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Maximum Induced Subgraph of an Augmented Cube

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***Abstract*—**Let $max_{\xi_G}(m)$ denote the maximum number of edges in a subgraph of graph G induced by m nodes. The n-dimensional augmented cube, denoted as AQ_n, a variation of the hypercube, possesses some properties superior to those of the hypercube. We study the cases when G is the augmented cube AQ_n. In this paper, we show that $max_{\xi_{AQ_n}}(m) = \sum_{i=0}^{r}(p_i + 2i - \frac{1}{2})2^{p_i}$, where $p_0 > p_1 > \cdots > p_r$ are nonnegative integers defined by $m = \sum_{i=0}^{r} 2^{p_i}$ and $m \geq 2$. We then apply this formula to find the bisection width of AQ_n.

***Keywords*—**Interconnection network, Augmented cube, Induced subgraph, Bisection width.

I. Introduction

THEtopology of an interconnection network is conveniently represented by an undirected simple graph $G = (V, E)$, where $V(G)$ and $E(G)$ is the vertex set and the edge set of G, respectively. For graph terminology and notation not defined here we refer the reader to [8]. There are a lot of interconnection network topologies proposed in literature [4]. Among these topologies, the n-dimensional hypercube, denoted by Q_n, is a popular one. Many variants of the hypercube have been proposed. The augmented cube, proposed by Choudum and Sunitha [3], is one of such variations. An n-dimensional augmented cube AQ_n can be formed as an extension of Q_n by adding some links. For any positive integer n, AQ_n is a vertex transitive, $(2n-1)$-regular, and $(2n-1)$-connected graph with 2^n vertices. AQ_n retains all favorable properties of Q_n since $Q_n \subset AQ_n$. Moreover, AQ_n possesses some embedding properties that Q_n does not. Previous works relating to the augmented cube can be found in [1], [2], [5], [6], [7], [9].

Let $max_{\xi_G}(m)$ denote the maximum number of edges in a subgraph of graph G induced by m nodes. Determining $max_{\xi_G}(m)$ for typical graph G not only is interesting in its own right, but the result has applications in the evaluation of bandwidth and fault tolerant of G [11]. Abdel-Ghaffar [10] solved this problem for hypercube and Yang et al. [12] solved it for recursive circulant graph $G(2^n, 4)$ which is one of various of hypercubes. In this paper, we show that $max_{\xi_{AQ_n}}(m) = \sum_{i=0}^{r}(p_i + 2i - \frac{1}{2})2^{p_i}$, where $p_0 > p_1 > \cdots > p_r$ are nonnegative integers defined by $m = \sum_{i=0}^{r} 2^{p_i}$ and $m \geq 2$. We then apply this formula to find the bisection width of AQ_n.

The rest of this paper is organized as follows: In Section II, provides formal definition of AQ_n. A useful function is given and study its properties in Section III. By exploiting these properties, we show $max_{\xi_{AQ_n}}(m) = \sum_{i=0}^{r}(p_i + 2i - \frac{1}{2})2^{p_i}$ in Section IV. Finally, the formula is applied to determine the bisection width of AQ_n in Section V.

II. Preliminaries

Let $G = (V, E)$ be a graph, and $V(G)$ and $E(G)$ denote vertex set and edge set of graph G, respectively. For $U \subseteq V(G)$, the subgraph of G induced by U, denoted by $G[U]$, is a graph with vertex set U and all the edges of G with both vertices in U. An*m-induced subgraph* of a graph is one that is induced by m vertices. A *maximum m-inducedsubgraph* of a graph is one that has the maximum number of edges. Let $max_{\xi_G}(m)$ denote the maximum number of edges in an m-induced subgraph of graph G. Let $\xi(U)$ denote the number of edges of $G[U]$. For a pair of disjoint vertex subsets U_1 and U_2 of graph G, let $\xi(U_1, U_2)$ denote the number of edges joining U_1 and U_2.

