e \quad (2)$$

where $\delta \boldsymbol{\varepsilon}^e$ is the elastic strain increment, $\delta \boldsymbol{\sigma}$ is the stress increment, \mathbf{D}^e is the elastic stiffness matrix

$$\mathbf{D}^e = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \quad (3)$$

and K and G are bulk and shear moduli, respectively.

The moduli are then defined as

$$K = \frac{vp'}{\kappa} \quad (4)$$

$$G = \frac{3(1-2\nu)vp'}{2(1+\nu)\kappa} \quad (5)$$

where v is the specific volume ($v = 1 + e$), ν is the Poisson's ratio, p' is the mean effective stress and κ is the slope of the elastic unloading-reloading line in $(v: \ln p')$ plane.

In the bounding surface theory, it is common to define a purely elastic region as a region bounded by the loading surface. However, deformation of clays is not purely elastic [19]. Thus in this study, a purely elastic region is omitted such that all deformation is viscoplastic.

B. Viscoplastic Properties

The main components of the bounding surface viscoplastic model are:

- The loading surface
- The bounding surface
- The flow rule
- The hardening rule

In the present model, viscoplastic strain occurs when the stress state lies on or within the bounding surface. The image point is selected using a mapping rule such that the unit normal vectors to the loading surface and the bounding surface are the same.

1. Bounding Surface

The bounding surface has a tear drop shape and always encloses the loading surface, see Fig. 1. In the deviatoric and mean stress plane, the bounding surface is expressed as $F(\bar{p}', \bar{q}, \bar{p}'_c) = 0$. Stress conditions on the bounding surface are denoted using a superimposed bar throughout.

The function adopted for the bounding surface is in the form

$$F(\bar{p}', \bar{q}, \bar{p}'_c) = \left(\frac{\bar{q}}{M_{cs}\bar{p}'} \right)^N - \frac{\ln(\bar{p}'_c/\bar{p}')}{\ln R} = 0 \quad (6)$$

where M_{cs} is the slope of the critical state line (CSL) in the $(q \sim p')$ plane. M_{cs} may be one of two constants, depending on whether compressive ($\bar{q} > 0$) or extensive ($\bar{q} < 0$) loading is occurring.

The parameter \bar{p}'_c controls the size of F and is a function of viscoplastic volumetric strain ϵ_p^{vp} and viscoplastic volumetric strain rate $\dot{\epsilon}_p^{vp}$, the material constant R represents the ratio between \bar{p}'_c and the value of \bar{p}' at the intercept of F with the CSL in the $(q \sim p')$ plane, and the material constant N controls the curvature of the surface.

2. Loading Surface

The effective stress σ' is always located on the loading surface. For the first time loading, the loading surface is of the same shape with the bounding surface and homologous to the bounding surface about the origin of the $(q \sim p')$ plane. The function for the loading surface takes the form

$$f(p', q, p'_c) = \left(\frac{q}{M_{cs}p'} \right)^N - \frac{\ln(p'_c/p')}{\ln R} = 0 \quad (7)$$

where p'_c is the hardening parameter controlling the size of the loading surface and is a function of ϵ_p^{vp} and $\dot{\epsilon}_p^{vp}$.

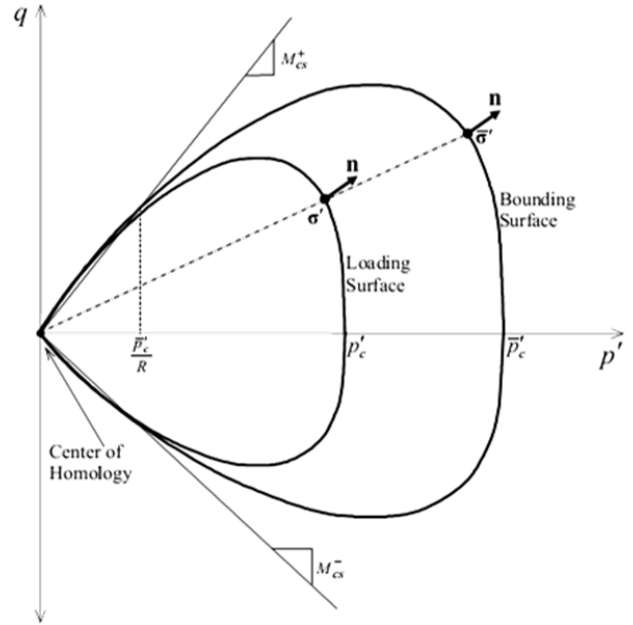


Fig. 1 Bounding surface, loading surface and mapping rule for the first time loading (after [16])

