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Identification of Nonlinear Systems Structured by Hammerstein-Wiener Model

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***Abstract*—**Standard Hammerstein-Wiener models consist of a linear subsystem sandwiched by two memoryless nonlinearities. The problem of identifying Hammerstein-Wiener systems is addressed in the presence of linear subsystem of structure totally unknown and polynomial input and output nonlinearities. Presently, the system nonlinearities are allowed to be noninvertible. The system identification problem is dealt by developing a two-stage frequency identification method. First, the parameters of system nonlinearities are identified. In the second stage, a frequency approach is designed to estimate the linear subsystem frequency gain. All involved estimators are proved to be consistent.

***Keywords*—**Nonlinear system identification, Hammerstein systems, Wiener systems, frequency identification.

I. Introduction

HAMMERSTEIN-WIENER systems consist of a series connection including a linear subsystem element sandwiched by two nonlinear blocs (Fig. 1). Clearly, this model structure is a generalization of Hammerstein and Wiener models and so it is expected to feature a superior modeling capability. This has been confirmed by several practical applications e.g. RF power amplifiers [1], submarine detection systems [2], ionospheric dynamics [3].

As a matter of fact, Hammerstein-Wiener systems are more difficult to identify than the simpler Hammerstein and Wiener models [4]. The complexity of the former lies in the fact that these systems involve two internal signals not accessible to measurements, whereas the latter only involve one. Then, it is not surprising that only a few methods are available that deal with Hammerstein-Wiener system identification.

The available methods have been developed following four main approaches i.e. blind methods [4], [5], frequency methods [6], iterative nonlinear optimization procedures [7]-[9], and stochastic methods [10].

Most previous works have focused on Hammerstein-Wiener systems with memoryless input nonlinearity and invertible output nonlinearity (e.g. [7], [9], and [11]).

In this paper, the problem of identifying Hammerstein-Wiener (Fig. 1) systems is addressed, for simplicity, in the continuous-time. Unlike many previous works, the model structure of the linear subsystem is entirely unknown. Furthermore, the system nonlinearities are not required to be invertible. The proposed identification methods also differ by the type of assumptions, made on both the system dynamic and the input signals, and the nature of convergence analysis results.

The major difficulty of the identification problem lies, on one hand, in the fact that the internal signals v and w are not accessible to measurement, on the other hand, in the form of input and output nonlinearities. In the present study, a frequency domain identification approach is designed based on the frequency geometric. First, the identification of system nonlinearities can be achieved by using a set of constant points. Then, a set of points of the linear subsystem frequency gain locus is estimated using a pre- ad post-controller [13], [14]. Doing so, the augmented system, including both the pre- and post-controller, boils down to a linear system with transfer function $G(s)$. Then, the estimation of the linear element can be coped with using available methods. Finally, we note that the signals used are easy to generate.

The outline of the remaining part of this paper consists of 4 sections. The identification problem is formally described in Section II. The identification scheme for input and output nonlinearities will be discussed in Section III and the linear subsystem identification is coped with in Section IV.

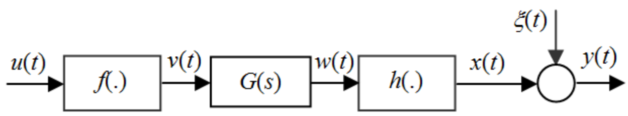

Fig. 1 Mapping Hammerstein-Wiener Model structure

II. Assumption and Problem Formulation

We are interested in systems that can be described by the Hammerstein-Wiener model (Fig. 1) with backlash input nonlinearity; the above model is analytically described by the following equations

$$y(t) = x(t) + \xi(t) = h(w) + \xi(t)\,; \;\; v = f(u) \tag{1a}$$

$$w(t) = g(t) * v(t) \;\text{ with }\; g(t) = L^{-1}\big(G(s)\big) \tag{1b}$$

where * refers to the convolution operation and L to Laplace transform; $g(t)$ denotes the impulse response of linear subsystem. The only measurable signals are the system input $u(t)$ and output $y(t)$. The noise $\xi(t)$ is supposed to be ergodic end it is a zero-mean stationary sequence of independent random variables.

The system is subject to the following assumptions:

A.1 The input nonlinearity $f(.)$ is polynomial; its degree is increased by a known integer n.

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A.2 The linear subsystem $G(s)$ is supposed to be asymptotically stable and $G(0) \neq 0$.