Let $n \geq 1$ be an integer. The graph of the n-dimensional augmented cube [3], denoted by AQ_n has 2^n vertices, each labeled by an n-bit binary string $V(AQ_n) = \{u_1u_2\ldots u_n \mid u_i \in \{0,1\}\}$. AQ_1 is the graph K_2 with vertex set $\{0,1\}$. For $n \geq 2$, AQ_n can be recursively constructed by two copies of AQ_{n-1}, denoted by AQ_{n-1}^0 and AQ_{n-1}^1 and by adding 2^n edge between AQ_{n-1}^0 and AQ_{n-1}^1 as follows:

Let $V(AQ_{n-1}^0) = \{(0u_2u_3\ldots u_n) \mid u_i = 0$ or 1 for $2 \leq i \leq n\}$ and $V(AQ_{n-1}^1) = \{(1v_2v_3\ldots v_n) \mid u_i = 0$ or 1 for $2 \leq i \leq n\}$. A vertex $u = (0u_2u_3\ldots u_n)$ of AQ_{n-1}^0 is joined to a vertex

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$v=(1v_2v_3\dots v_n)$ of AQ^1_{n-1} if and only if either (i) $u_i=v_i$ for $2\le i\le n$; in this case, (u,v) is called a hypercube edge, or (ii) $u_i=\bar{v}_i$ for $2\le i\le n$; in this case, (u,v) is called a complement edge.

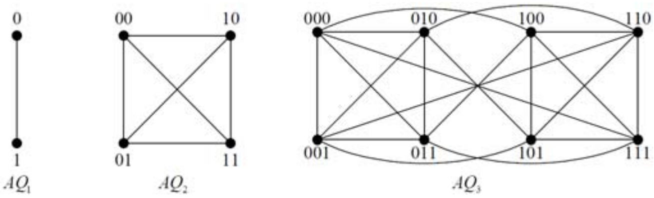

Fig.1 The augmented cubes: AQ_1, AQ_2, and AQ_3

The augmented cubes AQ_1, AQ_2, and AQ_3 are illustrated in Fig. 1. It is proved in [3] that AQ_n is a vertex transitive, $(2n-1)$ -regular, and $(2n-1)$ -connected graph with 2^n vertices for any positive integer n.

Any positive integer m can be uniquely represented by $m=\sum_{i=0}^{r}2^{p_i}$, where $p_0>p_1>\dots>p_r\ge 0$. We define a useful function

$$f(m)=\begin{cases}0 & : m\le 1\\ \sum_{i=0}^{r}(p_i+2i-\frac{1}{2})2^{p_i} & : m\ge 2\end{cases}$$

As an example, for $m=148=2^7+2^4+2^2$, we have $f(148)=(7+0-\frac{1}{2})2^7+(4+2-\frac{1}{2})2^4+(2+4-\frac{1}{2})2^2=942$

Theorem 1 *For any* $n\ge 1$ *and* $0<m\le 2^n$, *we have* $max_{\xi_{AQ_n}}(m)=f(m)$.

We drive several properties of the function $f(m)$ which are used to prove Theorem 1 in following sections and also give an explicit set U of vertices such that $\xi(U)=g(m)$.

III. Properties of $f(m)$

For a positive integer m, we define $l(m)=\lfloor log_2 m\rfloor$ and $m'=m-2^{l(m)}$.Obviously, $2^{l(m)}\le m<2^{l(m)+1}$ and $0\le m'<\frac{m}{2}$.

Proposition 1 *Let* m *be a positive. Then,* $f(m)=f(2^{l(m)})+f(m')+2m'$

Proof. We may write $m=2^{p_0}+2^{p_1}+\dots+2^{p_r}$ for some integer $r\ge 0$ and $p_0>p_1>\dots>p_r\ge 0$. Clearly, $l(m)=p_0$. From the definition of $f(m)$, $f(m)=(2l(m)-1)2^{l(m)-1}+\sum_{i=1}^{r}(p_i+2i-\frac{1}{2})2^{p_i}$.