3. Image Point

The image point ($\bar{\sigma}'$) is selected using a radial mapping rule such that a straight line through the centre of homology and σ' intersects the bounding surface at $\bar{\sigma}'$ having the same unit normal vector as σ' on the loading surface [20].

The unit normal vector at the image point defining the direction of loading is given as

$$\mathbf{n} = \frac{\partial f / \partial \sigma'}{\|\partial f / \partial \sigma'\|} = \frac{\partial F / \partial \bar{\sigma}'}{\|\partial F / \partial \bar{\sigma}'\|} \quad (8)$$

Differentiating (8) with respect to \bar{p}' and \bar{q} , we obtain

$$\frac{\partial F}{\partial \bar{p}'} = -\frac{N}{\bar{p}'} \left(\frac{\bar{q}}{M_{cs}\bar{p}'} \right)^N + \frac{1}{\bar{p}' \ln R} = a \quad (9)$$

$$\frac{\partial F}{\partial \bar{q}} = N \left(\frac{1}{M_{cs}\bar{p}'} \right)^N \bar{q}^{N-1} = \frac{N}{M_{cs}\bar{p}'} \left(\frac{\bar{q}}{M_{cs}\bar{p}'} \right)^{N-1} = b \quad (10)$$

The components of $\mathbf{n} = [n_p, n_q]^T$ at σ' are defined as

$$n_p = \frac{a}{\sqrt{a^2 + b^2}} \quad (11)$$

$$n_q = \frac{b}{\sqrt{a^2 + b^2}} \quad (12)$$

For 3-D stress state, the vector $\partial F / \partial \bar{\sigma}'$ is written as

$$\frac{\partial F}{\partial \bar{\sigma}'} = \frac{\partial F}{\partial \bar{p}'} \frac{\partial \bar{p}'}{\partial \bar{\sigma}'} + \frac{\partial F}{\partial \bar{q}} \frac{\partial \bar{q}}{\partial \bar{\sigma}'} + \frac{\partial F}{\partial \bar{\theta}} \frac{\partial \bar{\theta}}{\partial \bar{\sigma}'} \quad (13)$$

where $\partial F / \partial \bar{p}'$, $\partial F / \partial \bar{q}$ and $\partial F / \partial \bar{\theta}$ are evaluated by differentiating the generalized form of (6) with respect to \bar{p}' , \bar{q} and $\bar{\theta}$ as

$$\frac{\partial F}{\partial \bar{p}'} = -\frac{N}{\bar{p}'} \left(\frac{\bar{q}}{M_{cs}(\bar{\theta})\bar{p}'} \right)^N + \frac{1}{\bar{p}' \ln R} \quad (14)$$

$$\frac{\partial F}{\partial \bar{q}} = \frac{N}{M_{cs}(\bar{\theta})\bar{p}'} \left(\frac{\bar{q}}{M_{cs}(\bar{\theta})\bar{p}'} \right)^{N-1} \quad (15)$$

$$\frac{\partial F}{\partial \bar{\theta}} = \frac{\partial F}{\partial M_{cs}(\bar{\theta})} \frac{\partial M_{cs}(\bar{\theta})}{\partial \bar{\theta}} = -\frac{3N}{4} \left(\frac{\bar{q}}{M_{cs}(\bar{\theta})\bar{p}'} \right)^N \left(\frac{(1-\alpha^4)\cos 3\bar{\theta}}{1+\alpha^4-(1-\alpha^4)\sin 3\bar{\theta}} \right) \quad (16)$$

where M_{cs} is a function of $\bar{\theta}$

$$M_{cs}(\bar{\theta}) = M_{max} \left(\frac{2\alpha^4}{1+\alpha^4-(1-\alpha^4)\sin 3\bar{\theta}} \right)^{1/4} \quad (17)$$