A.3 The output nonlinearity $h(.)$ is polynomial of known degree *m*.

Except for the above assumption *f*(.), $G(s)$ and $h(.)$ are arbitrary. In particular, the static output nonlinearity is of unknown structure outside the subinterval where the identification of system nonlinearities is carried out. The linear subsystem may be continuous- or discrete-time.

We aim at designing an identification scheme that is able to provide: the estimates of the input and output nonlinearities (*f* and *h*); for any frequency ω, provide the estimates of $\left(\angle G(j\omega), |G(j\omega)|\right)$;

Remark 1. The considered identification problem does not have a unique solution: if $(f(u), G(s), h(w))$ represents a solution then, any model of the form $(f(u)/k_1, G(s)/k_2$, $h(k_1 k_2 w))$ is also a solution (where k_1 and k_2 are any nonzero real). This naturally leads to the question: what particular model should we focus. This question will be answered later.

III. Identification of System Nonlinearities

In this section, we seek the estimation of the input and output nonlinearities. First, let $I = [u_m \;\; u_M]$ the working interval within which the identification of system nonlinearities is carried out. Within the interval I, the output nonlinearity *h*(.) is assumed to be polynomial and arbitrary elsewhere. Accordingly, the Hammerstein-Wiener system is successively excited by N constant inputs $\{U_1; \ldots; U_N\}$, where the selected abscissas are arbitrarily chosen in the working interval I and *N* should verify the following condition

$$N > n + m \qquad (2)$$

Then, it follows A1 that, the input nonlinearity is polynomial function, it can be written as follows

$$f(x) = a_n x^n + \ldots + a_1 x + a_0 \qquad (3)$$

Presently, the output nonlinearity takes also the following form (from A3)

$$h(x) = b_m x^m + \ldots + b_1 x + b_0 \qquad (4)$$

On the other hand, let $a = [a_0 \ldots a_n]^T$ and $b = [b_0 \ldots b_m]^T$ be the coefficients vector corresponding to the input and output nonlinearities respectively. Then, the resulted polynomial function $p = hof$ is of degree *nm* and entails the following structure

$$p(x) = h \circ f^*(x) = \alpha_{nm} x^{nm} + \ldots + \alpha_1 x + \alpha_0 \qquad (5)$$

where $f^*(x) = G(0) f(x)$. Let $\theta = [\alpha_0 \ldots \alpha_{nm}]^T$ is the coefficients vector of the composed polynomial function *p*(.).

Presently, to determine a set of points belonging to the polynomial function *p*(.), apply the input sequence

$$u(t) = U_j \text{ for all } t \in [(j-1)LT_r \;\; jLT_r] \text{ with } j = 1 \ldots N \qquad (6)$$

where T_r should be comparable to the system rise time. Then, as the system is asymptotically stable, its step response settles down (i.e. gets very close to final value) after a transient period of LT_r seconds with $L \geq 1$. The number of points N is arbitrary but must satisfied (2). It follows from (1a) and (6) that, the internal signal *v*(*t*) takes N constant values and turns out to be

$$v(t) = f(U_j) = V_j \text{ for all } t \in [(j-1)LT_r \;\; jLT_r] \qquad (7)$$

where $j = 1 \ldots N$. Accordingly, for constant excitations U_j applied within the interval $[(j-1)LT_r \;\; jLT_r]$ ($j = 1 \ldots N$), as the linear subsystem $G(s)$ is asymptotically stable, it follows that the steady-state of the internal signal *w*(*t*) is constant. Then, it is readily obtained from (1a)-(7), the undisturbed output *x*(*t*) takes N constant values, in the steady state, that can be expressed as follows

$$X_j = p(U_j) = h \circ f^*(U_j) \text{ for } j = 1 \ldots N \qquad (8)$$

Finally, notice that the steady-state undisturbed output X_j $(j = 1 \ldots N)$ can simply be estimated using the fact that $y(t) = x(t) + \xi(t)$. The above results suggest the following estimator for X_j

$$\hat{X}_j(L) = \frac{1}{LT_r} \int_{(j-1)LT_r}^{jLT_r} y(t)\, dt \text{ for } j = 1 \ldots N \qquad (9)$$

where T_r should be comparable to the system rise time and $L \geq 1$. Since *N* is of high integer, usually higher than *nm+1*, the optimal estimate $\hat{\theta}(L, N) = [\hat{\alpha}_0(L,N) \ldots \hat{\alpha}_{nm}(L,N)]^T$ of θ is calculated based on the least-squares estimate.