Since $m'=2^{p_1}+2^{p_2}+\dots+2^{p_r}$, we also have $f(m')=\sum_{i=1}^{r}[p_i+2(i-1)-\frac{1}{2}]2^{p_i}$.

We conclude from the above that $f(m)=(2l(m)-1)2^{l(m)-1}+f(m')+\sum_{i=1}^{r}2\times 2^{p_i}=f(2^{l(m)})+f(m')+2m'$ because $f(2^{l(m)})=(2l(m)-1)2^{l(m)-1}$.

Proposition 2 *For any positive integers* m_1 *and* m_2, *we have* $f(m_1+m_2)\ge f(m_1)+f(m_2)+2min\{m_1,m_2\}$.

Proof. Clearly equality holds for $m_1=1$ or $m_2=1$. The proof is by induction on m_1+m_2. Without loss of generality, we may assume that $m_1\ge m_2\ge 2$. In particular, we want to prove that $f(m_1+m_2)\ge f(m_1)+f(m_2)+2m_2$, where the induction hypothesis implies that

$$f(m_1'+m_2)\ge f(m_1')+f(m_2)+2min\{m_1',m_2\} \quad (1)$$

$$f(m_1'+m_2')\ge f(m_1')+f(m_2')+2min\{m_1',m_2'\} \quad (2)$$

Notice that $2^{l(m_1)}\le m_1\le m_1+m_2\le 2m_1<2^{l(m_1)+2}$ and, in particular, $l(m_1+m_2)$ equals either $l(m_1)$ or $l(m_1)+1$. We consider all possible cases:

Case 1: $l(m_1+m_2)=l(m_1)$

In this case, $(m_1+m_2)'=m_1+m_2-2^{l(m_1+m_2)}=m_1+m_2-2^{l(m_1)}=m_1'+m_2$. Proposition 1 gives $f(m_1)=(2l(m_1)-1)2^{l(m_1)-1}+f(m_1')+2m_1'$ and

$$f(m_1+m_2)=(2l(m_1+m_2)-1)2^{l(m_1+m_2)-1}+f((m_1+m_2)')+2(m_1+m_2)'$$
$$=(2l(m_1)-1)2^{l(m_1)-1}+f(m_1'+m_2)+2(m_1'+m_2)$$

Hence,

$$f(m_1+m_2)=f(m_1)-f(m_1')+f(m_1'+m_2)+2m_2$$
$$\ge f(m_1)+f(m_2)+2min\{m_1',m_2\}+2m_2$$
$$\ge f(m_1)+f(m_2)+2m_2$$

, where the first inequality follows from (1).

Case 2: $l(m_1+m_2)=l(m_1)+1$ and $l(m_1)=l(m_2)$

In this case, $(m_1+m_2)'=(m_1+m_2)-2^{l(m_1+m_2)}=m_1+m_2-2^{l(m_1)+1}$ $=m_1-2^{l(m_1)}+m_2-2^{l(m_2)}=m_1'+m_2'$. Proposition 1 gives $f(m_1)=(2l(m_1)-1)2^{l(m_1)-1}+f(m_1')+2m_1'$, $f(m_2)=(2l(m_2)-1)2^{l(m_2)-1}+f(m_2')+2m_2'$ and

$$f(m_1+m_2)=(2l(m_1+m_2)-1)2^{l(m_1+m_2)-1}+f((m_1+m_2)')+2(m_1+m_2)'$$
$$=(2l(m_1)+1)2^{l(m_1)}+f(m_1'+m_2')+2m_1'+2m_2'.$$

Since $l(m_1)=l(m_2)$ and $m_1\ge m_2\ge 2$ implies $m_1'\ge m_2'\ge 0$, we have

$$f(m_1+m_2)=f(m_1)+f(m_2)+2^{l(m_1)+1}+f(m_1'+m_2')-f(m_1')-f(m_2'),$$
$$\ge f(m_1)+f(m_2)+2^{l(m_1)+1}+2m_2'=f(m_1)+f(m_2)+2m_2$$

where the inequality follows from (2).