4. Flow Rule

The flow rule describes the inelastic deformation. The viscoplastic strain increment $\delta \boldsymbol{\varepsilon}^{vp}$ is expressed by the flow rule as

$$\delta \boldsymbol{\varepsilon}^{vp} = \delta \lambda \mathbf{m} = \delta \lambda \frac{\partial g}{\partial \bar{p}'} \quad (18)$$

where $\delta \lambda$ is the viscoplastic multiplier, \mathbf{m} is the direction of the viscoplastic flow, governed by the gradient of the viscoplastic potential and g is the viscoplastic potential defining the direction of viscoplastic strains.

The viscoplastic potential defines the direction of viscoplastic strain increments. It is the ratio between the incremental viscoplastic volumetric strain and the incremental viscoplastic shear strain

$$d = \frac{\delta \boldsymbol{\varepsilon}_p^{vp}}{\delta \boldsymbol{\varepsilon}_q^{vp}} = \bar{t} A \left(M_{cs} - \frac{q}{p'} \right) \quad (19)$$

where A is a material constant dependent on the mechanism and amount of energy dissipation, \bar{t} is the loading direction multiplier with $\bar{t} = +1$ for compression ($\bar{q} > 0$) and $\bar{t} = -1$ for extension ($\bar{q} < 0$).

The viscoplastic potential g is obtained by integrating (19) with respect to p' and q .

For $A = 1$:

$$g(p', q, p_0) = \bar{t} \left[q + M_{cs} p' \ln \left(\frac{p'}{p_0} \right) \right] \quad (20)$$

For $A \neq 1$:

$$g(p', q, p_0) = \bar{t} \left[q + \frac{A M_{cs} p'}{A-1} \left(\left(\frac{p'}{p_0} \right)^{A-1} - 1 \right) \right] \quad (21)$$

in which p_0 is a dummy variable controlling the size of the viscoplastic potential.

Vectors of viscoplastic flow (at stress point $\boldsymbol{\sigma}'$) $\mathbf{m} = [m_p, m_q]^T$ with its components are defined as

$$m_p = \frac{\partial g / \partial p'}{\|\partial g / \partial \boldsymbol{\sigma}'\|} = \frac{d}{\sqrt{1+d^2}} \quad (22)$$

$$m_q = \frac{\partial g / \partial q}{\|\partial g / \partial \boldsymbol{\sigma}'\|} = \frac{\bar{t}}{\sqrt{1+d^2}} \quad (23)$$

5. Hardening Rule

The hardening rule of conventional plasticity satisfies consistency condition. In this study, the consistent viscoplasticity follows the mathematical framework and strategies of classical plasticity formulations. That means the stress point is always on or inside the yield surface and viscoplastic deformation occurs when the stress point lies in the yield surface.

The hardening parameter \bar{p}'_c is a function of the viscoplastic strain tensor ($\boldsymbol{\varepsilon}^{vp}$) and the viscoplastic strain rate tensor ($\dot{\boldsymbol{\varepsilon}}^{vp}$). Consequently, it can be written as $\bar{p}'_c(\boldsymbol{\varepsilon}^{vp}, \dot{\boldsymbol{\varepsilon}}^{vp})$. Applying consistency condition to the yield surface, we have

$$\delta F = \left(\frac{\partial F}{\partial \boldsymbol{\sigma}'} \right)^T \delta \boldsymbol{\sigma}' + \frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}^{vp}} \frac{\partial g}{\partial \bar{p}'} \delta \lambda + \frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \dot{\boldsymbol{\varepsilon}}^{vp}} \frac{\partial g}{\partial \bar{p}'} \delta \dot{\lambda} \quad (24)$$

Set

$$h_b = -\frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}^{vp}} \frac{\partial g}{\partial \bar{p}'} \quad (25)$$

$$\xi_b = -\frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \dot{\boldsymbol{\varepsilon}}^{vp}} \frac{\partial g}{\partial \bar{p}'} \quad (26)$$

Substitute back to (24), we have the differential equation

$$\delta F = \mathbf{n}^T \delta \boldsymbol{\sigma}' - h_b \delta \lambda - \xi_b \delta \dot{\lambda} = 0 \quad (27)$$