TABLE I
NONLINEARITIES IDENTIFICATION

Stage 1	Apply the piecewise signal defined by (6). Record the system output $y(t)$ for $\left[0 \;\; NLT_r\right]$.
Stage 2	Compute the filtered values $\hat{X}_j(L)$ for $j = 1\ldots N$ using the estimator (9).
Stage 3	* Calculate the least-squares estimates $\hat{\theta}(L,N) = \left[\hat{\alpha}_0(L,N) \ldots \hat{\alpha}_{nm}(L,N)\right]^T = \hat{\bar{\theta}}(L,N)\hat{\alpha}_{nm}(L,N)$ using the set of points $\left\{\left(U_j, \hat{X}_j(L)\right);\; j = 1\ldots N\right\}$ * Deduce the estimate $\hat{\bar{\theta}}(L,N) = \left[\hat{\bar{\alpha}}_0(L,N) \ldots \hat{\bar{\alpha}}_{nm}(L,N)\right]^T$
Stage 4	Set $q_0 = x^n$ and $\hat{\bar{a}}_n(L,N) = 1$ Take $\beta_1 = 0$ denoting the coefficient of x^{nm-1} in $(q_0)^m = x^{nm}$. Then $\hat{\bar{a}}_{n-1}(L,N) = \dfrac{\hat{\alpha}_{nm-1}(L,N) - \beta_1}{m} = \dfrac{\hat{\alpha}_{nm-1}(L,N)}{m}$ Do the following steps for $k = 2\ldots n-1$ * Let $q_{k-1} = q_{k-2} + \hat{\bar{a}}_{n-k}(L,N)x^{n-k+1}$. * Compute $(q_{k-1})^m$. * Deduce β_k and determine $\hat{\bar{a}}_{n-k}(L,N) = \dfrac{\hat{\alpha}_{nm-k}(L,N) - \beta_k}{m}$
Stage 5	The coefficients vector corresponding to $\tilde{h}$ can be estimated by solving the system $A\,\tilde{b}(L,N) = c$, where $A(i+1, j+1)$ $(0 \le i, j \le m)$ is the coefficient of x^{in} in $\left(\hat{\bar{f}}_{L,N}(x)\right)^j = \left(q_n\right)^j = \left(q_{n-1}\right)^j$ and $c = \left[\hat{\bar{\alpha}}_0(L,N), \hat{\bar{\alpha}}_n(L,N), \ldots \hat{\bar{\alpha}}_{nm}(L,N)\right]^T$

Proposition 1. (see [12]).

1) Let take the polynomial $p = h \circ f^*$. The elements f^* and h can be obtained by using the algorithm of Table I.

2) The couple of components (f^*, h) is not unique, e.g.

$$x^n o x^m = x^m o x^n \text{ and } h \circ f^* = h(x+b) \circ (f^* - b).$$

Then, if the model $(f(.), G(s), h(.))$ is solution of the above identification problem whatever, then it is readily checked that, any model $(\bar{f}(.), \bar{G}(s), \bar{h}(.))$ defined as follows is also solution

$$\bar{f}(x) = k_1 f^*(x) - k_1 k_0 \tag{10a}$$

$$\bar{G}(s) = \frac{G(s)}{k_1 k_2} \tag{10b}$$

$$\bar{h}(x) = h\left(k_2\,(x + k_0)\right) \tag{10c}$$

whatever the real triplet (k_0, k_1, k_2) with k_1 and k_2 are nonzero real scalar.

To solve this problem, it will prove judicious to focus on the model $(\bar{f}(.), \bar{G}(s), \bar{h}(.))$ characterized by the following properties:

First, consider the polynomial function that entails the following structure

$$\bar{p}(x) = \frac{1}{\alpha_{nm}} p(x) = \tilde{h} \circ \bar{f}(x) = x^{nm} + \ldots + \bar{\alpha}_1 x + \bar{\alpha}_0 \tag{11}$$

where $\tilde{h}(x) = \bar{h}(x)/\alpha_{nm}$ and $\bar{\alpha}_k = \alpha_k / \alpha_{nm}$ ($k = 0\ldots nm$).