Case 3: $l(m_1+m_2)=l(m_1)+1$ and $l(m_1)>l(m_2)$

In this case, $(m_1+m_2)'=(m_1+m_2)-2^{l(m_1+m_2)}=m_1+m_2-2^{l(m_1)+1}$ $=m_1-2^{l(m_1)}+m_2-2^{l(m_1)}=m_1'+m_2-2^{l(m_1)}$. Furthermore, as $2^{l(m_1)+1}=2^{l(m_1+m_2)}\le m_1+m_2<2^{l(m_1)+1}+2^{l(m_2)+1}\le 2^{l(m_1)+1}+2^{l(m_1)}$, we get $2^{l(m_1)}\le m_1+m_2-2^{l(m_1)}<2^{l(m_1)+1}$.

Since $m_1^{'} + m_2 = m_1 + m_2 - 2^{l(m_1)}$, we deduce that $l(m_1^{'} + m_2) = l(m_1)$ and $(m_1^{'} + m_2)^{'} = (m_1^{'} + m_2) - 2^{l(m_1^{'}+m_2)} = m_1^{'} + m_2 - 2^{l(m_1)}$.

Proposition 1 gives $f(m_1) = (2l(m_1)-1)2^{l(m_1)-1} + f(m_1^{'}) + 2m_1^{'}$,

$$f(m_1+m_2) = (2l(m_1+m_2)-1)2^{l(m_1+m_2)-1} + f((m_1+m_2)^{'}) + 2(m_1+m_2)^{'} = (2l(m_1)+1)2^{l(m_1)} + f(m_1^{'}+m_2-2^{l(m_1)}) + 2m_1^{'} + 2m_2 - 2^{l(m_1)+1},$$

and

$$f(m_1^{'}+m_2) = (2l(m_1^{'}+m_2)-1)2^{l(m_1^{'}+m_2)-1} + f((m_1^{'}+m_2)^{'}) + 2(m_1^{'}+m_2)^{'} = (2l(m_1)-1)2^{l(m_1)-1} + f(m_1^{'}+m_2-2^{l(m_1)}) + 2m_1^{'} + 2m_2 - 2^{l(m_1)+1}.$$

The above expressions for $f(m_1)$, $f(m_1+m_2)$, and $f(m_1^{'}+m_2)$ yield

$$\begin{aligned} f(m_1+m_2) &= f(m_1^{'}+m_2) + (2l(m_1)+3)2^{l(m_1)-1} \\ &= f(m_1) + f(m_1^{'}+m_2) - f(m_1^{'}) - 2m_1^{'} + 2^{l(m_1)+1}, \\ &\geq f(m_1) + f(m_2) + 2min\{m_1^{'}, m_2\} - 2m_1^{'} + 2^{l(m_1)+1} \\ &= f(m_1) + f(m_2) + 2min\{2^{l(m_1)}, m_2 - m_1^{'} + 2^{l(m_1)}\} \end{aligned}$$

where the inequality follows from (1). Since $m_1^{'} < m_1/2 < 2^{l(m_1)}$ and $m_2 < 2^{l(m_2)+1} \leq 2^{l(m_1)}$, we have $min\{2^{l(m_1)}, m_2 - m_1^{'} + 2^{l(m_1)}\} \geq min\{2^{l(m_1)}, m_2\} = m_2$.

Therefore, $f(m_1+m_2) \geq f(m_1) + f(m_2) + 2min\{m_1, m_2\}$.

IV. Proof of Theorem 1

A partition of a set S is a collection of disjoint subsets of S whose union equals S. Then the following lemma is obviously.