Following the conventional approach in bounding surface plasticity, the viscoplastic strain hardening modulus h is split into two components:

$$h = h_b + h_f \quad (28)$$

Similarly, the viscoplastic strain rate hardening modulus ξ is defined as

$$\xi = \xi_b + \xi_f \quad (29)$$

where h_b and ξ_b are the viscoplastic strain modulus and the viscoplastic strain rate modulus at stress point $\boldsymbol{\sigma}'$ on bounding surface, respectively; h_f and ξ_f are some arbitrary strain modulus and strain rate modulus at $\boldsymbol{\sigma}'$, defines as a function of the distance between $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}'$.

The strain hardening moduli are defined by applying consistency conditions to bounding surface

$$h_b = -\frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} \frac{\mathbf{m}_p}{\|\partial F / \partial \boldsymbol{\sigma}'\|} \quad (30)$$

$$h_f = \bar{t} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} \frac{p'}{\bar{p}'_c} \left[\frac{\bar{p}'_c}{p'_c} - 1 \right] k_m (\eta_p - \eta) \quad (31)$$

where \bar{p}'_c and p'_c define the sizes of the bounding surface and loading surface, respectively.

h_f is defined such that it is zero on the bounding surface and infinity at the point of stress reversal.

Each of components of differential values is expressed as follow

$$\frac{\partial F}{\partial \bar{p}'_c} = -\frac{1}{\bar{p}'_c \ln R} \quad (32)$$

$$\frac{\partial \bar{p}'_c}{\partial \varepsilon_p^{vp}} = \frac{v \bar{p}'_c}{\lambda - \kappa} \quad (33)$$

Substitute back to (31), we obtain

$$h_b = \frac{v}{(\lambda - \kappa) \ln R} \frac{m_p}{\|\partial F / \partial \bar{\sigma}'\|} \quad (34)$$

h_f is an arbitrary modulus at σ' , defined as a function of the distance between σ' and $\bar{\sigma}'$. It is assumed in the following form

$$h_f = \bar{\tau} \frac{\partial \bar{p}'_c}{\partial \varepsilon_p^{vp}} \frac{p'}{\bar{p}'_c} \left[\frac{\bar{p}'_c}{p'} - 1 \right] k_m (\eta_p - \eta) \quad (35)$$

where \bar{p}'_c and p'_c define the sizes of bounding surface and loading surface, η_p is the slope of the peak strength line in ($q \sim p'$) plane, k_m is material parameter controlling the steepness of the response in the ($q \sim \varepsilon_q$) plane, k is material parameter to define the peak strength line.

Similarly, the strain rate hardening moduli are defined as follow

$$\xi_b = -\frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \varepsilon_p^{vp}} \frac{m_p}{\|\partial F / \partial \bar{\sigma}'\|} \quad (36)$$

$$\xi_f = \bar{\tau} \frac{\partial \bar{p}'_c}{\partial \varepsilon_p^{vp}} \frac{p'}{\bar{p}'_c} \left[\frac{\bar{p}'_c}{p'} - 1 \right] k_m (\eta_p - \eta) \quad (37)$$

C. Stress-Strain Relationship

The numerical solution for the constitutive equations requires integration of the nonlinear differential equation (27). The finite difference approximation of the rate of change of the viscoplastic multiplier $\delta \lambda$ can be written as

$$\delta \lambda = \frac{\delta \lambda^{t+\Delta t} - \delta \lambda^t}{\Delta t} \quad (38)$$

in which, Δt is the time increment, $\delta \lambda^t$ and $\delta \lambda^{t+\Delta t}$ are the viscoplastic multipliers at the previous and current time steps, respectively.