Let $\bar{\theta} = \left[\bar{\alpha}_0 \ldots \bar{\alpha}_{nm}\right]^T$ is the coefficients vector of the polynomial function $\bar{p}(.)$.

Currently, we aim to obtain the elements of the polynomial function $\bar{p} = \tilde{h} \circ \bar{f}$, where $\bar{f}(0) = 0$. Let $\bar{a} = \left[\bar{a}_0 \ldots \bar{a}_n\right]^T$ be the coefficients vector corresponding to $\bar{f}(.)$. It is readily checked from (10a)-(11) that

$$\bar{a}_k = a_k / a_n \quad \text{for } k = 1\ldots n \tag{12a}$$

$$\bar{a}_0 = 0 \quad \text{and} \quad \bar{a}_n = 1 \tag{12b}$$

Let $\tilde{b} = \left[\tilde{b}_0 \ldots \tilde{b}_m\right]^T$ be the coefficients vector corresponding to $\tilde{h}(.)$. On the other hand, let us define the following function

$$q_k = x^n + \bar{a}_{n-1}x^{n-1} + \ldots + \bar{a}_{n-k}x^{n-k} \quad \text{for } k = 0\ldots n-1 \tag{13}$$

Knowing that $\bar{a}_n = 1$, it is easy to check that

$$q_0 = x^n \quad \text{and} \quad q_n = q_{n-1} = \bar{f}(x) \tag{14a}$$

and for $k = 1 \ldots n-1$

$$q_k = q_{k-1} + \bar{a}_{n-k}\, x^{n-k} \tag{14b}$$

Then, the $(k+1)^{th}$ coefficient of $(q_k)^m$ is the coefficient of x^{nm-k}. Let β_k denote the coefficient of x^{nm-k} in $(q_{k-1})^m$. Also, it is readily obtained [12]

$$\bar{\alpha}_{nm-k} = m\,\bar{a}_{n-k} + \beta_k \quad \text{for} \quad k = 1 \ldots n-1 \tag{15}$$

These ideas are formalized in the estimator of Table I.

IV. Linear Subsystem Identification

The problem of identifying the linear subsystem is dealt with in this section. The proposed solution is designed in two steps. First, an adequate controller is introduced. Many previous studies focused on compensating for input nonlinearity (e.g. [13], [14]). The obtained system representation is further transformed to cope with the unavailability of the internal signals $v(t)$ and $w(t)$.

At this point, the input and output nonlinearities ($f(.)$ and $h(.)$) are known. From the nonlinearity identification procedure (Table I), one gets estimates of input and output nonlinearities. For simplicity, we presently suppose that the estimated points have been exactly determined.

On the other hand, knowing that the input nonlinearity $f(.)$ is polynomial function, let $f^{-1}(.)$ designates its inverse. To get profit from this result, a controller (Fig. 2) will be introduced at the input of system (see e.g. [13]-[14]). Furthermore, using the fact that the output nonlinearity $h(.)$ is polynomial function, there exists a non-zero interval such that $h(.)$ is invertible and let $h^{-1}(.)$ designates its inverse. The second key idea is to introduce the inverse $h^{-1}(.)$ of $h(.)$ at the system output (Fig. 2).

Theoretically, the resulting system is equivalent to a linear subsystem with transfer function $G(s)$. In this context, an approach is designed based on the frequency method. The resulting system is submitted to a given sine input

$$u(t) = U\cos(\omega t) \tag{16}$$

for any frequency $\omega > 0$, the amplitude V is judiciously chosen Then, the problem of identifying $G(s)$ becomes a trivial issue.

Fig. 2 The system to be identified augmented with pre- and post-adaptive controller

V. Conclusion

We have developed a new frequency identification method to deal with continuous-time Hammerstein-Wiener systems, the problem is addressed in presence of polynomials input and output nonlinearities.

Accordingly, the nonlinear parts are determined first using the algorithm of Table I. The linear subsystem identification is coped with by using an adequate controller designed so that the resulting system becomes equivalent to a linear subsystem

The originality of the present study lies in the fact that the system is not necessarily parametric and of structure totally unknown. It is interesting to point that the estimation of linear subsystem and nonlinearities are performed separately.

Another feature of the method is the fact that the exciting signals are easily generated and the estimation algorithms can be simply implemented, compared with several published approaches. Finally, we note that the choice of interest frequency band is not required.

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