Lemma 1 *[12] Let U be a vertex subset of graph G. Let $\{U_0, U_1, \ldots, U_k\}$ be a partition of U. Then* $\xi(U) = \sum_{i=0}^{k}\xi(U_i) + \sum_{0\leq i<j\leq k}\xi(U_i, U_j)$.

Let U be a set of vertices on the AQ_n, let $U^{(a)} = U \cap V(AQ_{n-1}^{a})$ where $a = 0$ or 1. We have the following observation.

Lemma 2 *For a set U of vertices on AQ_n, $n > 1$, we have* $\xi(U) \leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\}$.

Proof. Since $\{U^{(0)}, U^{(1)}\}$ is a partition of U, by Lemma 1, $\xi(U) = \xi(U^{(0)}) + \xi(U^{(1)}) + |\xi(U^{(0)}, U^{(1)})|$. Without loss of generality, we may assume that $|U^{(0)}| \leq |U^{(1)}|$. One can observe that $U^{(0)}$ and $U^{(1)}$ are vertex subsets of AQ_{n-1}^{0} and AQ_{n-1}^{1} respectively. The proof is divided into two parts as follows.

Case 1: $|U^{(0)}| = 0$.

This implies $U = U^{(1)}$. It is obvious that $\xi(U^{(0)}) = 0$ and $min\{|U^{(0)}|, |U^{(1)}|\} = 0$. Thus $\xi(U) \leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\}$.

Case 2: $|U^{(0)}| \neq 0$.

By definition, every vertex of AQ_{n-1}^{0} connects to exactly two vertices of AQ_{n-1}^{1}. Hence, for any vertex $u \in U^{(0)}$, at most two vertices in $U^{(1)}$ are adjacent to u. Therefore, $\xi(U^{(0)}, U^{(1)}) \leq 2|U^{(0)}|$. As a result, $\xi(U) \leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\}$.

Lemma 3 *For any integer $n \geq 1$ and $0 \leq m \leq 2^n$, we have* $max_{\xi_{AQ_n}}(m) \leq f(m)$.

Proof. It suffices to show that $\xi(U) \leq f(m)$ for every set $U \in V(AQ_n)$. The proof is induction on n. It is obviously true for $n = 1,2$. Suppose the claim is true for $n = k$. Let U be an arbitrary set of m vertices in AQ_n. Thus $\{U^{(0)}, U^{(1)}\}$ is a partition of U, and $U^{(0)} \subseteq V(AQ_{n-1}^{0})$ and $U^{(1)} \subseteq V(AQ_{n-1}^{1})$. By Lemma 2, the induction hypothesis, and Proposition 2, we have

$$\begin{aligned} \xi(U) &\leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\} \\ &\leq f(|U^{(0)}|) + f(|U^{(1)}|) + 2min\{|U^{(0)}|, |U^{(1)}|\} \\ &\leq f(|U^{(0)}| + |U^{(1)}|) \\ &= f(m). \end{aligned}$$

Thus the lemma is proved.

Next, we give for any integer $n \geq 1$ and $0 \leq m \leq 2^n$, a set, denoted by $U_{m,n}$, of m vertices on the AQ_n for which $\xi(U_{m,n}) = f(m)$. The set $U_{m,n}$ is defined by

$U_{m,n} = \{(s_1 s_2 \cdots s_n) \in V(AQ_n) \mid \sum_{i=1}^{n} s_i 2^{i-1} < m\}$, i.e., $U_{m,n}$ consists of all vectors that are binary expansions of nonnegative integers less than m.

Lemma 4 *For any integer $n \geq 1$ and $0 \leq m \leq 2^n$, we have* $\xi(U_{m,n}) = f(m)$.

Proof. The proof is induction on n. Clearly the statement holds for $n = 1$. Suppose the claim is true for $n \leq k-1$. Now we consider the following three cases when $n = k$.