From (1), (2) and (18), we have

$$\delta \sigma = D^e \delta \varepsilon^e = D^e (\delta \varepsilon - \delta \varepsilon^{vp}) = D^e (\delta \varepsilon - \delta \lambda \frac{\partial g}{\partial \sigma}) \quad (39)$$

Substitute (38) and (39) into (27) yields

$$\delta F = \frac{\partial f^T}{\partial \sigma} D^e \left(\delta \varepsilon - \delta \lambda^{t+\Delta t} \frac{\partial g}{\partial \sigma} \right) - h \delta \lambda^{t+\Delta t} - \xi \frac{\delta \lambda^{t+\Delta t} - \delta \lambda^t}{\Delta t} = 0 \quad (40)$$

Hence, the solution for (40) can be expressed as

$$\delta \lambda^{t+\Delta t} = \frac{\frac{\partial f^T}{\partial \sigma} D^e \delta \varepsilon + \frac{\xi}{\Delta t} \delta \lambda^t}{\frac{\partial f^T}{\partial \sigma} D^e \frac{\partial g}{\partial \sigma} + h + \frac{\xi}{\Delta t}} \quad (41)$$

or equivalently,

$$\delta \lambda^{t+\Delta t} = \frac{n^T D^e \delta \varepsilon + \frac{\xi}{\Delta t} \delta \lambda^t}{n^T D^e m + h + \frac{\xi}{\Delta t}} \quad (42)$$

Substitute (42) into (39) and after some algebra, we have

$$\delta \sigma = \left[D^e - \frac{D^e m n^T D^e}{n^T D^e m + h + \frac{\xi}{\Delta t}} \right] \delta \varepsilon - \frac{D^e m \frac{\xi}{\Delta t} \delta \lambda^t}{n^T D^e m + h + \frac{\xi}{\Delta t}} \quad (43)$$

Defining

$$D^{vp} = D^e - \frac{D^e m n^T D^e}{n^T D^e m + h + \frac{\xi}{\Delta t}} \quad (44)$$

and

$$\sigma^{vp} = \frac{D^e m \frac{\xi}{\Delta t} \delta \lambda^t}{n^T D^e m + h + \frac{\xi}{\Delta t}} \quad (45)$$

The incremental elasto-viscoplastic stress-strain relation becomes

$$\delta \sigma = D^{vp} \delta \varepsilon - \sigma^{vp} \quad (46)$$

Equation (46) is expressing the incremental stress-strain relationship for the new proposed EVP model.

III. VALIDATION RESULTS

To demonstrate the performance of the proposed model in predicting time-dependent behavior of soils, the simulation results are compared with experimental data from the literature. Remoulded Fukakusa clay is analyzed. The physical properties and experimental data of remoulded Fukakusa clay were published in 1982 by Adachi & Oka [7]. The test specimens were preconsolidated at 49 kN/m² and isotropically consolidated under 392 kN/m² of effective cell pressure. The material constants used in the simulations were: $\kappa = 0.02$, $G = 36300$ kPa, $M_{CS} = 1.5$, $\lambda_0 = 0.1$, $N = 2.3$, $N_{BS} = 1.45$, $R_{BS} = 1.85$, $k = 2.0$, $k_d = 1.0$, $A = 1.0$, and $k_m = 0.0$. The initial conditions were $p' = 392$ kN/m² and $e = 0.72$. The strain rate hardening parameter, ξ , is assumed to be a linear function of the strain hardening modulus, h . By using the proposed model, simulation results for constant strain rate and creep tests are presented.

A. Strain-Rate Effects

The tests were carried out by using two different constant strain rates, 0.0835% and 0.00817%. The stress-strain curves and effective stress paths of 1 day consolidated samples under strain-rate controlled undrained triaxial conditions are shown in Figs. 2 and 3, respectively.

The strain rate hardening parameters, ξ , are 1000 and 200 times more than the strain hardening modulus, h , for constant strain rates of 0.0835% and 0.00817%, respectively. It can be seen from Fig. 2 that the maximum deviatoric stress increases with increase in the rate of straining. Fig. 3 shows that the lower the strain rate, the flatter the undrained stress path. It

can be observed that there is a good agreement between the experimental and numerical results. The results show increase in stiffness and strength of clay with increase in strain rate.

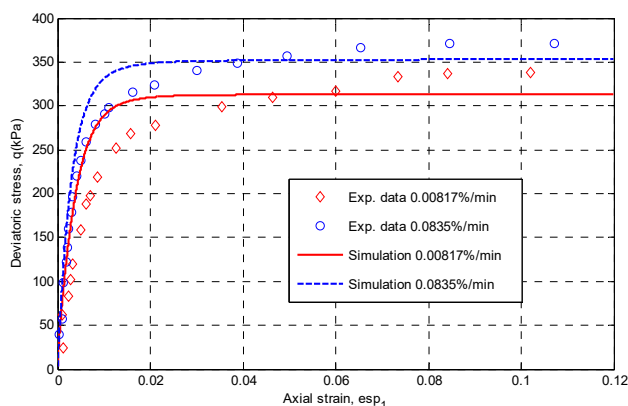


Fig. 2 Effect of strain rate on stress-strain response

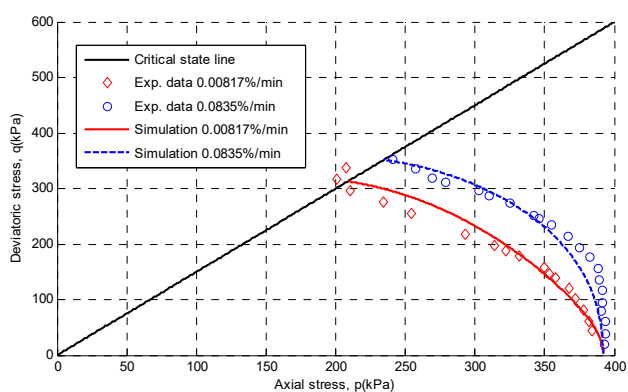


Fig. 3 Effect of strain rate on undrained stress path

B. Undrained Creep Test

The undrained creep behavior of Fukakusa clay is demonstrated in Fig. 4. First, specimens were isotropically consolidated to 392 kN/m². Then undrained creep tests were performed after sheared up to prescribed deviatoric stress levels of 0.2, 0.3, 0.4, 0.5 and 0.6 times of the value of $p' = 392$ kN/m². All creep tests were conducted for about 10000 minutes at each stress level.

The strain rate hardening parameters, ξ , are 800, 180, 100, 72 and 55 times more than the strain hardening modulus, h , for the creep tests with creep loadings of 0.2, 0.3, 0.4, 0.5 and 0.6 times of the ultimate value, respectively. Comparison between the experimental data and model simulations shows good agreement between experimental results and simulation results, demonstrating applicability of the proposed bounding surface viscoplastic model to describe the creep behavior of clay.

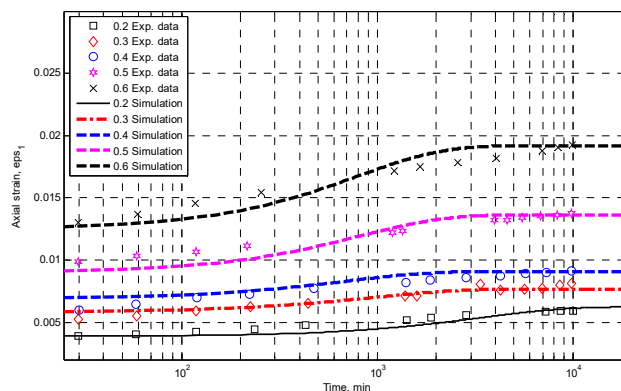


Fig. 4 Model simulation and experimental data of undrained creep tests on Remoulded Fukakusa clay with $p_0 = 392$ kN/m² and creep loadings of 0.2, 0.3, 0.4, 0.5 and 0.6 of $p' = 392$ kN/m²

IV. CONCLUSION

A bounding surface viscoplastic constitutive model is proposed for the analysis of time-dependent behavior of clay. The application of the model is demonstrated using several numerical examples. It is shown that the constitutive model proposed can successfully predict the time-dependent behavior of clay including strain rate dependency and undrained creep.