Case 1: $0 \leq m \leq 2^{k-1}$

In this case, $U_{m,k}^{(0)} = U_{m,k-1}$, $m = |U_{m,k}| = |U_{m,k}^{(0)}|$, and $U_{m,k}^{(1)}$ is empty. By Lemma 2, we have $\xi(U_{m,k}) = \xi(U_{m,k}^{(0)}) = \xi(U_{m,k-1})$. By induction hypothesis, $\xi(U_{m,k-1}) = f(m)$; this implies $\xi(U_{m,k}) = f(m)$.

Case 2: $2^{k-1} < m \leq 2^k$

In this case, $U_{m,k}^{(0)} = V(AQ_{k-1}^{0})$ and $|U_{m,k}^{(1)}| = m^{'}$ where $m^{'} = m - 2^{k-1}$. Thus for any vertex $u \in U_{m,k}^{(0)}$, there are exactly two vertices in $U_{m,k}^{(1)}$ adjacent to u. This implies $\xi(U_{m,k}^{(0)}, U_{m,k}^{(1)}) = 2|U_{m,k}^{(1)}| = 2m^{'}$.

Since $\{U_{m,k}^{(0)}, U_{m,k}^{(1)}\}$ is a partition of $U_{m,k}$, by Lemma 1, $\xi(U_{m,k}) = \xi(U_{m,k}^{(0)}) + \xi(U_{m,k}^{(1)}) + \xi(U_{m,k}^{(0)}, U_{m,k}^{(1)})$. By the induction hypothesis, we have

$$\xi(U_{m,k}) = \xi(U_{m,k}^{(0)}) + \xi(U_{m,k}^{(1)}) + \xi(U_{m,k}^{(0)}, U_{m,k}^{(1)})$$
$$= f(|U_{m,k}^{(0)}|) + f(|U_{m,k}^{(1)}|) + \xi(U_{m,k}^{(0)}, U_{m,k}^{(1)})$$
$$= f(2^{k-1}) + f(m') + 2m'.$$

Therefore, by Proposition 1, $\xi(U_{m,k}) = f(m)$ because $l(m) = k-1$.

Case 3: $m = 2^k$

In this case, $U_{m,k}$ contain all the vertices in the AQ_k and $\xi(U_{m,k}) = (2k-1)2^{k-1}$. By definition of $f(m)$, we have $f(2^k) = (k-\frac{1}{2})2^k = (2k-1)2^{k-1}$. Hence, $\xi(U_{m,k}) = f(m)$.

From Lemma 3 and Lemma 4, we have $max_{\xi_{AQ_n}}(m) = \xi(U_{m,n}) = f(m)$. Thus Theorem 1 is proved.

V. Application to Bisection Width

The bisection width of graph G, denoted by $bisection(G)$, is the minimum cardinality of an edge cut of G that splits G into two equally-size parts. The arm of this section is to determine the bisection width of AQ_n.

Lemma 5 *For a set U of vertices of n-regular graph G, we have* $\xi(U, V(G)-U) = n\times|U| - 2\xi(U)$.

Theorem 2 *For any integer n, we have* $bisection(AQ_n) = 2^n$

Proof. The proof is obviously true for $n = 1,2$. Suppose $n \geq 3$. For any set U of 2^{n-1} vertices of AQ_n, by Lemma 5 and Theorem 1 that

$$\xi(U, V(AQ_n)-U) = (2n-1)\times 2^{n-1} - 2\xi(U)$$
$$\geq (2n-1)\times 2^{n-1} - 2\times f(2^{n-1})$$
$$= (2n-1)\times 2^{n-1} - 2(2n-3)2^{n-2}$$
$$= 2^n.$$

Thus, $bisection(AQ_n) \geq 2^n$. On the other hand, let $U = V(AQ_{n-1}^0)$. Then $|U| = 2^{n-1}$ and $\xi(U, V(AQ_n)-U) = 2^n$. Therefore, we have $bisection(AQ_n) = 2^{n-1}$.

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