REFERENCES

- [1] K. Arulanandan, C. K. Shen, and R. B. Young, "Undrained creep behaviour of a coastal organic silty clay". *Geotechnique*. Vol. 21, no. 4, p. 359-375, 1971.
- [2] S. Leroueil, M. Kabbaj, F. Tavenas, and R. Bouchard, "Stress-strain-strain rate relation for the compressibility of sensitive natural clays". *Geotechnique*. Vol. 35, no. 2, p. 159-180, 1985.
- [3] A. Augustesen, M. Liingaard, and P. V. Lade, "Evaluation of time-dependent behavior of soils". *International Journal of Geomechanics*. Vol. 4, no. 3, p. 137-156, 2004.
- [4] M. Liingaard, A. Augustesen, and P. V. Lade, "Characterization of models for time-dependent behavior of soils". *International Journal of Geomechanics*. Vol. 4, no. 3, p. 157-177, 2004.
- [5] P. Perzyna, "On the constitutive equations for work-hardening and rate sensitive plastic materials". *Bulletin de l'Academie Polonaise des Sciences, Serie des Sciences Techniques*. Vol. 12, no. 4, p. 199-206, 1964.
- [6] P. Perzyna, *Fundamental problems in viscoplasticity*, in *Advances in Applied Mechanics*. 1966, Academic Press Inc. p. 243-377.
- [7] T. Adachi and F. Oka, "Constitutive equations for normally consolidated clay based on elasto-viscoplasticity". *Soils and Foundations*. Vol. 22, no. 4, p. 57-70, 1982.
- [8] F. Oka, T. Adachi, and Y. Okano, "Two-dimensional consolidation analysis using an elasto-viscoplastic constitutive equation". *International Journal for Numerical and Analytical Methods in Geomechanics*. Vol. 10, no. 1, p. 1-16, 1986.
- [9] V. N. Kaliakin and Y. F. Dafalias, "Theoretical aspects of the elastoplastic-viscoplastic bounding surface model for cohesive soils". *Soils and Foundations*. Vol. 30, no. 3, p. 11-24, 1990.
- [10] V. N. Kaliakin and Y. F. Dafalias, "Verification of the elastoplastic-viscoplastic bounding surface model for cohesive soils". *Soils and Foundations*. Vol. 30, no. 3, p. 25-36, 1990.
- [11] B. L. Kutter and N. Sathialingam, "Elastic - viscoplastic modelling of the rate-dependent behaviour of clays". *Geotechnique*. Vol. 42, no. 3, p. 427-441, 1992.
- [12] J. H. Yin and J. Graham, "Viscous-elastic-plastic modelling of one-dimensional time-dependent behaviour of clays". *Canadian geotechnical journal*. Vol. 26, no. 2, p. 199-209, 1989.
- [13] J.-H. Yin and J. Graham, "Equivalent times and one-dimensional elastic viscoplastic modelling of time-dependent stress-strain behaviour of clays". *Canadian Geotechnical Journal*. Vol. 31, no. 1, p. 42-52, 1994.

- [14] J.-H. Yin and J. Graham, "Elastic viscoplastic modelling of the time-dependent stress-strain behaviour of soils". *Canadian Geotechnical Journal*. Vol. 36, no. 4, p. 736-745, 1999.
- [15] J.-H. Yin, J.-G. Zhu, and J. Graham, "A new elastic viscoplastic model for time-dependent behaviour of normally and overconsolidated clays: Theory and verification". *Canadian Geotechnical Journal*. Vol. 39, no. 1, p. 157-173, 2002.
- [16] N. Khalili, M. A. Habte, and S. Valliappan, "A bounding surface plasticity model for cyclic loading of granular soils". *International Journal for Numerical Methods in Engineering*. Vol. 63, no. 14, p. 1939-1960, 2005.
- [17] W. M. Wang, L. J. Sluys, and R. de Borst, "Viscoplasticity for instabilities due to strain softening and strain-rate softening". *International Journal for Numerical Methods in Engineering*. Vol. 40, no. 20, p. 3839-64, 1997.
- [18] A. Carosio, K. Willam, and G. Etse, "On the consistency of viscoplastic formulations". *International Journal of Solids and Structures*. Vol. 37, no. 48, p. 7349-7369, 2000.
- [19] D. M. Wood, *Soil behaviour and critical state soil mechanics* 1990: Cambridge England ; : Cambridge University Press, New York 462.
- [20] A. R. Russell and N. Khalili, "A bounding surface plasticity model for sands exhibiting particle crushing". *Canadian Geotechnical Journal*. Vol. 41, no. 6, p. 1179-1192, 2004.